VECTOR PRODUCT

A DEFINITION

A.1 CALCULATING THE VECTOR PRODUCT

Ex 1: For $\overrightarrow{a} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, calculate:

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Ex 2: For $\overrightarrow{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{c} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$, calculate:

$$\overrightarrow{b} \times \overrightarrow{c} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Ex 3: For $\overrightarrow{c} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ and $\overrightarrow{d} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, calculate:

$$\overrightarrow{c} \times \overrightarrow{d} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

Ex 4: For $\overrightarrow{u} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ and $\overrightarrow{v} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$, calculate:

$$\overrightarrow{u} \times \overrightarrow{v} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

A.2 VERIFYING PROPERTIES OF THE VECTOR PRODUCT

Ex 5: Suppose $\overrightarrow{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$.

1. Find $\overrightarrow{a} \times \overrightarrow{b}$.
2. Hence determine $\overrightarrow{a} \cdot (\overrightarrow{a} \times \overrightarrow{b})$ and $\overrightarrow{b} \cdot (\overrightarrow{a} \times \overrightarrow{b})$.
3. Explain your results.

Ex 6: $\overrightarrow{i}$, $\overrightarrow{j}$, and $\overrightarrow{k}$ are the base unit vectors in a 3D orthonormal system.

1. Find $\overrightarrow{i} \times \overrightarrow{i}$, $\overrightarrow{j} \times \overrightarrow{j}$, and $\overrightarrow{k} \times \overrightarrow{k}$.
2. Find $\overrightarrow{i} \times \overrightarrow{j}$ and $\overrightarrow{j} \times \overrightarrow{i}$.

Ex 7: For $\overrightarrow{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, prove that $\overrightarrow{a} \times \overrightarrow{a} = \overrightarrow{0}$.

Ex 8: For $\overrightarrow{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, prove that $\overrightarrow{a} \times \overrightarrow{b} = -\overrightarrow{b} \times \overrightarrow{a}$.

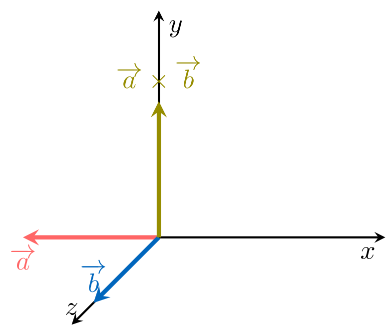
Ex 9: Let $\overrightarrow{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\overrightarrow{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, and $\overrightarrow{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ be three vectors in space. Prove the distributive property of the vector product:

$$\overrightarrow{a} \times (\overrightarrow{b} + \overrightarrow{c}) = (\overrightarrow{a} \times \overrightarrow{b}) + (\overrightarrow{a} \times \overrightarrow{c})$$

B GEOMETRIC INTERPRETATION

B.1 APPLYING THE RIGHT-HAND RULE

MCQ 10: The diagram below illustrates three vectors, $\overrightarrow{a}$, $\overrightarrow{b}$, and their vector product $\overrightarrow{a} \times \overrightarrow{b}$.

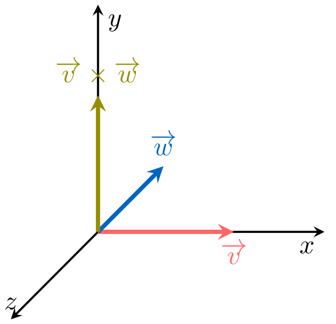


According to the right-hand rule, is the direction of the vector product $\overrightarrow{a} \times \overrightarrow{b}$ correctly illustrated?

☐ Yes

☐ No

MCQ 11: The diagram below illustrates three vectors, $\overrightarrow{v}$, $\overrightarrow{w}$, and their vector product $\overrightarrow{v} \times \overrightarrow{w}$.

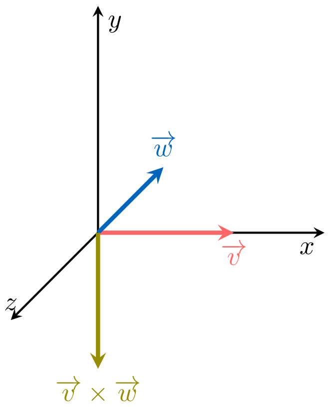


According to the right-hand rule, is the direction of the vector product $\vec{v} \times \vec{w}$ correctly illustrated?

☐ Yes

☐ No

MCQ 12: The diagram below illustrates three vectors, $\vec{v}$, $\vec{w}$, and their vector product $\vec{v} \times \vec{w}$.

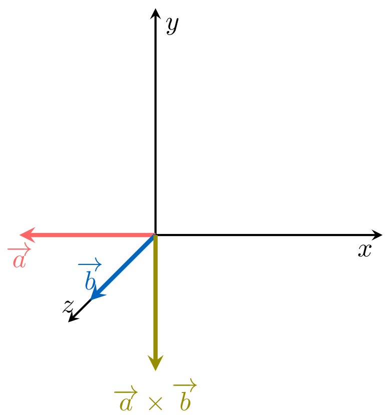


According to the right-hand rule, is the direction of the vector product $\vec{v} \times \vec{w}$ correctly illustrated?

☐ Yes

☐ No

MCQ 13: The diagram below illustrates three vectors, $\vec{a}$, $\vec{b}$, and their vector product $\vec{a} \times \vec{b}$.



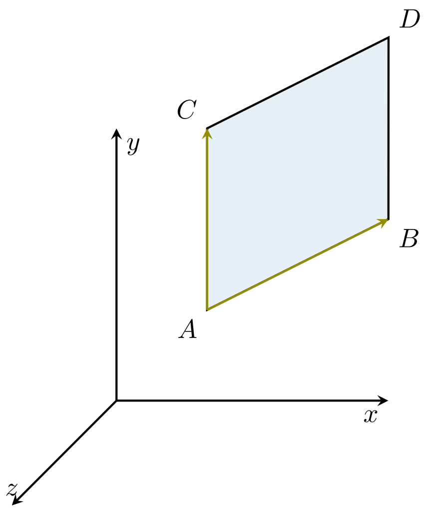
According to the right-hand rule, is the direction of the vector product $\vec{a} \times \vec{b}$ correctly illustrated?

☐ Yes

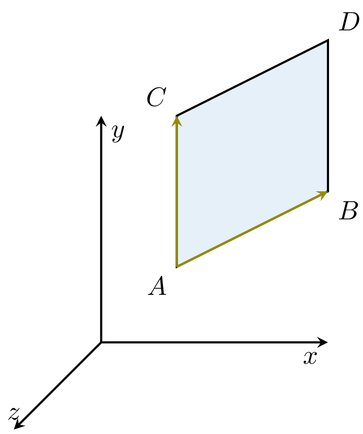
☐ No

B.2 CALCULATING AREA USING THE VECTOR PRODUCT

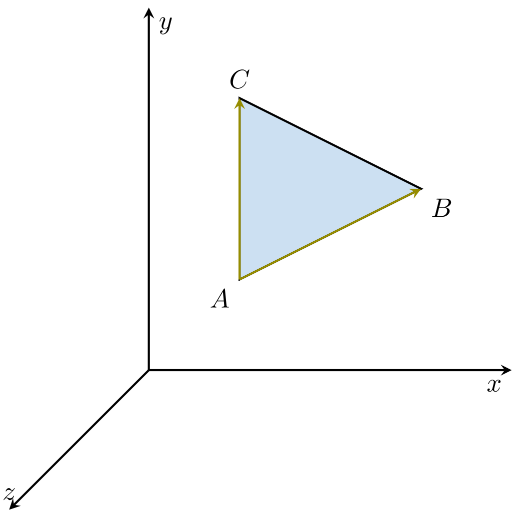
Ex 14: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$. Calculate the area of the parallelogram with adjacent sides $\overline{AB}$ and $\overline{AC}$.



Ex 15: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$. Calculate the area of the parallelogram with adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



Ex 16: Consider the points $A(1,1,1)$, $B(3,2,1)$, and $C(1,3,3)$.
Calculate the area of the triangle ABC.



Ex 17: Consider the points $A(0,0,0)$, $B(-1,2,3)$, and $C(1,2,6)$.
Calculate the area of the triangle ABC.