VOLUME

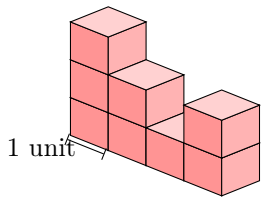
A DEFINITION

Definition Volume

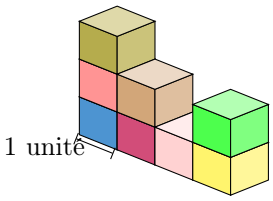
The **volume** of a shape is the amount of space it takes up in three dimensions. We measure it in cubic units, like cubic centimeters or cubic meters.

To find the volume, imagine filling the shape with small cubes, like building blocks. Count how many cubes fit inside the shape.

Ex: Find the volume:



Answer:



Volume = 8 cube units

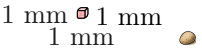
B UNITS OF VOLUME

Discover: We can measure volume using different units like blocks or small boxes. But these units are not the same for everyone. Your friend might have a bigger block than yours, making it hard to compare. To solve this, mathematicians created universal units like the cubic centimeter and cubic meter, so everyone can measure and compare volumes the same way.

Definition **Units of Volume**

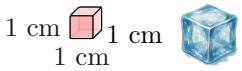
- Cubic Millimeter $\left(\text{mm}^3\right)$: A very small unit of volume, about the size of a tiny grain of sand.

$$1\ \text{mm}^3 = 1\ \text{mm} \times 1\ \text{mm} \times 1\ \text{mm}$$



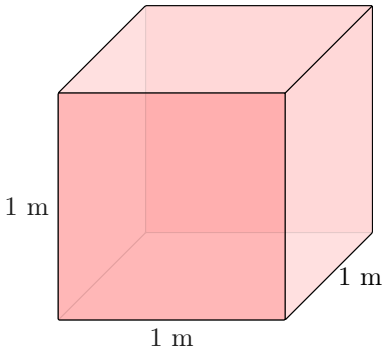
- Cubic Centimeter $\left(\text{cm}^3\right)$: A small unit of volume, about the size of a small ice cube.

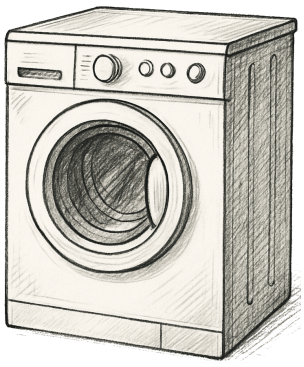
$$1\ \text{cm}^3 = 1\ \text{cm} \times 1\ \text{cm} \times 1\ \text{cm}$$



- Cubic Meter $\left(\text{m}^3\right)$: A larger unit of volume, about the space it takes for a washing machine.

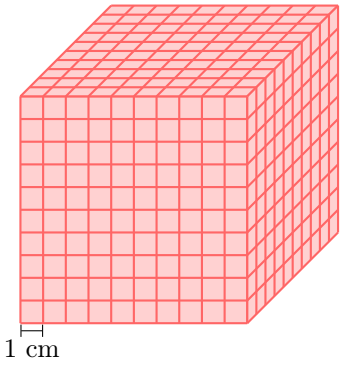
$$1\ \text{m}^3 = 1\ \text{m} \times 1\ \text{m} \times 1\ \text{m}$$





C CONVERSION OF VOLUME UNITS

Discover: Let's explore how volume units are related by looking at a small cube. Below is a cube that measures 1 cm by 1 cm by 1 cm, which is 1 cm^3. We divide it into smaller cubes, each 1 mm by 1 mm by 1 mm (1 mm^3). Let's count how many 1 mm^3 cubes fit inside 1 cm^3.



Each small square shown above is 1 mm^2. There are 10 small squares along the length and 10 along the width, so there are $10 \times 10 = 100$ small squares in total for one layer. Since the cube is also 1 cm (10 mm) high, there are 10 such layers. Therefore, 1 cm^3 contains $100 \times 10 = 1000$ mm^3.

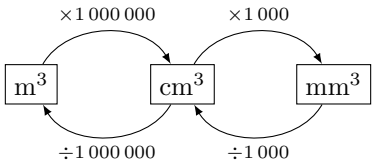
$$\begin{aligned} 1\,\text{cm}^3 &= 1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm} \\ &= 10\,\text{mm} \times 10\,\text{mm} \times 10\,\text{mm} \quad (1\,\text{cm} = 10\,\text{mm}) \\ &= 1\,000\,\text{mm}^3 \end{aligned}$$

Proposition **Conversion of Volume Units**

- $1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3$
- $1\,\text{cm}^3 = 1\,000\,\text{mm}^3$

Method **Converting Using Multiplication or Division**

- Use **multiplication** to go from a larger unit to a smaller one (like cubic meters to cubic centimeters).
- Use **division** to go from a smaller unit to a larger one (like cubic centimeters to cubic meters).



Method **Converting Using a Table**

To convert between units of volume, we can use a conversion table. For example, to convert 10.5 cubic meters to cubic centimeters:

1. Write the units in the table: $\mathbf{m^3}$, $\mathbf{cm^3}$, $\mathbf{mm^3}$.

m^3						cm^3			mm^3		

2. Place the number in the column of the unit you start with.

m^3						cm^3			mm^3		
	1	0.	5								

3. Fill in zeros in the columns to the right until you reach the unit you want to convert to.

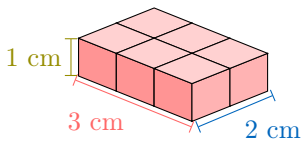
m^3						cm^3			mm^3		
	1	0.	5	0	0	0	0	0			

4. Read the number in the column of the target unit.

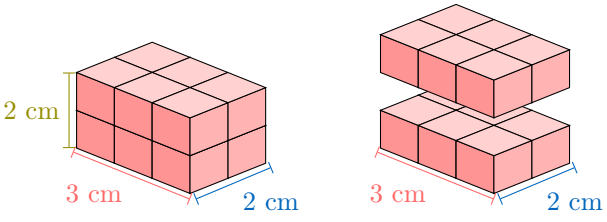
So, 10.5 $\mathbf{m^3}$ = 10 500 000 $\mathbf{cm^3}$.

D VOLUME OF A RECTANGULAR CUBOID

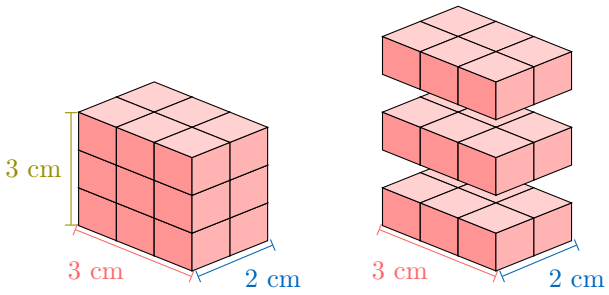
Discover: To calculate the volume of a rectangular cuboid, we can count the cubes inside, but that takes a long time. Let's look at how the volume changes when we make the box taller, one step at a time, to find an easier way. We'll also see how counting layer by layer can help us understand the pattern.

- 

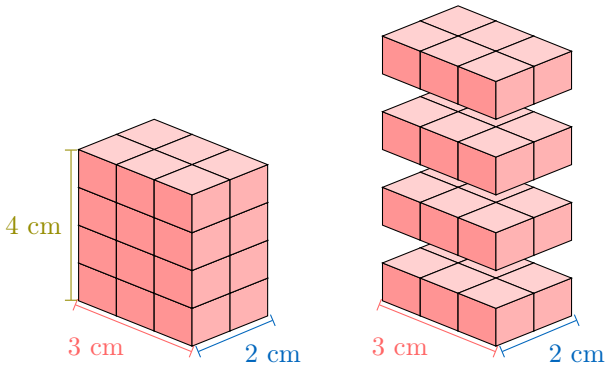
$$\text{Volume} = 3 \times 2 \times 1$$
$$= 6\,\text{cm}^3$$

- 

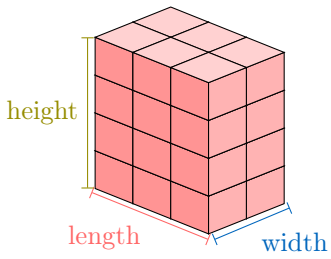
$$\begin{aligned}\text{Volume} &= (3 \times 2) + (3 \times 2)\\ &= (3 \times 2) \times 2\\ &= 12\,\text{cm}^3\end{aligned}$$

- 

$$\begin{aligned}\text{Volume} &= (3 \times 2) + (3 \times 2) + (3 \times 2)\\ &= (3 \times 2) \times 3\\ &= 18\,\text{cm}^3\end{aligned}$$

- 

$$\begin{aligned}\text{Volume} &= (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2)\\ &= (3 \times 2) \times 4\\ &= 24\,\text{cm}^3\end{aligned}$$

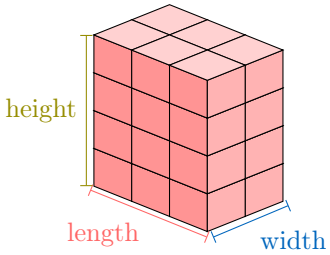
- 

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

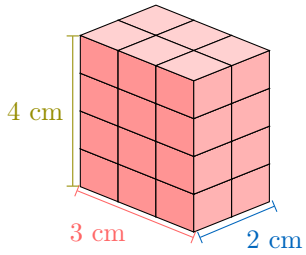
Proposition Volume of a Rectangular Cuboid

The volume of a rectangular prism is multiplying its length, width, and height:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$



Ex: Find the volume:



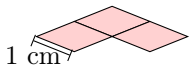
Answer:

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 3 \times 2 \times 4 \\ &= 24\,\text{cm}^3\end{aligned}$$

E VOLUMES OF SOLIDS WITH UNIFORM CROSS-SECTION

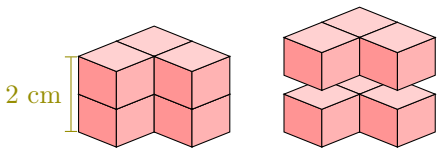
Discover: Counting the cubes inside a solid to find its volume can be time-consuming, especially for solids with a uniform cross-section. Instead, we can explore a faster method by examining the solid layer by layer to identify a pattern for calculating the volume.

- **Area of the Base:**



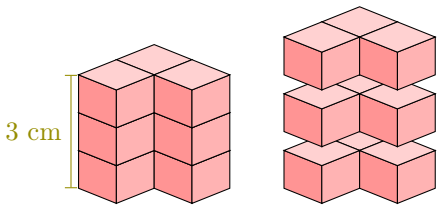
$$\text{Area} = 3\,\text{cm}^3$$

- **Volume with Height of 2 cm:**



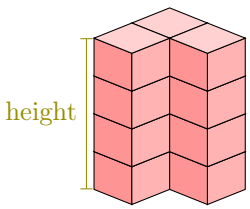
$$\begin{aligned}\text{Volume} &= \text{Area} + \text{Area} \\ &= \text{Area} \times 2 \\ &= 6\,\text{cm}^3\end{aligned}$$

- **Volume with Height of 3 cm:**



$$\begin{aligned}\text{Volume} &= \text{Area} + \text{Area} + \text{Area} \\ &= \text{Area} \times 3 \\ &= 9\,\text{cm}^3\end{aligned}$$

- **General Formula:**

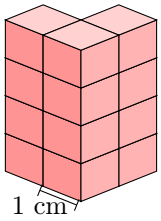


$$\text{Volume} = \text{Area} \times \text{height}$$

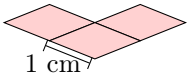
Proposition Volume of Solid of Uniform Cross-Section

Volume = area of end × height

Ex: Find the volume of the figure



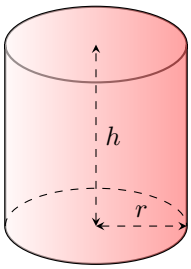
Answer:

- Area of  $= 3\,\text{cm}^2$
- height $= 4\,\text{cm}$
-

$$\begin{aligned} V &= \text{area of end} \times \text{height} \\ &= 3\,\text{cm}^2 \times 4\,\text{cm} \\ &= 12\,\text{cm}^3 \end{aligned}$$

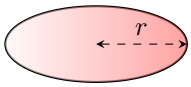
Proposition Volume of a Cylinder

Volume of a cylinder $= \pi r^2 h$



Proof

A cylinder has a uniform cross-section.

- Area of end = Area of  = Area of a circle $= \pi r^2$
- height $= h$
-

$$\begin{aligned} V &= \text{area of end} \times \text{height} \\ &= \pi r^2 \times h \end{aligned}$$

F CAPACITY

We often need to measure liquids like water, milk, or juice. Instead of using cubic centimeters, there's an **easier way** to talk about these amounts: we use a unit called a **liter**. For example, instead of saying "1000 cubic centimeters," we can simply say **"1 liter"**. This makes it much easier to understand how much liquid we have!

Definition **Liter**

A **liter** is a unit we use to measure the volume of liquids.

- 1 liter is the volume of a cube that measures 10 cm on each side.

$$1\,\text{L} = 1000\,\text{cm}^3 \text{ and } 1\,000\,\text{L} = 1\,\text{m}^3$$

- We write it with the symbol **L** (a capital "L").
- A smaller unit, the **centiliter** (cL), is often used for smaller volumes:

$$1\,\text{L} = 100\,\text{cL}$$

- An even smaller unit, the **milliliter** (mL), is used for very small volumes:

$$1\,\text{L} = 1\,000\,\text{mL} \quad \text{and} \quad 1\,\text{cL} = 10\,\text{mL}$$

Ex:

- A big water bottle holds about 1 liter of water, which is 100 centiliters:

- A small soda can holds about 0.33 liters of soda, which is 33 centiliters: