VECTOR PRODUCT

A DEFINITION

A.1 CALCULATING THE VECTOR PRODUCT

Ex 1: For $\overrightarrow{a} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix}1\\2\\3\end{pmatrix}$, calculate:

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix}\boxed{5}\\\boxed{-7}\\\boxed{3}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{a} \times \overrightarrow{b} &= \begin{pmatrix}a_2b_3 - a_3b_2\\a_3b_1 - a_1b_3\\a_1b_2 - a_2b_1\end{pmatrix}\\&= \begin{pmatrix}(1)(3) - (-1)(2)\\(-1)(1) - (2)(3)\\(2)(2) - (1)(1)\end{pmatrix}\\&= \begin{pmatrix}3 - (-2)\\-1 - 6\\4 - 1\end{pmatrix}\\&= \begin{pmatrix}5\\-7\\3\end{pmatrix}\end{aligned}$$

Ex 2: For $\overrightarrow{b} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\overrightarrow{c} = \begin{pmatrix}2\\0\\4\end{pmatrix}$, calculate:

$$\overrightarrow{b} \times \overrightarrow{c} = \begin{pmatrix}\boxed{8}\\\boxed{2}\\\boxed{-4}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{b} \times \overrightarrow{c} &= \begin{pmatrix}b_2c_3 - b_3c_2\\b_3c_1 - b_1c_3\\b_1c_2 - b_2c_1\end{pmatrix}\\&= \begin{pmatrix}(2)(4) - (3)(0)\\(3)(2) - (1)(4)\\(1)(0) - (2)(2)\end{pmatrix}\\&= \begin{pmatrix}8 - 0\\6 - 4\\0 - 4\end{pmatrix}\\&= \begin{pmatrix}8\\2\\-4\end{pmatrix}\end{aligned}$$

Ex 3: For $\overrightarrow{c} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$ and $\overrightarrow{d} = \begin{pmatrix}1\\3\\-2\end{pmatrix}$, calculate:

$$\overrightarrow{c} \times \overrightarrow{d} = \begin{pmatrix}\boxed{-6}\\\boxed{0}\\\boxed{-3}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{c} \times \overrightarrow{d} &= \begin{pmatrix}c_2d_3 - c_3d_2\\c_3d_1 - c_1d_3\\c_1d_2 - c_2d_1\end{pmatrix}\\&= \begin{pmatrix}(0)(-2) - (2)(3)\\(2)(1) - (-1)(-2)\\(-1)(3) - (0)(1)\end{pmatrix}\\&= \begin{pmatrix}0 - 6\\2 - 2\\-3 - 0\end{pmatrix}\\&= \begin{pmatrix}-6\\0\\-3\end{pmatrix}\end{aligned}$$

Ex 4: For $\overrightarrow{u} = \begin{pmatrix}2\\-1\\0\end{pmatrix}$ and $\overrightarrow{v} = \begin{pmatrix}3\\1\\-4\end{pmatrix}$, calculate:

$$\overrightarrow{u} \times \overrightarrow{v} = \begin{pmatrix}\boxed{4}\\\boxed{8}\\\boxed{5}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{u} \times \overrightarrow{v} &= \begin{pmatrix}u_2v_3 - u_3v_2\\u_3v_1 - u_1v_3\\u_1v_2 - u_2v_1\end{pmatrix}\\&= \begin{pmatrix}(-1)(-4) - (0)(1)\\(0)(3) - (2)(-4)\\(2)(1) - (-1)(3)\end{pmatrix}\\&= \begin{pmatrix}4 - 0\\0 - (-8)\\2 - (-3)\end{pmatrix}\\&= \begin{pmatrix}4\\8\\5\end{pmatrix}\end{aligned}$$

A.2 VERIFYING PROPERTIES OF THE VECTOR PRODUCT

Ex 5: Suppose $\overrightarrow{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix}-1\\3\\-1\end{pmatrix}$.

1. Find $\overrightarrow{a} \times \overrightarrow{b}$.
2. Hence determine $\overrightarrow{a} \cdot (\overrightarrow{a} \times \overrightarrow{b})$ and $\overrightarrow{b} \cdot (\overrightarrow{a} \times \overrightarrow{b})$.
3. Explain your results.

Answer:

1. **Find $\vec{a} \times \vec{b}$**

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}-1\\3\\-1\end{pmatrix}\\ &= \begin{pmatrix}(2)(-1)-(3)(3)\\(3)(-1)-(1)(-1)\\(1)(3)-(2)(-1)\end{pmatrix}\\ &= \begin{pmatrix}-2-9\\-3-(-1)\\3-(-2)\end{pmatrix}\\ &= \begin{pmatrix}-11\\-2\\5\end{pmatrix}\end{aligned}$$

2. **Determine the scalar products** Using the result from part 1:

$$\begin{aligned}\vec{a} \cdot (\vec{a} \times \vec{b}) &= \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}-11\\-2\\5\end{pmatrix}\\ &= (1)(-11)+(2)(-2)+(3)(5)\\ &= -11-4+15\\ &= 0\end{aligned}$$

And for the second one:

$$\begin{aligned}\vec{b} \cdot (\vec{a} \times \vec{b}) &= \begin{pmatrix}-1\\3\\-1\end{pmatrix} \cdot \begin{pmatrix}-11\\-2\\5\end{pmatrix}\\ &= (-1)(-11)+(3)(-2)+(-1)(5)\\ &= 11-6-5\\ &= 0\end{aligned}$$

3. **Explanation** The results are both zero. This is because, by definition, the vector product $\vec{a} \times \vec{b}$ produces a vector that is orthogonal (perpendicular) to both $\vec{a}$ and $\vec{b}$. The scalar product of two orthogonal vectors is always zero. The calculations in part 2 verify this fundamental property.

Ex 6: $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the base unit vectors in a 3D orthonormal system.

1. Find $\vec{i} \times \vec{i}$, $\vec{j} \times \vec{j}$, and $\vec{k} \times \vec{k}$.
2. Find $\vec{i} \times \vec{j}$ and $\vec{j} \times \vec{i}$.

Answer: The base unit vectors are $\vec{i} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{j} = \begin{pmatrix}0\\1\\0\end{pmatrix}$, and $\vec{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$.

1. **Cross product of a vector with itself**

$$\vec{i} \times \vec{i} = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}(0)(0)-(0)(0)\\(0)(1)-(1)(0)\\(1)(0)-(0)(1)\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} = \vec{0}$$

Similarly, $\vec{j} \times \vec{j} = \vec{0}$ and $\vec{k} \times \vec{k} = \vec{0}$.

2. **Cross product of two different base vectors**

$$\vec{i} \times \vec{j} = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}(0)(0)-(0)(1)\\(0)(0)-(1)(0)\\(1)(1)-(0)(0)\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} = \vec{k}$$

$$\vec{j} \times \vec{i} = \begin{pmatrix}0\\1\\0\end{pmatrix} \times \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}(1)(0)-(0)(0)\\(0)(1)-(0)(0)\\(0)(0)-(1)(1)\end{pmatrix} = \begin{pmatrix}0\\0\\-1\end{pmatrix} = -\vec{k}$$

Ex 7: For $\vec{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$, prove that $\vec{a} \times \vec{a} = \vec{0}$.

Answer: We apply the definition of the vector product of $\vec{a}$ with itself.
Let $\vec{b} = \vec{a}$, so $b_1 = a_1$, $b_2 = a_2$, and $b_3 = a_3$.

$$\begin{aligned}\vec{a} \times \vec{a} &= \begin{pmatrix}a_2a_3-a_3a_2\\a_3a_1-a_1a_3\\a_1a_2-a_2a_1\end{pmatrix}\\ &= \begin{pmatrix}0\\0\\0\end{pmatrix}\\ &= \vec{0}\end{aligned}$$

Since the multiplication of real numbers is commutative ($ab = ba$), each component simplifies to zero.

Ex 8: For $\vec{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, prove that $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

Answer:

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \times \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}\\ &= \begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}\\ &= \begin{pmatrix}-(a_3b_2-a_2b_3)\\-(a_1b_3-a_3b_1)\\-(a_2b_1-a_1b_2)\end{pmatrix}\\ &= -\begin{pmatrix}a_3b_2-a_2b_3\\a_1b_3-a_3b_1\\a_2b_1-a_1b_2\end{pmatrix}\\ &= -\left(\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \times \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\right)\\ &= -\left(\vec{b} \times \vec{a}\right)\end{aligned}$$

Therefore, we have shown that $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

Ex 9: Let $\vec{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$, $\vec{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, and $\vec{c} = \begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix}$ be three vectors in space. Prove the distributive property of the vector product:

$$\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$$

Answer: We will prove this property by expanding both sides of the equation by their components.

- **Left-Hand Side (LHS):** First, we find the sum $\vec{b} + \vec{c}$:

$$\vec{b} + \vec{c} = \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix}$$

  Now we calculate $\vec{a} \times (\vec{b} + \vec{c})$:

$$\begin{aligned}
\vec{a} \times (\vec{b} + \vec{c}) &= \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix} \\
&= \begin{pmatrix} a_2(b_3 + c_3) - a_3(b_2 + c_2) \\ a_3(b_1 + c_1) - a_1(b_3 + c_3) \\ a_1(b_2 + c_2) - a_2(b_1 + c_1) \end{pmatrix} \\
&= \begin{pmatrix} a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2 \\ a_3b_1 + a_3c_1 - a_1b_3 - a_1c_3 \\ a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1 \end{pmatrix}
\end{aligned}$$

- **Right-Hand Side (RHS):** First, we find the two separate cross products:

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \quad \text{and} \quad \vec{a} \times \vec{c} = \begin{pmatrix} a_2c_3 - a_3c_2 \\ a_3c_1 - a_1c_3 \\ a_1c_2 - a_2c_1 \end{pmatrix}$$

  Now we add them together:

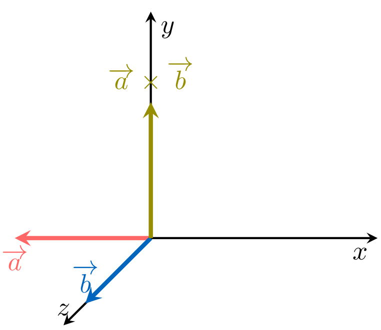
$$\begin{aligned}
(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) &= \begin{pmatrix} (a_2b_3 - a_3b_2) + (a_2c_3 - a_3c_2) \\ (a_3b_1 - a_1b_3) + (a_3c_1 - a_1c_3) \\ (a_1b_2 - a_2b_1) + (a_1c_2 - a_2c_1) \end{pmatrix} \\
&= \begin{pmatrix} a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2 \\ a_3b_1 + a_3c_1 - a_1b_3 - a_1c_3 \\ a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1 \end{pmatrix}
\end{aligned}$$

Since the resulting vectors for the LHS and RHS are identical, we have proven that $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$.

B GEOMETRIC INTERPRETATION

B.1 APPLYING THE RIGHT-HAND RULE

MCQ 10: The diagram below illustrates three vectors, $\vec{a}$, $\vec{b}$, and their vector product $\vec{a} \times \vec{b}$.

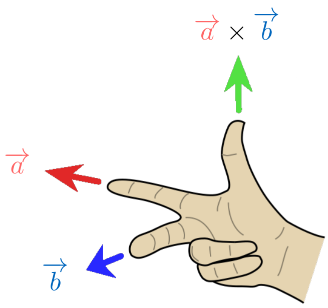


According to the right-hand rule, is the direction of the vector product $\vec{a} \times \vec{b}$ correctly illustrated?

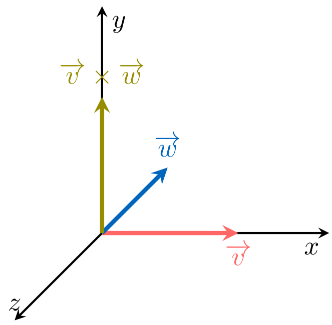
☒ Yes

☐ No

Answer: Yes. By pointing the fingers of the right hand in the direction of $\vec{a}$ (along the negative x-axis) and curling them toward the direction of $\vec{b}$ (along the positive z-axis), the thumb points in the direction of the positive y-axis.



MCQ 11: The diagram below illustrates three vectors, $\vec{v}$, $\vec{w}$, and their vector product $\vec{v} \times \vec{w}$.

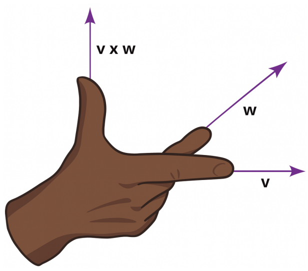


According to the right-hand rule, is the direction of the vector product $\vec{v} \times \vec{w}$ correctly illustrated?

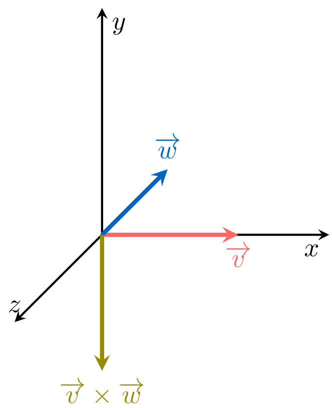
☒ Yes

☐ No

Answer: Yes. By pointing the fingers of the right hand in the direction of $\vec{v}$ (along the positive x-axis) and curling them toward the direction of $\vec{w}$ (along the negative z-axis), the thumb correctly points in the direction of the positive y-axis.



MCQ 12: The diagram below illustrates three vectors, $\vec{v}$, $\vec{w}$, and their vector product $\vec{v} \times \vec{w}$.

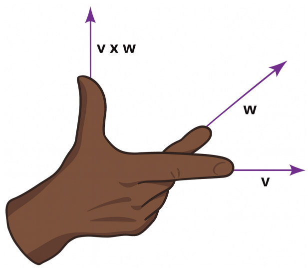


According to the right-hand rule, is the direction of the vector product $\vec{v} \times \vec{w}$ correctly illustrated?

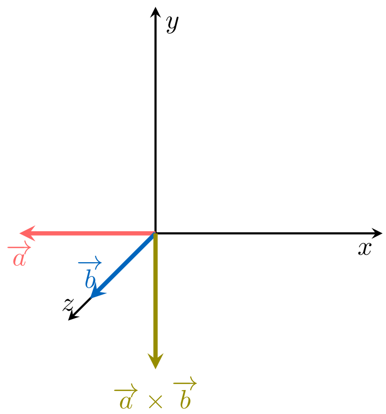
☐ Yes

☒ No

Answer: No. By pointing the fingers of the right hand in the direction of $\overrightarrow{v}$ (along the positive x-axis) and curling them toward the direction of $\overrightarrow{w}$ (along the negative z-axis), the thumb points in the direction of the positive y-axis.



MCQ 13: The diagram below illustrates three vectors, $\overrightarrow{a}$, $\overrightarrow{b}$, and their vector product $\overrightarrow{a} \times \overrightarrow{b}$.

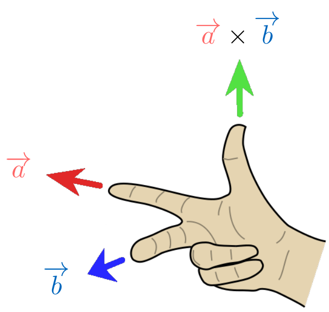


According to the right-hand rule, is the direction of the vector product $\overrightarrow{a} \times \overrightarrow{b}$ correctly illustrated?

☐ Yes

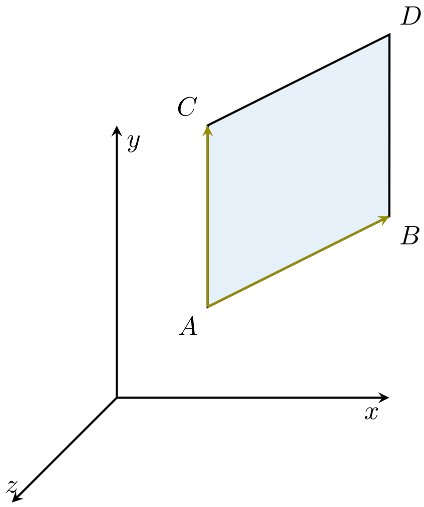
☒ No

Answer: No. By pointing the fingers of the right hand in the direction of $\overrightarrow{a}$ (along the negative x-axis) and curling them toward the direction of $\overrightarrow{b}$ (along the positive z-axis), the thumb points in the direction of the positive y-axis.



B.2 CALCULATING AREA USING THE VECTOR PRODUCT

Ex 14: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$. Calculate the area of the parallelogram with adjacent sides $\overline{AB}$ and $\overline{AC}$.



Answer: The area of a parallelogram defined by two vectors is equal to the magnitude of their vector product: Area $= |\overrightarrow{AB} \times \overrightarrow{AC}|$.

1. **Determine the component vectors**

$$\overrightarrow{AB} = \begin{pmatrix} 3-1 \\ 2-1 \\ 1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 1-1 \\ 3-1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

2. **Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$**

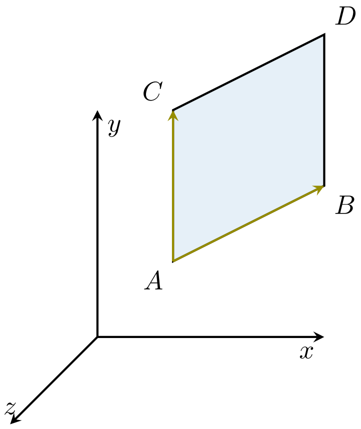
$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} (1)(2)-(0)(2) \\ (0)(0)-(2)(2) \\ (2)(2)-(1)(0) \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \end{aligned}$$

3. **Calculate the magnitude of the resulting vector**

$$\begin{aligned} \text{Area} &= |\overrightarrow{AB} \times \overrightarrow{AC}| \\ &= \left\| \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \right\| \\ &= \sqrt{2^2 + (-4)^2 + 4^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

The area of the parallelogram is 6 square units.

Ex 15: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$. Calculate the area of the parallelogram with adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



Answer: The area of a parallelogram defined by two vectors is equal to the magnitude of their vector product: Area $= |\overrightarrow{AB} \times \overrightarrow{AC}|$.

1. **Determine the component vectors**

$$\overrightarrow{AB} = \begin{pmatrix} 3-1 \\ 2-1 \\ 1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 1-1 \\ 3-1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

2. **Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$**

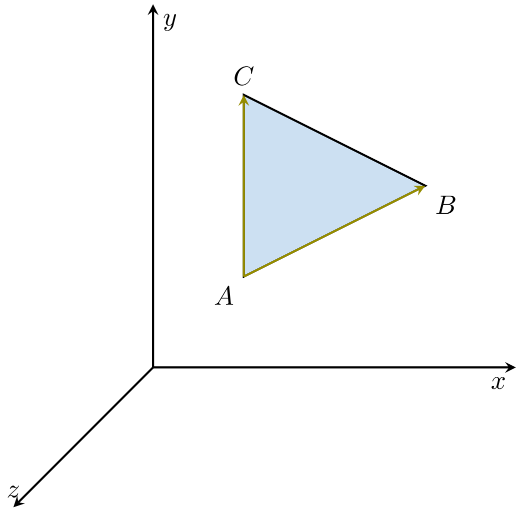
$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} (1)(2)-(0)(2) \\ (0)(0)-(2)(2) \\ (2)(2)-(1)(0) \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \end{aligned}$$

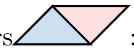
3. **Calculate the magnitude of the resulting vector**

$$\begin{aligned} \text{Area} &= |\overrightarrow{AB} \times \overrightarrow{AC}| \\ &= \left\| \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \right\| \\ &= \sqrt{2^2 + (-4)^2 + 4^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

The area of the parallelogram is 6 square units.

Ex 16: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$. Calculate the area of the triangle ABC.



Answer: The area of a triangle defined by two vectors is half the magnitude of their vector product, since this triangle is half of the parallelogram built on those vectors  : Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$.

1. **Determine the component vectors**

$$\overrightarrow{AB} = \begin{pmatrix} 3-1 \\ 2-1 \\ 1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 1-1 \\ 3-1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

2. **Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$**

$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} (1)(2)-(0)(2) \\ (0)(0)-(2)(2) \\ (2)(2)-(1)(0) \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \end{aligned}$$

3. **Calculate the magnitude and the area** The magnitude of the cross product vector is:

$$\begin{aligned} |\overrightarrow{AB} \times \overrightarrow{AC}| &= \left\| \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \right\| \\ &= \sqrt{2^2 + (-4)^2 + 4^2} \\ &= \sqrt{4+16+16} = \sqrt{36} = 6 \end{aligned}$$

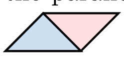
The area of the triangle is half of this magnitude:

$$\text{Area of Triangle} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \times 6 = 3$$

The area of the triangle ABC is 3 square units.

Ex 17: Consider the points $A(0,0,0)$, $B(-1,2,3)$, and $C(1,2,6)$. Calculate the area of the triangle ABC.

Answer: The area of a triangle defined by two vectors is half the magnitude of their vector product, since this triangle is half of the parallelogram built on those vectors: Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$.



1. **Determine the component vectors** Since A is the origin, the components of the vectors are simply the coordinates of the points B and C.

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$$

2. **Calculate the vector product** $\overrightarrow{AB} \times \overrightarrow{AC}$

$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} (2)(6) - (3)(2) \\ (3)(1) - (-1)(6) \\ (-1)(2) - (2)(1) \end{pmatrix} \\ &= \begin{pmatrix} 12 - 6 \\ 3 - (-6) \\ -2 - 2 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 9 \\ -4 \end{pmatrix} \end{aligned}$$

3. **Calculate the magnitude and the area** The magnitude of the cross product vector is:

$$\begin{aligned} |\overrightarrow{AB} \times \overrightarrow{AC}| &= \left\| \begin{pmatrix} 6 \\ 9 \\ -4 \end{pmatrix} \right\| \\ &= \sqrt{6^2 + 9^2 + (-4)^2} \\ &= \sqrt{36 + 81 + 16} = \sqrt{133} \end{aligned}$$

The area of the triangle is half of this magnitude:

$$\text{Area of Triangle} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{\sqrt{133}}{2}$$

The area of the triangle ABC is $\frac{\sqrt{133}}{2}$ square units.