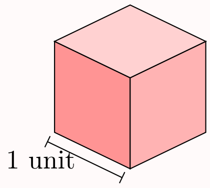
VOLUME

A DEFINITION

Definition Volume

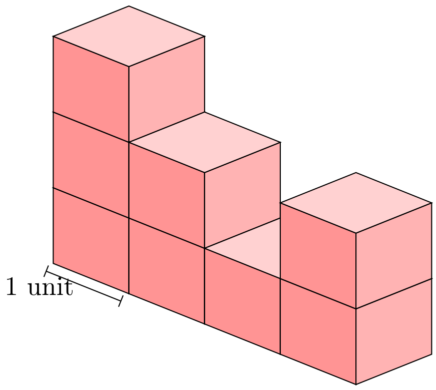
The **volume** of an object is the amount of space it takes up. We measure volume by counting how many **cubic units** can fit inside it. A cubic unit is a cube with sides that are 1 unit long.



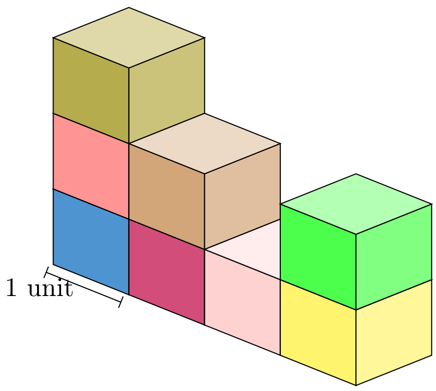
Method Counting Cubes to Find Volume

To find the volume of a shape made of blocks, simply count the total number of blocks (cubic units) it is made from. A good strategy is to count the blocks in each layer.

Ex: Find the volume of the shape below.



Answer: We can find the volume by counting the cubes in the shape. Each small cube has a volume of 1 cubic unit.



There are 8 cubes in total, so:

$$\text{Volume} = 8 \text{ cubic units}$$

B UNITS OF VOLUME

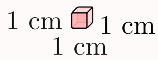
Definition Units of Volume

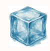
- **Cubic Millimeter** (mm^3): the volume of a cube with sides 1 m long. This is about the size of a tiny grain of sand.

$$1 \text{ mm}^3 = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm}$$

- **Cubic Centimeter** (cm^3): The volume of a cube with sides 1 cm long. This is about the size of an ice cube.

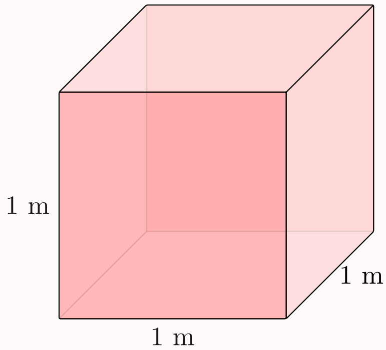
$$1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$$

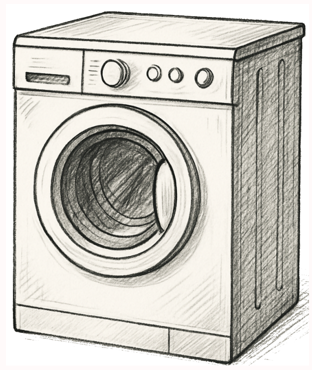




- **Cubic Meter** (m^3): the volume of a cube with sides 1 m long. This is about the volume of a washing machine.

$$1 \text{ m}^3 = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$$





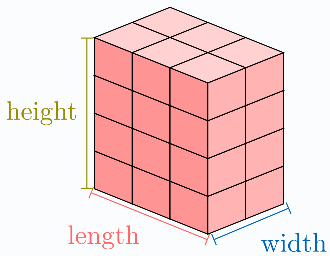
C VOLUME OF A RECTANGULAR CUBOID

Proposition Volume of a Rectangular Cuboid

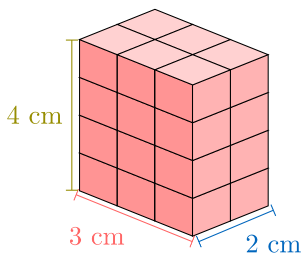
The volume of a rectangular cuboid (also called a rectangular prism) is found by multiplying its length, width, and height:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$V = l \times w \times h$$



Ex: Find the volume of this rectangular cuboid.



Answer: Using the formula for the volume of a rectangular cuboid:

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 3 \times 2 \times 4 \\ &= 24 \, \text{cm}^3\end{aligned}$$

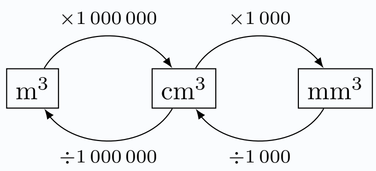
D CONVERSION OF VOLUME UNITS

Proposition Conversion of Volume Units

- $1\,\text{cm}^3 = (10 \times 10 \times 10)\,\text{mm}^3 = \mathbf{1\,000}\,\text{mm}^3$
- $1\,\text{m}^3 = (100 \times 100 \times 100)\,\text{cm}^3 = \mathbf{1\,000\,000}\,\text{cm}^3$

Method Converting Using Multiplication or Division

- Use **multiplication** to go from a larger unit to a smaller one (like cubic meters to cubic centimeters).
- Use **division** to go from a smaller unit to a larger one (like cubic centimeters to cubic meters).



Method Converting Using a Table

For volume, each unit in the place value table is split into **three columns**. Let's convert **10.5** m^3 to cm^3.

1. **Draw the volume conversion table.** Each unit has three columns.

		m^3						cm^3			mm^3

2. **Place the number in the table.** The rule is: the digit in the **ones place** goes into the **right-hand column** of the starting unit. For $10.5\,\text{m}^3$, the ones digit is **0**, so it goes in the right-hand column of m^3.

		m^3						cm^3			mm^3
	1	**0**	**5**								

3. **Move the decimal point** to the right side of your target unit's columns. Our target is cm^3. Fill any empty columns with zeros.

		m^3						cm^3			mm^3
	1	0	5	**0**	**0**	**0**	**0**	**0.**			

4. **Read the final number.**

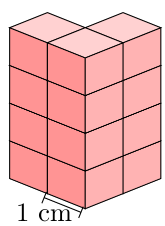
$$\text{So, } 10.5\,\text{m}^3 = 10\,500\,000\,\text{cm}^3.$$

E VOLUMES OF SOLIDS WITH UNIFORM CROSS-SECTION

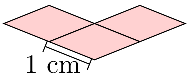
Proposition Volume of Solid of Uniform Cross-Section

$$\text{Volume} = \text{area of base (cross-section)} \times \text{height}$$

Ex: Find the volume of the figure.



Answer:

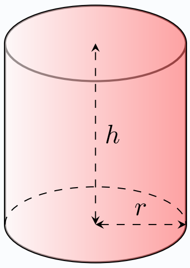
- Area of 

$= 3\,\text{cm}^2$
- height $= 4\,\text{cm}$
-

$$\begin{aligned} V &= \text{area of base} \times \text{height} \\ &= 3\,\text{cm}^2 \times 4\,\text{cm} \\ &= 12\,\text{cm}^3 \end{aligned}$$

Proposition Volume of a Cylinder

$$V = \pi r^2 h$$

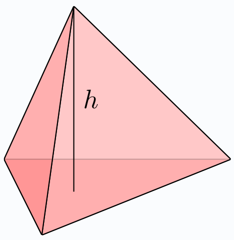


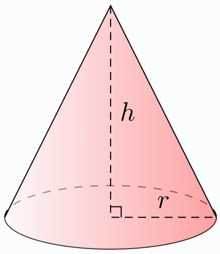
F VOLUMES OF TAPERED SOLIDS AND SPHERES

Proposition Volume of a Pyramid or Cone

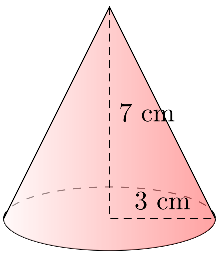
The volume of any tapered solid (a pyramid or a cone) that comes to a point (apex) is one-third of the area of its base multiplied by its perpendicular height.

$$V = \frac{1}{3} \times A_{\text{base}} \times h$$





Ex: Find the volume of the cone.



Answer:

1. **Find the area of the circular base:**

$$\begin{aligned} A_{\text{base}} &= \pi r^2 \\ &= \pi(3)^2 \\ &= 9\pi\,\text{cm}^2 \end{aligned}$$

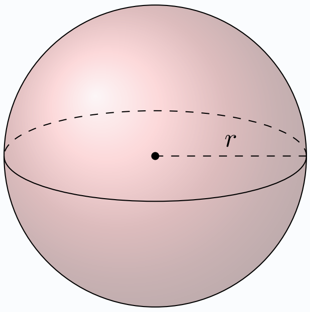
2. **Calculate the volume of the cone:**

$$\begin{aligned} V &= \frac{1}{3} \times A_{\text{base}} \times h \\ &= \frac{1}{3} \times 9\pi \times 7 \\ &= 21\pi\,\text{cm}^3 \\ &\approx 65.97\,\text{cm}^3 \end{aligned}$$

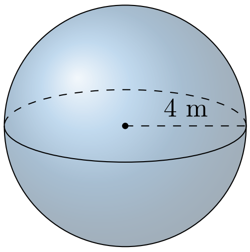
Proposition Volume of a Sphere

The volume of a sphere with radius r is given by the formula:

$$V = \frac{4}{3}\pi r^3$$



Ex: Find the volume of the sphere.



Answer:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi (4)^3 \\ &= \frac{4}{3}\pi (64) \\ &= \frac{256}{3}\pi \text{ m}^3 \\ &\approx 268.08 \text{ m}^3 \end{aligned}$$

G CAPACITY

Definition Liter

A **liter** is a unit we use to measure the volume (capacity) of liquids.

- 1 **liter** is the volume of a cube that measures 10 cm on each side.

$$1\,\text{L} = 1\,000\,\text{cm}^3 \quad \text{and} \quad 1\,000\,\text{L} = 1\,\text{m}^3$$

- We write it with the symbol **L** (a capital "L").
- A smaller unit, the **centiliter** (cL), is often used for smaller volumes: $1\,\text{L} = 100\,\text{cL}$.
- An even smaller unit, the **milliliter** (mL), is used for very small volumes:

$$1\,\text{L} = 1\,000\,\text{mL} \quad \text{and} \quad 1\,\text{cL} = 10\,\text{mL} \quad \text{and} \quad 1\,\text{mL} = 1\,\text{cm}^3$$

Ex:

- A big water bottle holds about 1 L of water, which is 100 cL or 1 000 mL:

- A small soda can holds about 0.33 L, which is 33 cL (about 330 mL): 

H DENSITY

Definition Density

The **density** ρ of a substance is its mass per unit volume:

$$\rho = \frac{m}{V}.$$

Common units: kg/m^3 or g/cm^3.

Proposition Rearranging the Density Formula

$$m = \rho V \qquad \text{and} \qquad V = \frac{m}{\rho}.$$

Method Unit Conversions

$$1\ \text{g}/\text{cm}^3 = 1000\ \text{kg}/\text{m}^3 \qquad \text{and} \qquad 1\ \text{kg}/\text{m}^3 = 0.001\ \text{g}/\text{cm}^3.$$

Ex: An aluminium block measures 3 cm $\times$ 2 cm $\times$ 4 cm. Aluminium has density $\rho = 2.7\ \text{g}/\text{cm}^3$. Find its mass.

Answer: Volume $V = 3 \cdot 2 \cdot 4 = 24\ \text{cm}^3$. Then

$$m = \rho V = 2.7 \times 24 = 64.8\ \text{g}.$$

I SURFACE AREA

Definition Surface Area

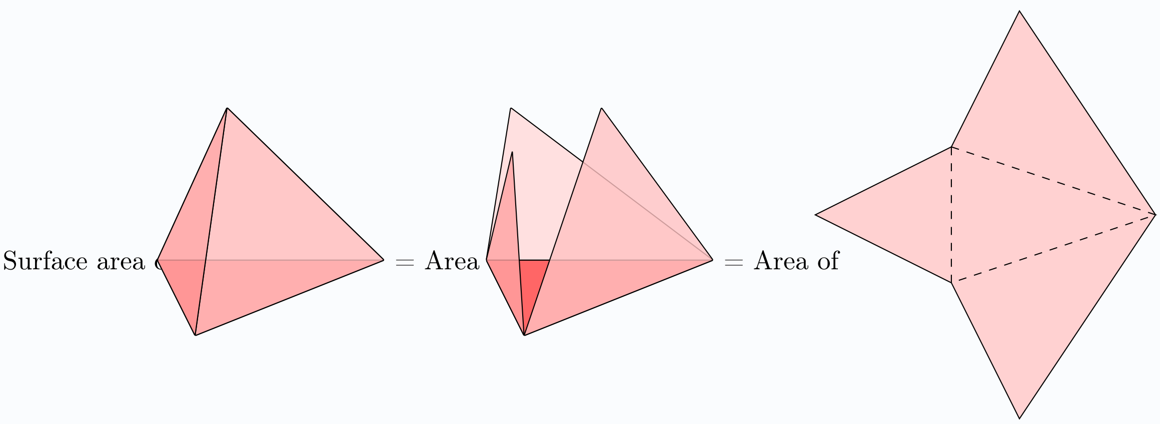
The **surface area** of a 3D shape is the total area of all its faces. It is measured in square units (cm^2, m^2, ...).

Proposition Surface area of Polyhedra

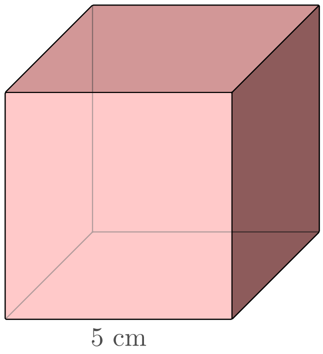
The surface area of a three-dimensional figure with plane faces is the sum of the areas of the faces.

Method Collapse the 3D figure into its net

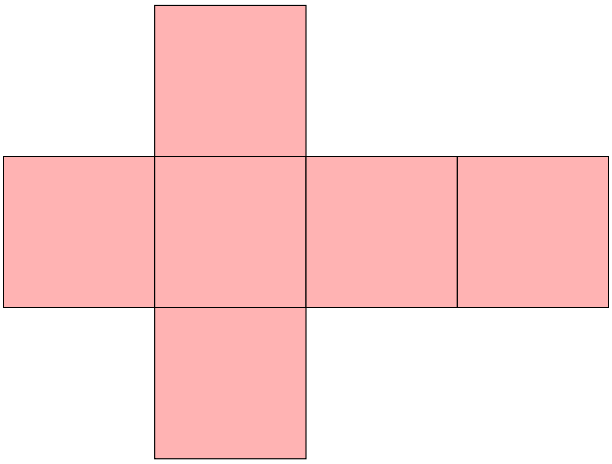
If we collapse the figure into its net, the surface area is the area of the net.



Ex: Find the surface area of the cube:

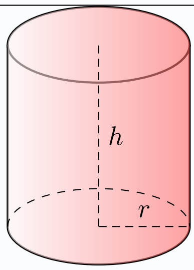
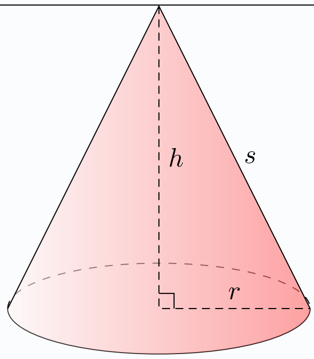
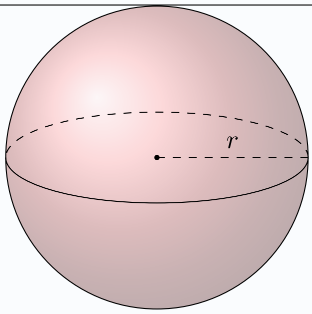


Answer: A cube has 6 identical square faces. The area of one face is the side length squared (s^2).

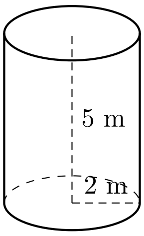


$$\begin{aligned}\text{Surface Area} &= 6 \times (\text{Area of one square}) \\ &= 6 \times s^2 \\ &= 6 \times 5^2 \\ &= 6 \times 25 \\ &= 150\,\text{cm}^2\end{aligned}$$

Proposition **Surface area of solids with curved surfaces**

Name	Solid	Surface area
Cylinder		$A =$ curved surface $+ 2$ circular ends $= 2\pi rh + 2\pi r^2$
Cone		$A =$ curved surface $+$ circular base $= \pi rs + \pi r^2$
Sphere		$A = 4\pi r^2$

Ex: Find the surface area of the cylinder:



Answer:

$$\begin{aligned} A &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(2)(5) + 2\pi(2)^2 \\ &= 20\pi + 8\pi \\ &= 28\pi \,\text{m}^2 \\ &\approx 87.96\,\text{m}^2 \end{aligned}$$