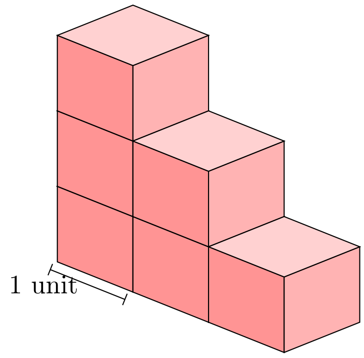
VOLUME

A DEFINITION

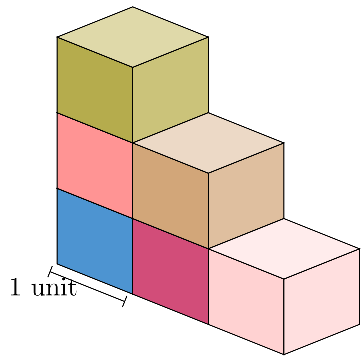
A.1 FINDING VOLUME OF A SHAPE

Ex 1: What is the volume of the red solid?



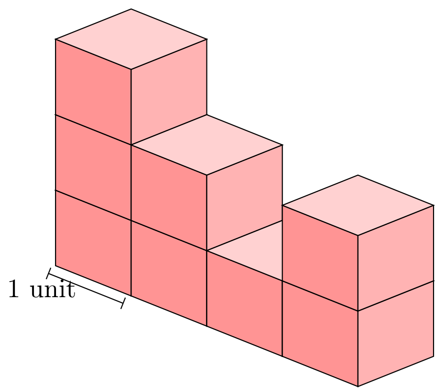
$V =$ 6 cubic units

Answer: To find the volume, we count the number of unit cubes inside the shape.



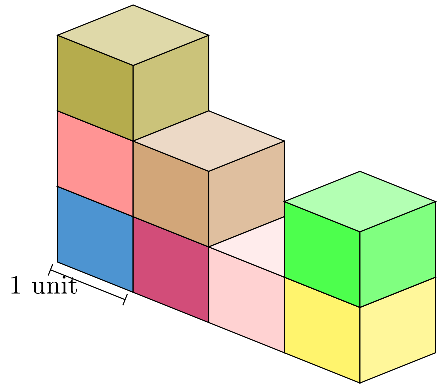
The volume is 6 cubic units.

Ex 2: What is the volume of the red solid?



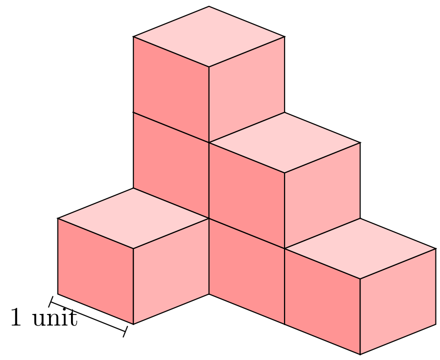
$V =$ 8 cubic units

Answer: To find the volume, we count the number of unit cubes inside the shape.



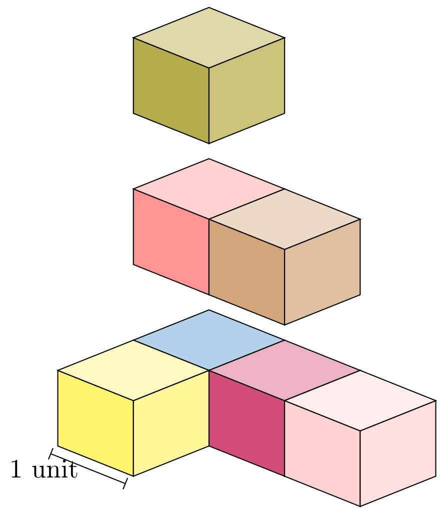
The volume is 8 cubic units.

Ex 3: What is the volume of the red solid?



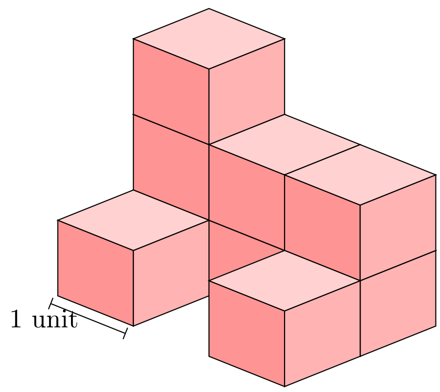
$V =$ 7 cubic units

Answer: To find the volume, we count the number of unit cubes inside the shape.



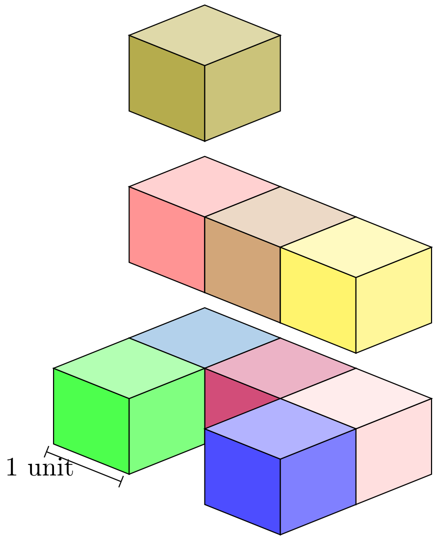
The volume is 7 cubic units.

Ex 4: What is the volume of the red solid?



$V = \boxed{9}$ cubic units

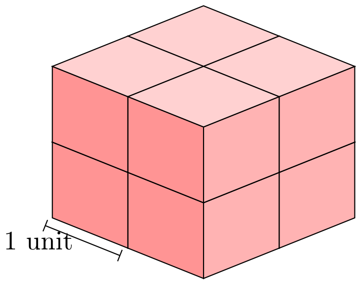
Answer: To find the volume, we count the number of unit cubes in each layer and then we add them together.



The volume is 9 cubic units.

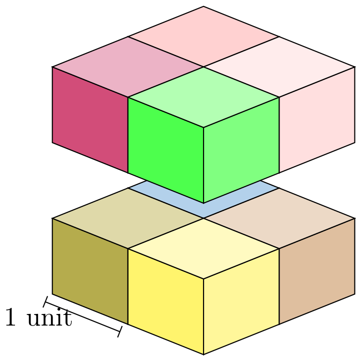
A.2 FINDING VOLUME OF A RECTANGULAR CUBOID

Ex 5: What is the volume of the red solid?



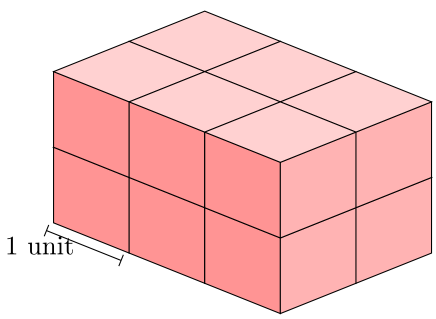
$V = \boxed{8}$ cubic units

Answer: To find the volume, we count the number of unit cubes in each layer and then we add them together.



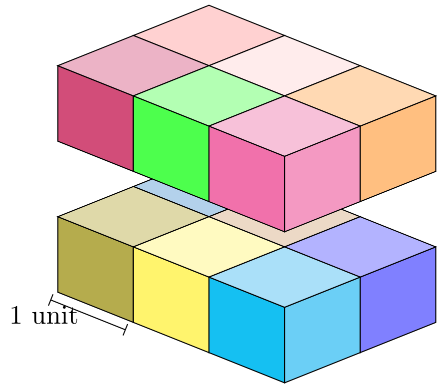
The volume is 8 cubic units.

Ex 6: What is the volume of the red solid?



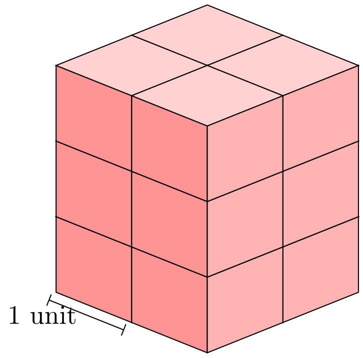
$V = \boxed{12}$ cubic units

Answer: To find the volume, we count the number of unit cubes in each layer and then we add them together.



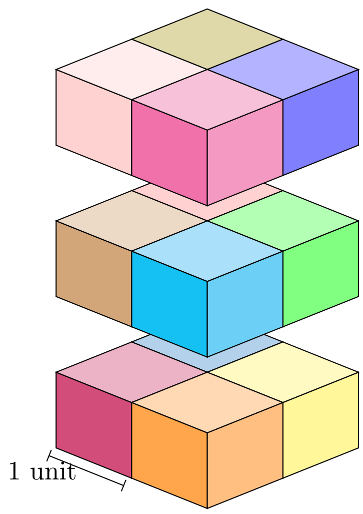
The volume is 12 cubic units.

Ex 7: What is the volume of the red solid?



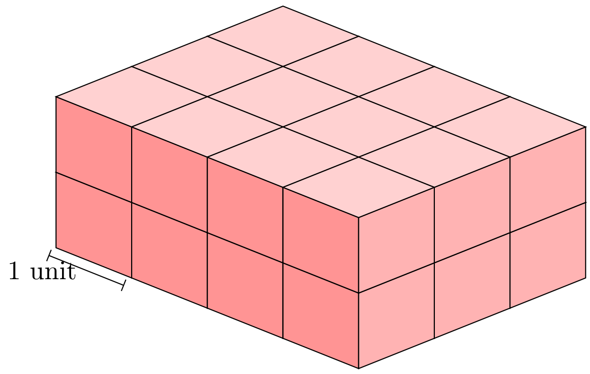
$V = \boxed{12}$ cubic units

Answer: To find the volume, we count the number of unit cubes in each layer and then we add them together.



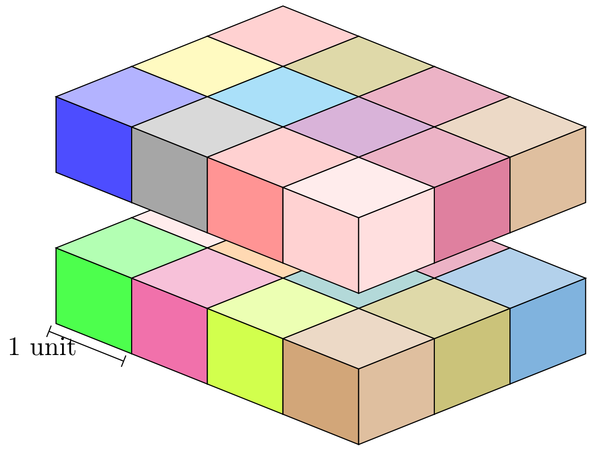
The volume is 12 cubic units.

Ex 8: What is the volume of the red solid?



$V = \boxed{24}$ cubic units

Answer: To find the volume, we count the number of unit cubes in each layer and then we add them together.



The volume is 24 cubic units.

B UNITS OF VOLUME

B.1 CHOOSING UNITS FOR VOLUME

MCQ 9: What unit will be used to measure the volume of your bedroom?
Choose 1 answer:

- ☐ Cubic millimeters
- ☐ Cubic centimeters
- ☒ Cubic meters

Answer: Cubic meters will be used to measure the volume of your bedroom because it's a larger unit, perfect for measuring bigger spaces like a room. Cubic millimeters and cubic centimeters are too small for such a large space.

MCQ 10: What unit will be used to measure the volume of a small toy block?
Choose 1 answer:

- ☐ Cubic millimeters
- ☒ Cubic centimeters
- ☐ Cubic meters

Answer: Cubic centimeters will be used to measure the volume of a small toy block because it's a smaller unit, perfect for measuring small objects like a toy block. Cubic millimeters are too tiny, and cubic meters are too large for such a small object.

MCQ 11: What unit will be used to measure the volume of a grain of rice?
Choose 1 answer:

- ☒ Cubic millimeters
- ☐ Cubic centimeters
- ☐ Cubic meters

Answer: Cubic millimeters will be used to measure the volume of a grain of rice because it's a very small unit, perfect for measuring tiny objects like a grain of rice. Cubic centimeters are too large, and cubic meters are much too big for such a small object.

MCQ 12: What unit will be used to measure the volume of a bottle of milk?
Choose 1 answer:

- ☐ Cubic millimeters
- ☒ Cubic centimeters
- ☐ Cubic meters

Answer: Cubic centimeters will be used to measure the volume of a bottle of milk because it's a smaller unit, perfect for measuring small objects like a bottle of milk. Cubic millimeters are too tiny, and cubic meters are too large for such a small object.

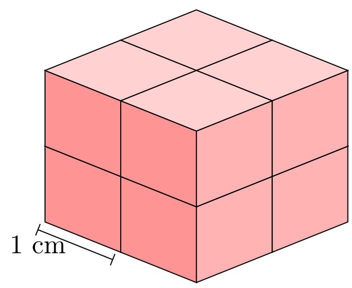
MCQ 13: What unit will be used to measure the volume of a swimming pool?
Choose 1 answer:

- ☐ Cubic millimeters
- ☐ Cubic centimeters
- ☒ Cubic meters

Answer: Cubic meters will be used to measure the volume of a swimming pool because it's a larger unit, perfect for measuring bigger spaces like a swimming pool. Cubic millimeters and cubic centimeters are too small for such a large space.

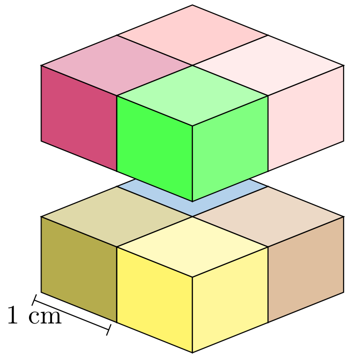
B.2 FINDING VOLUME OF A RECTANGULAR CUBOID

Ex 14: What is the volume of the red figure?



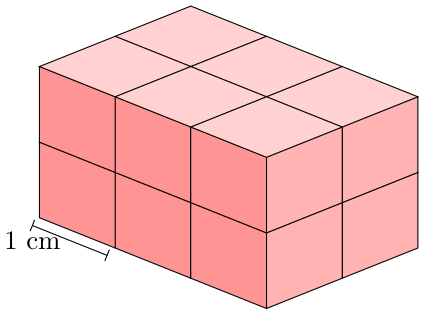
$V = \boxed{8}\ \boxed{\mathbf{cm^3}}$

Answer: To find the volume, we count the number of cubes inside the shape. Each cube is 1 cm by 1 cm by 1 cm, so each cube is 1 cm^3.



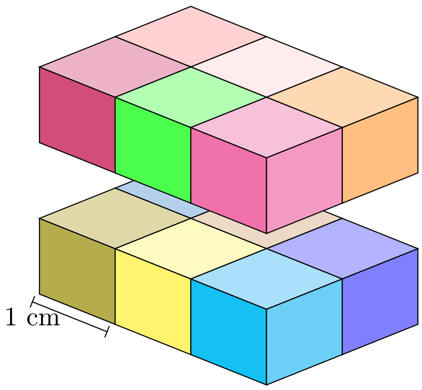
The volume is 4+4=8 cm^3.

Ex 15: What is the volume of the red figure?



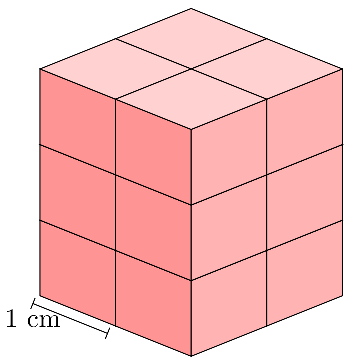
$V =$ 12 **cm**^{3}

Answer: To find the volume, we count the number of cubes inside the shape. Each cube is 1 cm by 1 cm by 1 cm, so each cube is 1 cm^3.



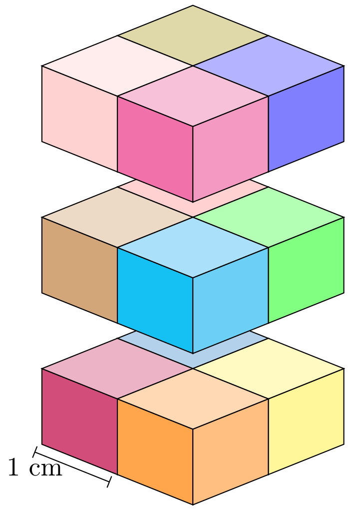
The volume is 6+6=12 cm^3.

Ex 16: What is the volume of the red figure?



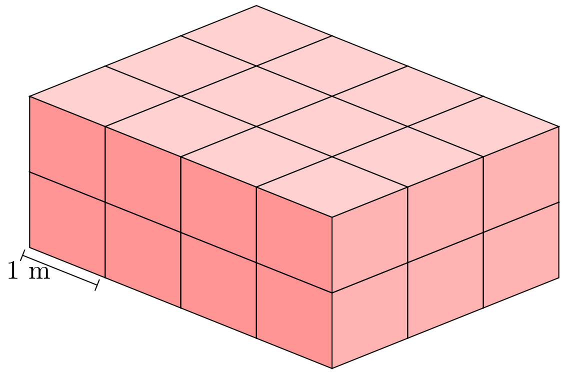
$V =$ 12 **cm**^{3}

Answer: To find the volume, we count the number of cubes inside the shape. Each cube is 1 cm by 1 cm by 1 cm, so each cube is 1 cm^3.



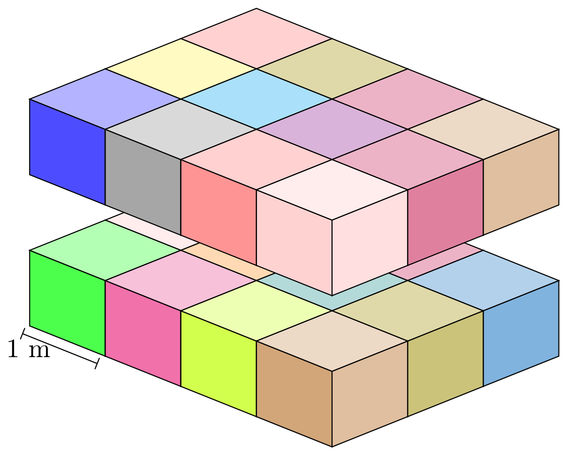
The volume is 4+4+4=12 cm^3.

Ex 17: What is the volume of the red figure?



$V =$ 24 **m**^{3}

Answer: To find the volume, we count the number of cubes inside the shape. Each cube is 1 m by 1 m by 1 m, so each cube is 1 m^3.

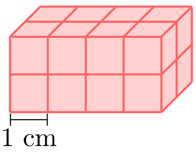


The volume is 12+12=24 m^3.

C VOLUME OF A RECTANGULAR CUBOID

C.1 FINDING VOLUMES OF A RECTANGULAR CUBOIDS

Ex 18: What is the volume of the red figure?

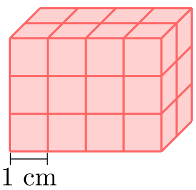


$V = \boxed{16}\ \text{cm}^3$

Answer: Length = 4 cm, width = 2 cm and height = 2 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 4\,\text{cm} \times 2\,\text{cm} \times 2\,\text{cm} \\ &= 16\,\text{cm}^3 \end{aligned}$$

Ex 19: What is the volume of the red figure?

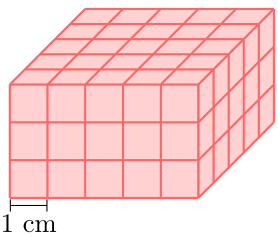


$V = \boxed{24}\ \text{cm}^3$

Answer: Length = 4 cm, width = 3 cm and height = 2 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 4\,\text{cm} \times 3\,\text{cm} \times 2\,\text{cm} \\ &= 24\,\text{cm}^3 \end{aligned}$$

Ex 20: What is the volume of the red figure?

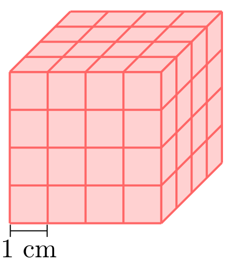


$V = \boxed{75}\ \text{cm}^3$

Answer: Length = 5 cm, width = 3 cm and height = 5 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 5\,\text{cm} \times 3\,\text{cm} \times 5\,\text{cm} \\ &= 75\,\text{cm}^3 \end{aligned}$$

Ex 21: What is the volume of the red figure?

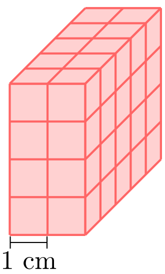


$V = \boxed{64}\ \text{cm}^3$

Answer: Length = 4 cm, width = 4 cm and height = 4 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 4\,\text{cm} \times 4\,\text{cm} \times 4\,\text{cm} \\ &= 64\,\text{cm}^3 \end{aligned}$$

Ex 22: What is the volume of the red figure?

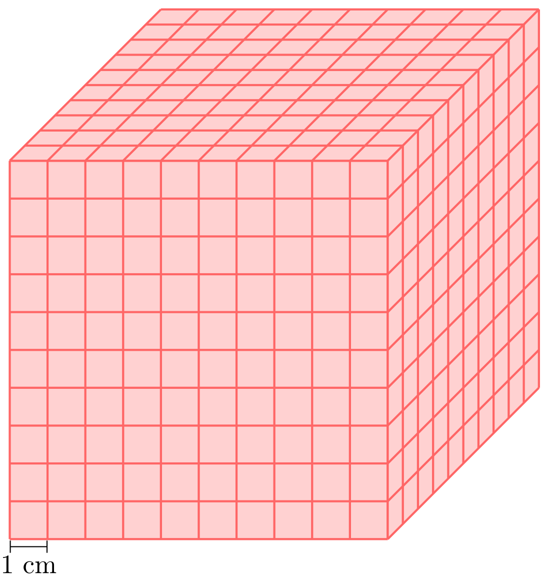


$V = \boxed{40}\ \text{cm}^3$

Answer: Length = 2 cm, width = 4 cm and height = 5 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 2\,\text{cm} \times 4\,\text{cm} \times 5\,\text{cm} \\ &= 40\,\text{cm}^3 \end{aligned}$$

Ex 23: What is the volume of the red figure?



$V = \boxed{1000}\ \text{cm}^3$

Answer: Length = 10 cm, width = 10 cm and height = 10 cm.

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 10\,\text{cm} \times 10\,\text{cm} \times 10\,\text{cm} \\ &= 1000\,\text{cm}^3 \end{aligned}$$

C.2 SOLVING PROBLEMS

Ex 24: A rectangular swimming pool is 8 m long, 5 m wide, and 2 m deep. The water costs 10 dollars per cubic meter.
What is the volume of the swimming pool?

$$V = \boxed{80}\ \mathrm{m}^3$$

What is the cost to fill the swimming pool with water?

$$\boxed{800}\ \text{dollars}$$

Answer:

- The volume of the rectangular swimming pool is:

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 8\,\mathrm{m} \times 5\,\mathrm{m} \times 2\,\mathrm{m} \\ &= 80\,\mathrm{m}^3 \end{aligned}$$

- The cost to fill the swimming pool with water is calculated by:

$$\begin{aligned} \text{Cost} &= \text{Volume} \times \text{cost per m}^3 \\ &= 80\,\mathrm{m}^3 \times 10\,\text{dollars per m}^3 \\ &= 800\,\text{dollars} \end{aligned}$$

Ex 25: A container has a volume of $20\,\mathrm{m}^3$. A box is 2 m long, 1 m wide, and 0.5 m high.
What is the volume of the box?

$$V = \boxed{1}\ \mathrm{m}^3$$

How many boxes can fit inside the container?

$$\boxed{20}\ \text{boxes}$$

Answer:

- The volume of the box is:

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 2\,\mathrm{m} \times 1\,\mathrm{m} \times 0.5\,\mathrm{m} \\ &= 1\,\mathrm{m}^3 \end{aligned}$$

- The number of boxes that can fit inside the container is calculated by:

$$\begin{aligned} \text{Number of boxes} &= \text{Volume of container} \div \text{Volume of one box} \\ &= 20\,\mathrm{m}^3 \div 1\,\mathrm{m}^3 \\ &= 20\,\text{boxes} \end{aligned}$$

Ex 26: A storage room has a volume of $150\,\mathrm{m}^3$. A water tank is 5 m long, 2 m wide, and 3 m high.
What is the volume of the water tank?

$$V = \boxed{30}\ \mathrm{m}^3$$

How many water tanks can fit inside the storage room?

$$\boxed{5}\ \text{water tanks}$$

Answer:

- The volume of the water tank is:

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 5\,\mathrm{m} \times 2\,\mathrm{m} \times 3\,\mathrm{m} \\ &= 30\,\mathrm{m}^3 \end{aligned}$$

- The number of water tanks that can fit inside the storage room is calculated by:

$$\begin{aligned} \text{Number of water tanks} &= \text{Volume of room} \div \text{Volume of one tan} \\ &= 150\,\mathrm{m}^3 \div 30\,\mathrm{m}^3 \\ &= 5\,\text{water tanks} \end{aligned}$$

Ex 27: A rectangular fish tank is 2 m long, 1 m wide, and 1 m deep. The water costs 15 dollars per cubic meter.
What is the volume of the fish tank?

$$V = \boxed{2}\ \mathrm{m}^3$$

What is the cost to fill the fish tank with water?

$$\boxed{30}\ \text{dollars}$$

Answer:

- The volume of the rectangular fish tank is:

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 2\,\mathrm{m} \times 1\,\mathrm{m} \times 1\,\mathrm{m} \\ &= 2\,\mathrm{m}^3 \end{aligned}$$

- The cost to fill the fish tank with water is calculated by:

$$\begin{aligned} \text{Cost} &= \text{Volume} \times \text{cost per m}^3 \\ &= 2\,\mathrm{m}^3 \times 15\,\text{dollars per m}^3 \\ &= 30\,\text{dollars} \end{aligned}$$

D CONVERSION OF VOLUME UNITS

D.1 CONVERTING VOLUME UNITS

Ex 28: Convert:

$$3\,\mathrm{cm}^3 = \boxed{3000}\ \mathrm{mm}^3.$$

Answer:

- *Multiplication Method:*

$$\begin{aligned} 3\,\mathrm{cm}^3 &= 3 \times 1\,000\,\mathrm{mm}^3 \quad (1\,\mathrm{cm}^3 = 1\,000\,\mathrm{mm}^3) \\ &= 3\,000\,\mathrm{mm}^3 \end{aligned}$$

- *Conversion Table Method:*

		m^3						cm^3			mm^3
								3	0	0	0

So,

$$3\,\mathrm{cm}^3 = 3\,000\,\mathrm{mm}^3$$

Ex 29: Convert:

$$12\,000\,\text{mm}^3 = \boxed{12}\ \text{cm}^3.$$

Answer:

- *Division Method:*

$$\begin{aligned}12\,000\,\text{mm}^3 &= 12\,000 \div 1000\,\text{cm}^3 \quad (1000\,\text{mm}^3 = 1\,\text{cm}^3)\\ &= 12\,\text{cm}^3\end{aligned}$$

- *Conversion Table Method:*

m^3						cm^3			mm^3		
							1	2	0	0	0

So,

$$12\,000\,\text{mm}^3 = 12\,\text{cm}^3$$

Ex 30: Convert:

$$4\,\text{m}^3 = \boxed{4000000}\ \text{cm}^3.$$

Answer:

- *Multiplication Method:*

$$\begin{aligned}4\,\text{m}^3 &= 4 \times 1\,000\,000\,\text{cm}^3 \quad (1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3)\\ &= 4\,000\,000\,\text{cm}^3\end{aligned}$$

- *Conversion Table Method:*

m^3						cm^3			mm^3		
		4	0	0	0	0	0	0			

So,

$$4\,\text{m}^3 = 4\,000\,000\,\text{cm}^3$$

Ex 31: Convert:

$$15\,000\,000\,\text{cm}^3 = \boxed{15}\ \text{m}^3.$$

Answer:

- *Division Method:*

$$\begin{aligned}15\,000\,000\,\text{cm}^3 &= 15\,000\,000 \div 1\,000\,000\,\text{m}^3\ (1\,000\,000\,\text{cm}^3 = 1\,\text{m}^3)\\ &= 15\,\text{m}^3\end{aligned}$$

- *Conversion Table Method:*

m^3						cm^3			mm^3		
	1	5	0	0	0	0	0	0			

So,

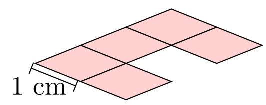
$$15\,000\,000\,\text{cm}^3 = 15\,\text{m}^3$$

E VOLUMES OF SOLIDS WITH UNIFORM CROSS-SECTION

E.1 CALCULATING VOLUMES STEP-BY-STEP

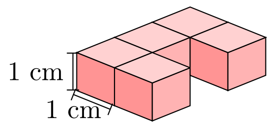
Ex 32:

1. Calculate the area of this figure :



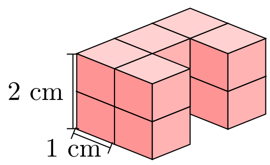
$$\text{Area of base} = \boxed{5}\ \text{cm}^2$$

2. Calculate the volume of this solid:



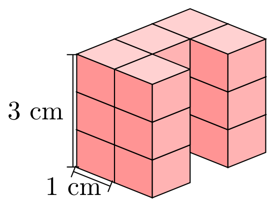
$$\text{Volume} = \boxed{5}\ \text{cm}^3$$

3. Calculate the volume of this solid:



$$\text{Volume} = \boxed{10}\ \text{cm}^3$$

4. Calculate the volume of this solid:



$$\text{Volume} = \boxed{15}\ \text{cm}^3$$

Answer:

1. **Area of the Base:**

$$\text{Area of base} = 5\,\text{cm}^2$$

2. **Volume with Height of 1 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 5\,\text{cm}^2 \times 1\,\text{cm}\\ &= 5\,\text{cm}^3\end{aligned}$$

3. **Volume with Height of 2 cm:**

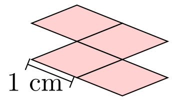
$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 5\,\text{cm}^2 \times 2\,\text{cm}\\ &= 10\,\text{cm}^3\end{aligned}$$

4. **Volume with Height of 3 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 5\,\text{cm}^2 \times 3\,\text{cm} \\ &= 15\,\text{cm}^3\end{aligned}$$

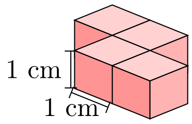
Ex 33:

1. Calculate the area of this figure:



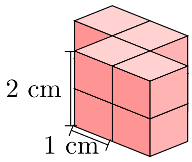
Area of base $= \boxed{4}\ \text{cm}^2$

2. Calculate the volume of this solid:



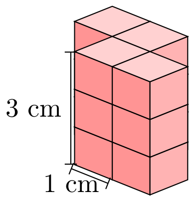
Volume $= \boxed{4}\ \text{cm}^3$

3. Calculate the volume of this solid:



Volume $= \boxed{8}\ \text{cm}^3$

4. Calculate the volume of this solid:



Volume $= \boxed{12}\ \text{cm}^3$

Answer:

1. **Area of the Base:**

$$\text{Area of base} = 4\,\text{cm}^2$$

2. **Volume with Height of 1 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 4\,\text{cm}^2 \times 1\,\text{cm} \\ &= 4\,\text{cm}^3\end{aligned}$$

3. **Volume with Height of 2 cm:**

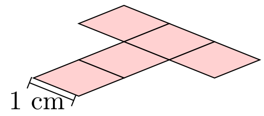
$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 4\,\text{cm}^2 \times 2\,\text{cm} \\ &= 8\,\text{cm}^3\end{aligned}$$

4. **Volume with Height of 3 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 4\,\text{cm}^2 \times 3\,\text{cm} \\ &= 12\,\text{cm}^3\end{aligned}$$

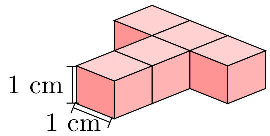
Ex 34:

1. Calculate the area of this figure:



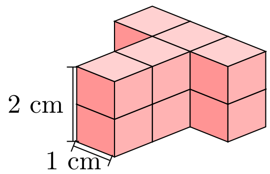
Area of base $= \boxed{5}\ \text{cm}^2$

2. Calculate the volume of this solid:



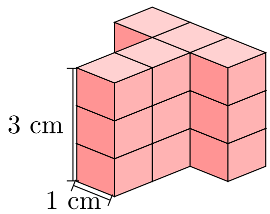
Volume $= \boxed{5}\ \text{cm}^3$

3. Calculate the volume of this solid:



Volume $= \boxed{10}\ \text{cm}^3$

4. Calculate the volume of this solid:



Volume $= \boxed{15}\ \text{cm}^3$

Answer:

1. **Area of the Base:**

$$\text{Area of base} = 5\,\text{cm}^2$$

2. **Volume with Height of 1 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height} \\ &= 5\,\text{cm}^2 \times 1\,\text{cm} \\ &= 5\,\text{cm}^3\end{aligned}$$

3. **Volume with Height of 2 cm:**

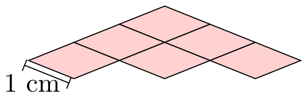
$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 5\,\text{cm}^2 \times 2\,\text{cm}\\ &= 10\,\text{cm}^3\end{aligned}$$

4. **Volume with Height of 3 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 5\,\text{cm}^2 \times 3\,\text{cm}\\ &= 15\,\text{cm}^3\end{aligned}$$

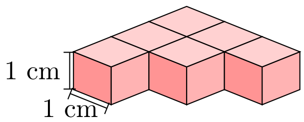
Ex 35:

1. Calculate the area of this figure:



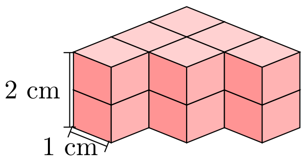
Area of base = [6] cm^2

2. Calculate the volume of this solid:



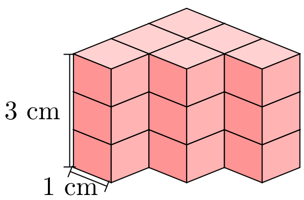
Volume = [6] cm^3

3. Calculate the volume of this solid:



Volume = [12] cm^3

4. Calculate the volume of this solid:



Volume = [18] cm^3

Answer:

1. **Area of the Base:**

$$\text{Area of base} = 6\,\text{cm}^2$$

2. **Volume with Height of 1 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 6\,\text{cm}^2 \times 1\,\text{cm}\\ &= 6\,\text{cm}^3\end{aligned}$$

3. **Volume with Height of 2 cm:**

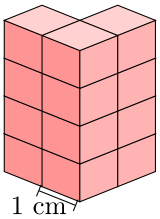
$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 6\,\text{cm}^2 \times 2\,\text{cm}\\ &= 12\,\text{cm}^3\end{aligned}$$

4. **Volume with Height of 3 cm:**

$$\begin{aligned}\text{Volume} &= \text{Area of base} \times \text{height}\\ &= 6\,\text{cm}^2 \times 3\,\text{cm}\\ &= 18\,\text{cm}^3\end{aligned}$$

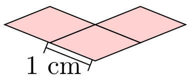
E.2 CALCULATING VOLUMES OF SOLIDS MADE OF CUBES

Ex 36: Find the volume of the solid:

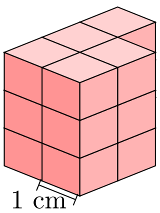


$V =$ [12] cm^3

Answer:

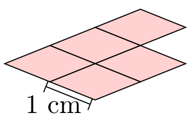
- Area of  = 3 cm^2
- height = 4 cm
- V = area of end × height
  $= 3\ \text{cm}^2 \times 4\ \text{cm}$
  $= 12\ \text{cm}^3$

Ex 37: Find the volume of the solid:

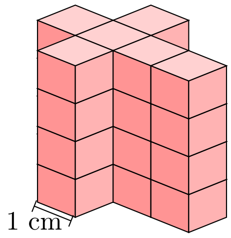


$V =$ [15] cm^3

Answer:

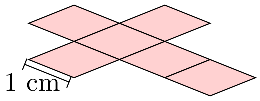
- Area of  = 5 cm^2
- height = 3 cm
- V = area of base × height
  $= 5\ \text{cm}^2 \times 3\ \text{cm}$
  $= 15\ \text{cm}^3$

Ex 38: Find the volume of the solid:



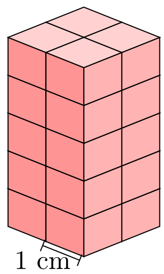
$V = \boxed{24}\ \text{cm}^3$

Answer:

- Area of 

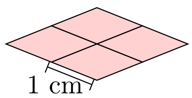
$= 6\ \text{cm}^2$
- height $= 4$ cm
- $V =$ area of base $\times$ height
  $= 6\ \text{cm}^2 \times 4\ \text{cm}$
  $= 24\ \text{cm}^3$

Ex 39: Find the volume of the solid:



$V = \boxed{20}\ \text{cm}^3$

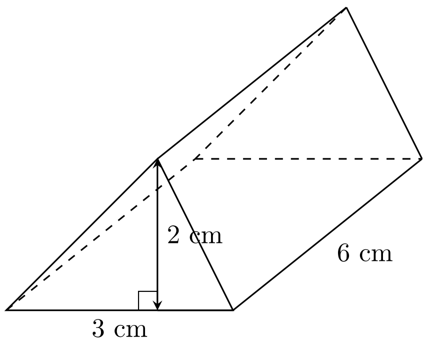
Answer:

- Area of 

$= 4\ \text{cm}^2$
- height $= 5$ cm
- $V =$ area of base $\times$ height
  $= 4\ \text{cm}^2 \times 5\ \text{cm}$
  $= 20\ \text{cm}^3$

E.3 FINDING VOLUMES OF SOLIDS WITH UNIFORM CROSS-SECTION

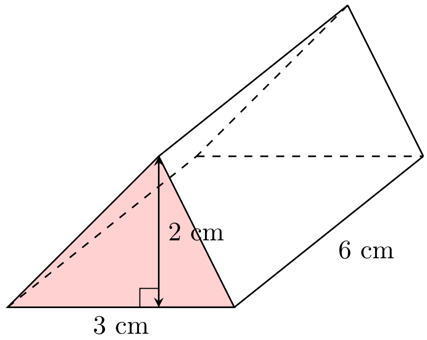
Ex 40: Find the volume of the solid:



$V = \boxed{18}\ \text{cm}^3$

Answer:

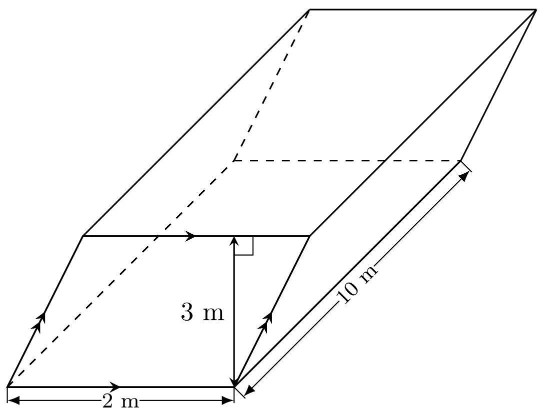
- The solid is a prism with a uniform cross-section. The end is a triangle.



- $$\begin{aligned}\text{Area of base} &= \text{Area of triangle}\\ &= \frac{b \times h}{2}\\ &= \frac{3 \times 2}{2}\\ &= 3\,\text{cm}^2\end{aligned}$$

- $$\begin{aligned}\text{Volume of prism} &= \text{Area of base} \times \text{height}\\ &= 3\,\text{cm}^2 \times 6\,\text{cm}\\ &= 18\,\text{cm}^3\end{aligned}$$

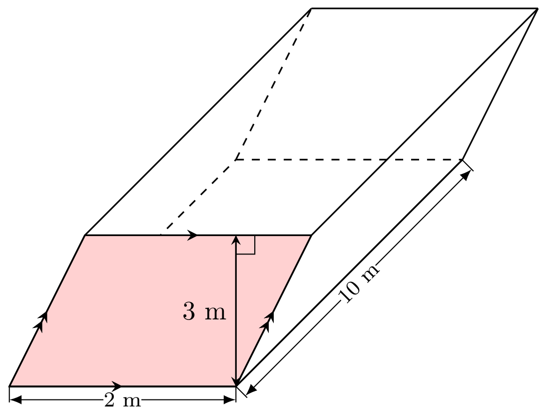
Ex 41: Find the volume of the solid:



$V = \boxed{60}\ \text{m}^3$

Answer:

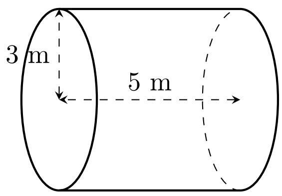
- The solid is a prism with a uniform cross-section. The end is a parallelogram.



- $$\begin{aligned} \text{Area of base} &= \text{Area of parallelogram} \\ &= b \times h \\ &= 2\,\mathrm{m} \times 3\,\mathrm{m} \\ &= 6\,\mathrm{m}^2 \end{aligned}$$

- $$\begin{aligned} \text{Volume of prism} &= \text{Area of base} \times \text{height} \\ &= 6\,\mathrm{m}^2 \times 10\,\mathrm{m} \\ &= 60\,\mathrm{m}^3 \end{aligned}$$

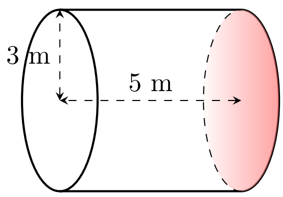
Ex 42: Find the volume of the solid (round to 1 decimal place):



$V = \boxed{141.4}\,\mathrm{m}^3$

Answer:

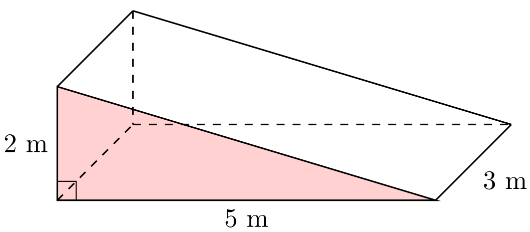
- The solid is a cylinder with a uniform cross-section. The end is a circle.



- $$\begin{aligned} \text{Area of base} &= \text{Area of circle} \\ &= \pi r^2 \\ &= \pi \times (3)^2 \\ &= 9\pi\,\mathrm{m}^2 \\ &\approx 28.2743\,\mathrm{m}^2 \end{aligned}$$

- $$\begin{aligned} \text{Volume of cylinder} &= \text{Area of base} \times \text{height} \\ &= 9\pi\,\mathrm{m}^2 \times 5\,\mathrm{m} \\ &= 45\pi\,\mathrm{m}^3 \\ &\approx 141.3717\,\mathrm{m}^3 \\ &\approx 141.4\,\mathrm{m}^3 \text{ (rounded to 1 decimal place)} \end{aligned}$$

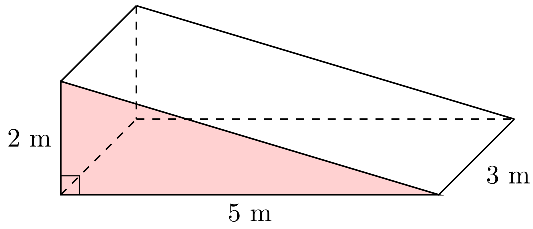
Ex 43: Find the volume of the solid:



$V = \boxed{15}\,\mathrm{m}^3$

Answer:

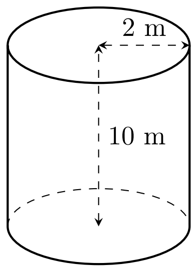
- The solid is a prism with a uniform cross-section. The end is a right-angled triangle.



- $$\begin{aligned} \text{Area of base} &= \text{Area of triangle} \\ &= \frac{b \times h}{2} \\ &= \frac{5 \times 2}{2} \\ &= 5\,\mathrm{m}^2 \end{aligned}$$

- $$\begin{aligned} \text{Volume of prism} &= \text{Area of base} \times \text{height} \\ &= 5\,\mathrm{m}^2 \times 3\,\mathrm{m} \\ &= 15\,\mathrm{m}^3 \end{aligned}$$

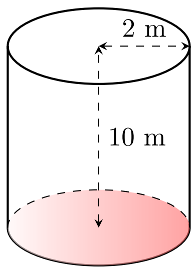
Ex 44: Find the volume of the solid (round to 1 decimal place):



$V = \boxed{125.7}\,\mathrm{m}^3$

Answer:

- The solid is a cylinder with a uniform cross-section. The end is a circle.

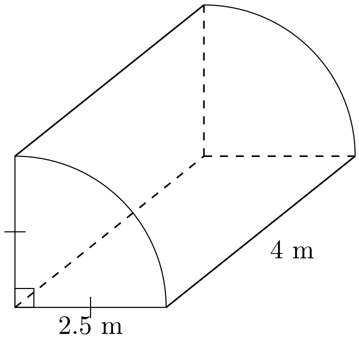


- $$\begin{aligned} \text{Area of base} &= \text{Area of circle} \\ &= \pi r^2 \\ &= \pi \times (2)^2 \\ &= 4\pi\,\mathrm{m}^2 \\ &\approx 12.5664\,\mathrm{m}^2 \end{aligned}$$

-

$$\begin{aligned}\text{Volume of cylinder} &= \text{Area of base} \times \text{height}\\ &= 4\pi\,\text{m}^2 \times 10\,\text{m}\\ &= 40\pi\,\text{m}^3\\ &\approx 125.6637\,\text{m}^3\\ &\approx 125.7\,\text{m}^3 \text{ (rounded to 1 decimal place)}\end{aligned}$$

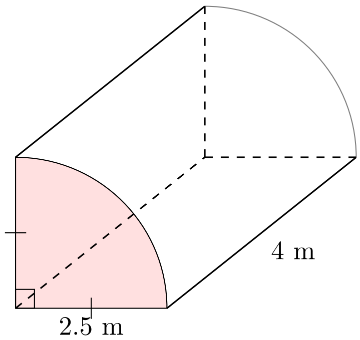
Ex 45: Find the volume of the solid (round to 1 decimal place):



$V =$ 19.6 m^3

Answer:

- The solid has a uniform cross-section. The end is a quarter-circle.



-

$$\begin{aligned}\text{Area of base} &= \text{Area of quarter-circle}\\ &= \frac{1}{4} \times \pi r^2\\ &= \frac{1}{4} \times \pi \times (2.5)^2\\ &= \frac{1}{4} \times \pi \times 6.25\\ &\approx 4.9087\,\text{m}^2\end{aligned}$$

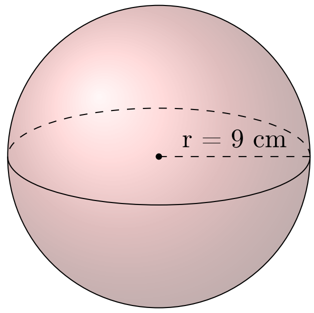
-

$$\begin{aligned}\text{Volume of prism} &= \text{Area of base} \times \text{height}\\ &= 4.9087\,\text{m}^2 \times 4\,\text{m}\\ &\approx 19.635\,\text{m}^3\\ &\approx 19.6\,\text{m}^3 \text{ (rounded to 1 decimal place)}\end{aligned}$$

F VOLUMES OF TAPERED SOLIDS AND SPHERES

F.1 CALCULATING VOLUMES OF TAPERED SOLIDS AND SPHERES: LEVEL 1

Ex 46: Find the volume of the sphere. (Leave your answer in terms of π)



$V =$ 972π cm^3

Answer:

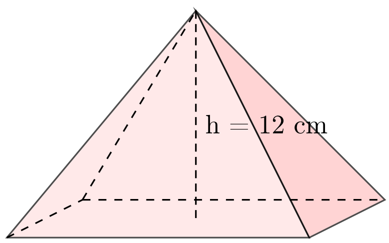
1. **Identify the radius (r):**

$$r = 9\,\text{cm}$$

2. **Calculate the volume of the sphere:**

$$\begin{aligned}V &= \frac{4}{3}\pi r^3\\ &= \frac{4}{3}\pi(9)^3\\ &= \frac{4}{3}\pi(729)\\ &= 972\pi\,\text{cm}^3\end{aligned}$$

Ex 47: Find the volume of the square-based pyramid.



$V =$ 400 cm^3

Answer:

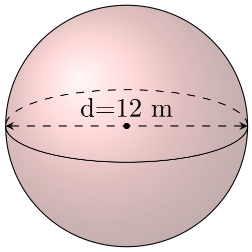
1. **Find the area of the square base:**

$$A_{\text{base}} = \text{side}^2 = (10)^2 = 100\,\text{cm}^2$$

2. **Calculate the volume of the pyramid:**

$$V = \frac{1}{3} \times A_{\text{base}} \times h = \frac{1}{3} \times 100 \times 12 = 400\,\text{cm}^3$$

Ex 48: Find the volume of the sphere. (Round to two decimal places)



$V \approx$ 904.78 m^3

Answer:

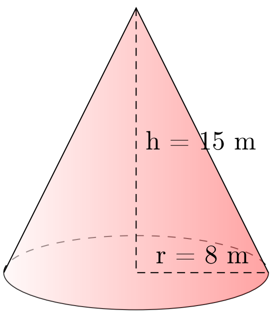
1. **Find the radius (r) from the diameter (d):**

$$r = \frac{d}{2} = \frac{12}{2} = 6\,\text{m}$$

2. **Calculate the volume of the sphere:**

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = 288\pi\,\text{m}^3 \approx 904.78\,\text{m}^3$$

Ex 49: Find the volume of the cone. (Round to two decimal places)



$V \approx$ 1005.31 m^3

Answer:

1. **Identify the area of the base and the height:**

$$A_{\text{base}} = \pi r^2 = \pi(8)^2 = 64\pi\,\text{m}^2$$

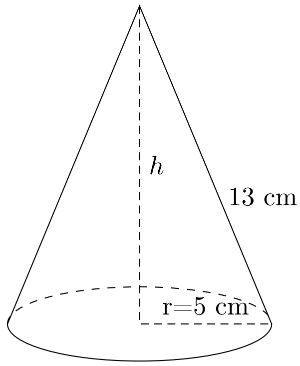
$$h = 15\,\text{m}$$

2. **Calculate the volume of the cone:**

$$V = \frac{1}{3} \times A_{\text{base}} \times h = \frac{1}{3} \times 64\pi \times 15 = 320\pi\,\text{m}^3 \approx 1005.31\,\text{m}^3$$

F.2 CALCULATING VOLUMES OF TAPERED SOLIDS AND SPHERES: LEVEL 2

Ex 50: Find the volume of a cone with a slant height of 13 cm and a radius of 5 cm. (Leave your answer in terms of π)



Answer:

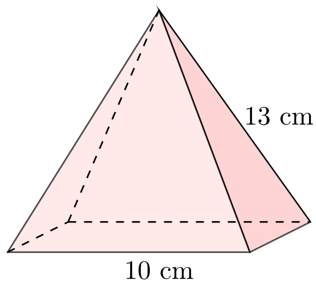
1. **Find the perpendicular height (h) using the Pythagorean theorem:** The radius (r), height (h), and slant height (s) form a right-angled triangle.

$$r^2 + h^2 = s^2$$
$$5^2 + h^2 = 13^2$$
$$25 + h^2 = 169$$
$$h^2 = 144$$
$$h = 12\,\text{cm}$$

2. **Calculate the volume of the cone:**

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5)^2(12) = 100\pi\,\text{cm}^3$$

Ex 51: A square-based pyramid has a base side length of 10 cm and a slant height (the height of each triangular face) of 13 cm. Calculate the volume of the pyramid.



Answer:

1. **Find the perpendicular height (h) of the pyramid:** The slant height (13 cm), the perpendicular height (h), and half the base length (5 cm) form a

right-angled triangle.

$$\begin{aligned}(\text{half base})^2 + h^2 &= (\text{slant height})^2\\ 5^2 + h^2 &= 13^2\\ 25 + h^2 &= 169\\ h^2 &= 144\\ h &= 12\,\text{cm}\end{aligned}$$

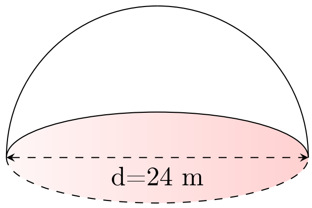
2. **Find the area of the base:**

$$A_{\text{base}} = \text{side}^2 = 10^2 = 100\,\text{cm}^2$$

3. **Calculate the volume of the pyramid:**

$$V = \frac{1}{3} \times A_{\text{base}} \times h = \frac{1}{3} \times 100 \times 12 = 400\,\text{cm}^3$$

Ex 52: Find the volume of the hemisphere with a diameter of 24 m. (Round to one decimal place)



Answer:

1. **Find the radius (r):**

$$\begin{aligned}r &= \frac{\text{diameter}}{2}\\ &= \frac{24}{2}\\ &= 12\,\text{m}\end{aligned}$$

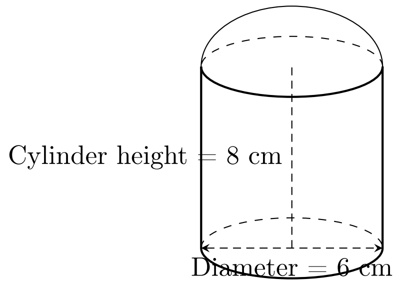
2. **Calculate the volume of a full sphere:**

$$\begin{aligned}V_{\text{sphere}} &= \frac{4}{3}\pi r^3\\ &= \frac{4}{3}\pi(12)^3\\ &= \frac{4}{3}\pi(1728)\\ &= 2304\pi\,\text{m}^3\end{aligned}$$

3. **Calculate the volume of the hemisphere:**

$$\begin{aligned}V_{\text{hemisphere}} &= \frac{1}{2} \times V_{\text{sphere}}\\ &= \frac{1}{2} \times 2304\pi\\ &= 1152\pi\,\text{m}^3\\ &\approx 3619.1\,\text{m}^3\end{aligned}$$

Ex 53: Find the volume of the composite solid, which consists of a cylinder and a hemisphere. (Round to one decimal place)



Answer:

1. **Find the radius (r):**

$$\begin{aligned}r &= \frac{\text{diameter}}{2}\\ &= \frac{6}{2}\\ &= 3\,\text{cm}\end{aligned}$$

2. **Calculate the volume of the cylinder part:**

$$\begin{aligned}V_{\text{cylinder}} &= \pi r^2 h\\ &= \pi(3)^2(8)\\ &= 72\pi\,\text{cm}^3\end{aligned}$$

3. **Calculate the volume of the hemisphere part:**

$$\begin{aligned}V_{\text{hemisphere}} &= \frac{1}{2} \times \left(\frac{4}{3}\pi r^3\right)\\ &= \frac{2}{3}\pi(3)^3\\ &= \frac{2}{3}\pi(27)\\ &= 18\pi\,\text{cm}^3\end{aligned}$$

4. **Add the volumes to find the total volume:**

$$\begin{aligned}V_{\text{total}} &= V_{\text{cylinder}} + V_{\text{hemisphere}}\\ &= 72\pi + 18\pi\\ &= 90\pi\,\text{cm}^3\\ &\approx 282.7\,\text{cm}^3\end{aligned}$$

G CAPACITY

G.1 CHOOSING UNITS FOR CAPACITY

MCQ 54: What unit best measures the capacity of a bathtub?
Choose 1 answer:

☐ 220 mL

☐ 2 200 mL

☒ 220 L

Answer: 220 L best measures the capacity of a bathtub because it's a larger unit, suitable for a big container like a bathtub. 220 mL and 2 200 mL are too small for such a large volume.

MCQ 55: What unit best measures the capacity of a dosage of medicine?
Choose 1 answer:

☒ 5 mL

☐ 0.5 L

☐ 5 L

Answer: 5 mL best measures the capacity of a dosage of medicine because it's a small unit, perfect for tiny amounts like a medicine dose. 0.5 L and 5 L are too large for such a small volume.

MCQ 56: What unit best measures the capacity of a wine glass?
Choose 1 answer:

☐ 150 L

☒ 15 cL

☐ 1.5 L

Answer: 15 cL best measures the capacity of a wine glass because it's a small unit, suitable for a small container like a wine glass. 150 L is much too large, and 1.5 L is also too big for such a small volume.

MCQ 57: What unit best measures the capacity of a soup bowl?
Choose 1 answer:

☒ 40 cL

☐ 40 mL

☐ 40 L

Answer: 40 cL best measures the capacity of a soup bowl because it's a suitable unit for a small container like a bowl. 40 mL is too small, and 40 L is too large for a typical soup bowl.

MCQ 58: What unit best measures the capacity of a car's fuel tank?
Choose 1 answer:

☐ 60 mL

☒ 60 L

☐ 600 L

Answer: 60 L best measures the capacity of a car's fuel tank because it's a larger unit, suitable for a big container like a fuel tank. 60 mL is much too small, and 600 L is too large for a typical car's fuel tank.

MCQ 59: What unit best measures the capacity of a pitcher?
Choose 1 answer:

☐ 2.5 mL

☒ 2.5 L

☐ 25 L

Answer: 2.5 L best measures the capacity of a pitcher because it's a suitable unit for a medium-sized container like a pitcher. 2.5 mL is too small, and 25 L is too large for a typical pitcher.

G.2 CONVERTING CAPACITY UNITS

Ex 60: Convert:

$$3\,\text{L} = \boxed{300}\,\text{cL}.$$

Answer:

$$\begin{aligned} 3\,\text{L} &= 3 \times 100\,\text{cL} \quad (1\,\text{L} = 100\,\text{cL}) \\ &= 300\,\text{cL} \end{aligned}$$

Ex 61: Convert:

$$1.5\,\text{L} = \boxed{150}\,\text{cL}.$$

Answer:

$$\begin{aligned} 1.5\,\text{L} &= 1.5 \times 100\,\text{cL} \quad (1\,\text{L} = 100\,\text{cL}) \\ &= 150\,\text{cL} \end{aligned}$$

Ex 62: Convert:

$$20\,\text{cL} = \boxed{0.2}\,\text{L}.$$

Answer:

$$\begin{aligned} 20\,\text{cL} &= 20 \div 100\,\text{L} \quad (100\,\text{cL} = 1\,\text{L}) \\ &= 0.2\,\text{L} \end{aligned}$$

Ex 63: Convert:

$$250\,\text{cL} = \boxed{2.5}\,\text{L}.$$

Answer:

$$\begin{aligned} 250\,\text{cL} &= 250 \div 100\,\text{L} \quad (100\,\text{cL} = 1\,\text{L}) \\ &= 2.5\,\text{L} \end{aligned}$$

Ex 64: Convert:

$$2\,\text{L} = \boxed{2000}\,\text{mL}.$$

Answer:

$$\begin{aligned} 2\,\text{L} &= 2 \times 1\,000\,\text{mL} \quad (1\,\text{L} = 1\,000\,\text{mL}) \\ &= 2\,000\,\text{mL} \end{aligned}$$

Ex 65: Convert:

$$30\,\text{mL} = \boxed{3}\,\text{cL}.$$

Answer:

$$\begin{aligned} 30\,\text{mL} &= 30 \div 10\,\text{cL} \quad (10\,\text{mL} = 1\,\text{cL}) \\ &= 3\,\text{cL} \end{aligned}$$

G.3 CONVERTING BETWEEN METRIC VOLUME AND CAPACITY UNITS

Ex 66: Convert:

$$5\,\text{m}^3 = \boxed{5000}\,\text{L}.$$

Answer:

$$\begin{aligned} 5\,\text{m}^3 &= 5 \times 1\,000\,\text{L} \quad (1\,000\,\text{L} = 1\,\text{m}^3) \\ &= 5\,000\,\text{L} \end{aligned}$$

Ex 67: Convert:

$$500\,\text{L} = \boxed{0.5}\,\text{m}^3.$$

Answer:

$$\begin{aligned} 500\,\mathrm{L} &= 500 \div 1\,000\,\mathrm{m}^3 \quad (1\,000\,\mathrm{L} = 1\,\mathrm{m}^3) \\ &= 0.5\,\mathrm{m}^3 \end{aligned}$$

Ex 68: Convert:

$$3.4\,\mathrm{m}^3 = \boxed{3400}\ \mathrm{L}.$$

Answer:

$$\begin{aligned} 3.4\,\mathrm{m}^3 &= 3.4 \times 1\,000\,\mathrm{L} \quad (1\,000\,\mathrm{L} = 1\,\mathrm{m}^3) \\ &= 3\,400\,\mathrm{L} \end{aligned}$$

Ex 69: Convert:

$$2\,\mathrm{L} = \boxed{0.002}\ \mathrm{m}^3.$$

Answer:

$$\begin{aligned} 2\,\mathrm{L} &= 2 \div 1\,000\,\mathrm{m}^3 \quad (1\,000\,\mathrm{L} = 1\,\mathrm{m}^3) \\ &= 0.002\,\mathrm{m}^3 \end{aligned}$$

H DENSITY

H.1 SOLVING PROBLEMS INVOLVING DENSITY

Ex 70: A solid gold bar is a rectangular prism with dimensions 5 cm by 10 cm by 2 cm. The density of gold is 19.3 g/cm³. What is the mass of the gold bar in kilograms?

Answer:

- **Step 1: Calculate the volume (V).**

$$\begin{aligned} V &= \text{length} \times \text{width} \times \text{height} \\ &= 10 \times 5 \times 2 \\ &= 100\,\mathrm{cm}^3 \end{aligned}$$

- **Step 2: Calculate the mass (m) using the density formula.**

$$\begin{aligned} m &= \rho \times V \\ &= 19.3\,\mathrm{g/cm}^3 \times 100\,\mathrm{cm}^3 \\ &= 1930\,\mathrm{g} \end{aligned}$$

- **Step 3: Convert the mass to kilograms.**

$$\begin{aligned} m &= 1930 div 1000 \\ &= 1.93\,\mathrm{kg} \end{aligned}$$

The mass of the gold bar is 1.93 kg.

Ex 71: A block of ice in the shape of a cube has a side length of 50 cm. Its mass is measured to be 114.5 kg. What is the density of the ice in g/cm³?

Answer:

- **Step 1: Calculate the volume (V) in cm³.**

$$\begin{aligned} V &= s^3 \\ &= (50\,\mathrm{cm})^3 \\ &= 125\,\mathrm{cm}^3 \end{aligned}$$

- **Step 2: Convert the mass (m) to grams.**

$$\begin{aligned} m &= 114.5 \times 1000 \\ &= 114{,}500\,\mathrm{g} \end{aligned}$$

- **Step 3: Calculate the density (ρ) in g/cm³.**

$$\begin{aligned} \rho &= \frac{m}{V} \\ &= \frac{114{,}500\,\mathrm{g}}{125{,}000\,\mathrm{cm}^3} \\ &= 0.916\,\mathrm{g/cm}^3 \end{aligned}$$

The density of the ice is 0.916 g/cm³.

Ex 72: A scientist has a 5.4 kg sample of aluminum. The density of aluminum is 2700 kg/m³. If the sample is a cylinder with a radius of 5 cm, what is its height in cm? (Round to one decimal place)

Answer:

- **Step 1: Find the volume (V) of the sample.** The units must be consistent. Let's work in kg and m.

$$\begin{aligned} V &= \frac{m}{\rho} \\ &= \frac{5.4\,\mathrm{kg}}{2700\,\mathrm{kg/m}^3} \\ &= 0.002\,\mathrm{m}^3 \end{aligned}$$

- **Step 2: Convert the radius to meters.**

$$\begin{aligned} r &= 5\,\mathrm{cm} \\ &= 0.05\,\mathrm{m} \end{aligned}$$

- **Step 3: Use the volume formula for a cylinder to find the height (h).**

$$\begin{aligned} V &= \pi r^2 h \\ h &= \frac{V}{\pi r^2} \\ &= \frac{0.002}{\pi(0.05)^2} \\ &= \frac{0.002}{\pi(0.0025)} \\ &\approx 0.2546\,\mathrm{m} \end{aligned}$$

- **Step 4: Convert the height back to centimeters.**

$$\begin{aligned} h &\approx 0.2546\,\mathrm{m} \times 100 \\ &\approx 25.5\,\mathrm{cm} \end{aligned}$$

The height of the cylinder is approximately 25.5 cm.

Ex 73: A solid sphere made of lead has a mass of 380 g. If the density of lead is 11.34 g/cm³, what is the radius of the sphere? (Round to one decimal place)

Answer:

- **Step 1: Find the volume (V) of the lead sphere.**

$$\begin{aligned} V &= \frac{m}{\rho} \\ &= \frac{380\,\text{g}}{11.34\,\text{g/cm}^3} \\ &\approx 33.51\,\text{cm}^3 \end{aligned}$$

- **Step 2: Use the volume formula for a sphere to find the radius (r).**

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ r^3 &= \frac{3V}{4\pi} \\ &\approx \frac{3 \times 33.51}{4\pi} \\ &\approx 8.00 \\ r &\approx \sqrt[3]{8.00} \\ &\approx 2.0\,\text{cm} \end{aligned}$$

The radius of the sphere is approximately 2.0 cm.

Ex 74: A cone has a radius of 10 cm, a height of 30 cm, and a mass of 7.85 kg. Calculate its density in g/cm**³**. Based on your result, is the material more likely to be glass ($\rho \approx 2.5$ g/cm**³**) or aluminum ($\rho = 2.7$ g/cm**³**)?

Answer:

- **Step 1: Calculate the volume (V) of the cone.**

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times (10)^2 \times 30 \\ &= 1000\pi\,\text{cm}^3 \\ &\approx 3141.59\,\text{cm}^3 \end{aligned}$$

- **Step 2: Convert the mass (m) to grams.**

$$\begin{aligned} m &= 7.85\,\text{kg} \times 1000 \\ &= 7850\,\text{g} \end{aligned}$$

- **Step 3: Calculate the density (ρ).**

$$\begin{aligned} \rho &= \frac{m}{V} \\ &= \frac{7850\,\text{g}}{1000\pi\,\text{cm}^3} \\ &\approx \frac{7850}{3141.59} \\ &\approx 2.50\,\text{g/cm}^3 \end{aligned}$$

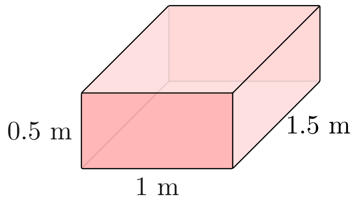
- **Step 4: Compare and identify the material.** The calculated density (2.50 g/cm**³**) is approximately equal to the density of glass.

The material is likely to be glass.

I SURFACE AREA

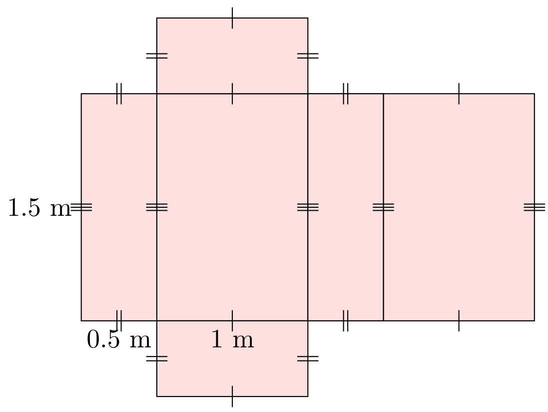
I.1 FINDING SURFACE AREAS

Ex 75: Find the surface area of the rectangular cuboid.



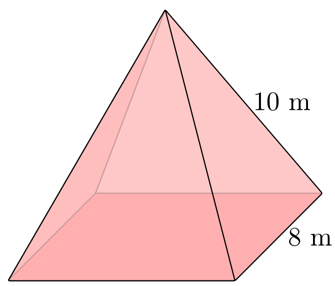
$S =$ 5.5 m^2

Answer:



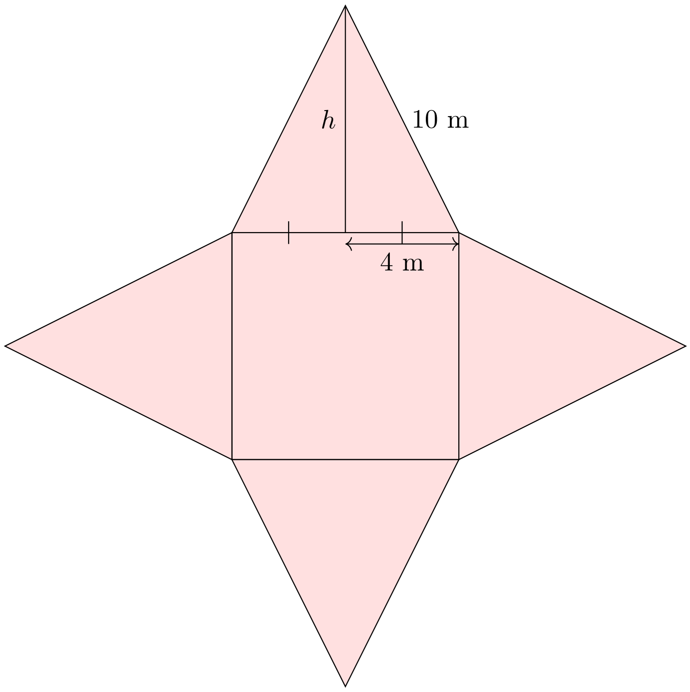
$$\begin{aligned} \text{Surface area} &= 2A_{1\text{ m x }0.5\text{ m}} + 2A_{1.5\text{ m x }1\text{ m}} + 2A_{1.5\text{ m x }0.5\text{ m}} \\ &= 2 \times 1 \times 0.5 + 2 \times 1.5 \times 1 + 2 \times 1.5 \times 0.5 \\ &= 5.5\ \text{m}^2 \end{aligned}$$

Ex 76: Find the surface area of the square-based pyramid.



$S \approx$ 137 m^2 (round to the nearest integer)

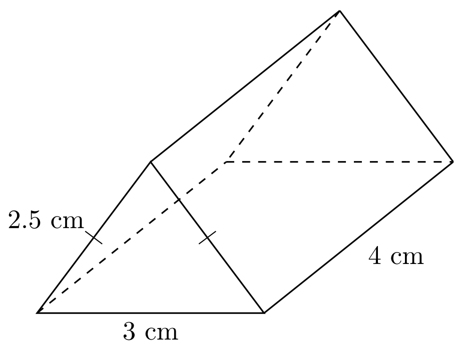
Answer:



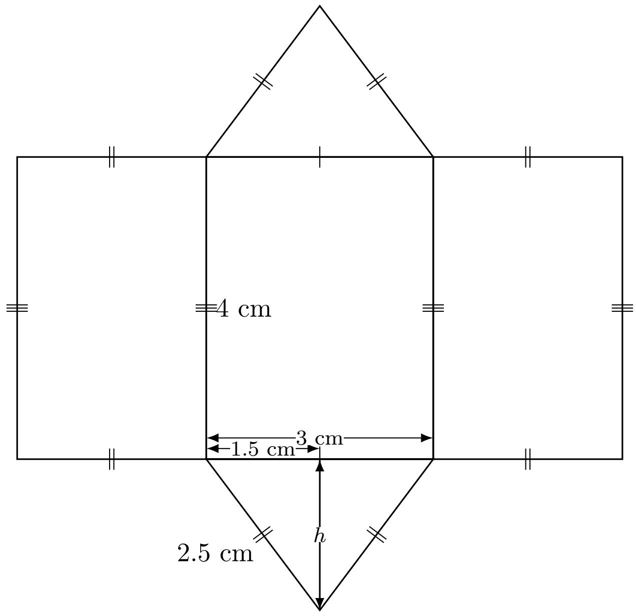
$$h^2 + 4^2 = 10^2 \quad \text{(Pythagoras theorem)}$$
$$h^2 + 16 = 100$$
$$h^2 = 84$$
$$h = \sqrt{84} \quad \text{since } h \geq 0$$

$$\begin{aligned} \text{Surface area} &= \text{Area of square } + 4\times \text{ Area of isoceles triangle} \\ &= 8\times 8 + 4\times \frac{1}{2}\times 4\times\sqrt{84} \\ &= 8\times 8 + 4\times \frac{1}{2}\times 4\times\sqrt{84} \\ &\approx 137 \text{ m}^2 \end{aligned}$$

Ex 77: Find the surface area of the triangular prism.



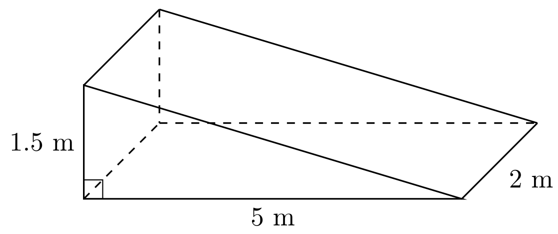
$S =$ 38 cm^2



Answer:

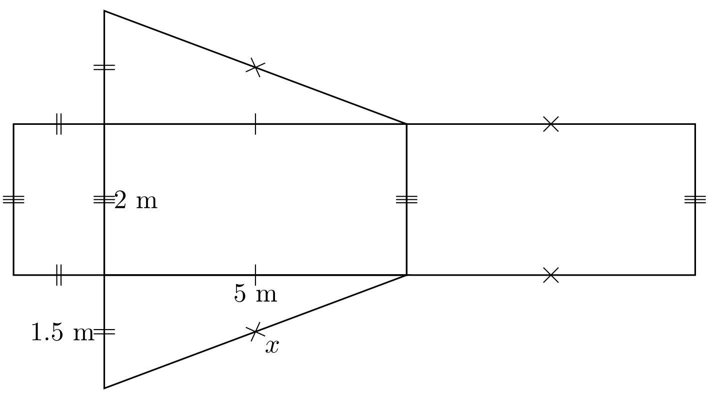
- $h^2 + (1.5)^2 = (2.5)^2$ (Pythagoras theorem)
  $h^2 = (2.5)^2 - (1.5)^2$
  $h^2 = 4$
  $h = \sqrt{4}$ as $h \geq 0$
  $h = 2$
- Surface area
  $= 2\times A_{\text{isocele triangle}} + 2\times A_{\text{left/right rectangles}} + A_{\text{center rectangle}}$
  $= 2\times \frac{3\times 2}{2} + 2\times 4\times 2.5 + 4\times 3$
  $= 38 \text{ cm}^2$

Ex 78: Find the surface area of the triangular prism.



$S =$ 30.9 m^2 (round to 1 decimal place)

Answer:



- $5^2 + (1.5)^2 = x^2$ (Pythagoras theorem)
  $x = \sqrt{5^2 + (1.5)^2}$
  $x \approx 5.22\,\text{m}$

-

$$\begin{aligned}\text{Surface area} &= 2\times A_{\text{right triangle}} + A_{\text{rectangle } 2\times 1.5}\\ &\quad + A_{\text{rectangle } 2\times 5} + A_{\text{rectangle } 2\times 5.22}\\ &\approx 2\times\frac{1.5\times 5}{2} + 2\times 1.5 + 2\times 5 + 2\times 5.22\\ &\approx 30.9\,\text{m}^2\end{aligned}$$