VECTOR PRODUCT

While the scalar product of two vectors results in a scalar, there is a second form of product, defined only in three dimensions, called the **vector product** or **cross product**. Given two vectors in three-dimensional space, this operation produces a third vector that is perpendicular to both of the original vectors. The cross product is a fundamental tool in physics (for example, in torque and angular momentum) and in mathematics (for calculating areas, volumes, and describing geometric orientations).

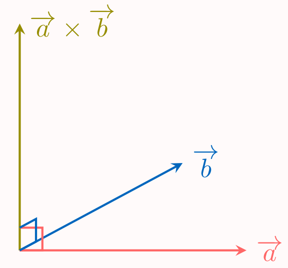
A DEFINITION

Definition Vector (Cross) Product

The **vector product** of two vectors in space $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is defined as:

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

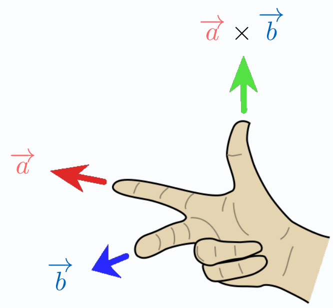
The result $\vec{a} \times \vec{b}$ is a vector in three-dimensional space that is perpendicular to both $\vec{a}$ and $\vec{b}$.



B GEOMETRIC INTERPRETATION

Proposition Right-Hand Rule

The direction of the vector product $\vec{a} \times \vec{b}$ is determined by the **right-hand rule**. If you curl the fingers of your right hand in the direction from vector $\vec{a}$ to vector $\vec{b}$, your thumb will point in the direction of $\vec{a} \times \vec{b}$.



Proposition Magnitude Formulas

- The magnitude (or length) of the vector product is given by:

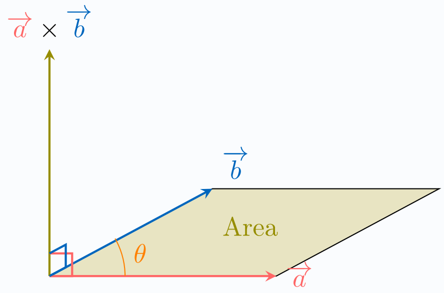
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

  where θ is the angle between vectors $\vec{a}$ and $\vec{b}$, with $0 \le \theta \le \pi$.

- Geometrically, the magnitude of the cross product equals the area of the parallelogram with sides represented by $\vec{a}$ and $\vec{b}$:

$$|\vec{a} \times \vec{b}| = \text{Area}.$$

  Consequently, the area of the triangle formed by $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.


$\overrightarrow{a} \times \overrightarrow{b}$
$\overrightarrow{b}$
Area
θ
$\overrightarrow{a}$