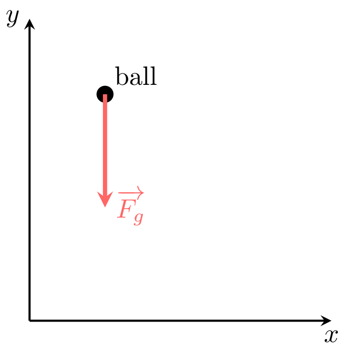
VECTOR APPLICATIONS IN PHYSICS

A NEWTON'S SECOND LAW

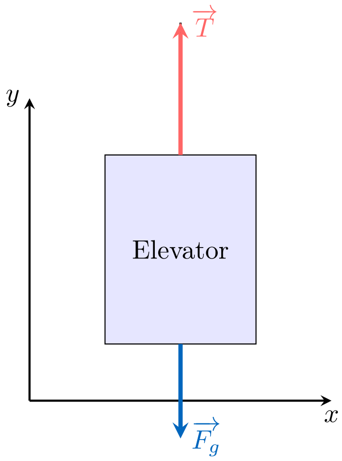
A.1 APPLYING NEWTON'S SECOND LAW

Ex 1: A ball of mass $m = 2$ kg is thrown into the air. Near the Earth's surface, the only significant force acting on it is the force of gravity, $\overrightarrow{F_g}$.



Assuming the y-axis points vertically upwards and the acceleration due to gravity is $g = 9.8$ m/s^2, calculate the gravitational force vector $\overrightarrow{F_g}$ and use Newton's Second Law to find the acceleration vector $\overrightarrow{a}$ of the ball.

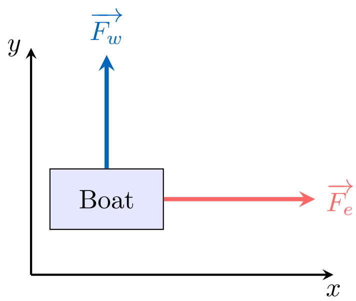
Ex 2: An elevator of mass $m = 500$ kg is supported by a cable that exerts an upward tension force $\overrightarrow{T}$ with a magnitude of 5900 N. The force of gravity, $\overrightarrow{F_g}$, acts downwards. The y-axis is oriented vertically upwards.



Using the acceleration due to gravity $g = 9.8$ m/s^2,

1. Calculate the force of gravity, $F_g = mg$, and write the vectors for the tension force, $\overrightarrow{T}$, and the gravitational force, $\overrightarrow{F_g}$.
2. Calculate the net force $\sum \overrightarrow{F}$ on the elevator.
3. Determine the acceleration vector $\overrightarrow{a}$ of the elevator.

Ex 3: A boat of mass $m = 200$ kg is moving on the surface of a lake. The boat's engine exerts a force $\overrightarrow{F_e}$ of 600 N towards the East (positive x-direction). The wind exerts a force $\overrightarrow{F_w}$ of 200 N towards the North (positive y-direction). Friction is negligible.



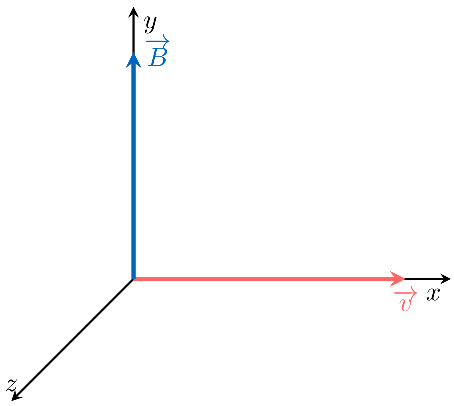
1. Write the column vectors for the engine force, $\overrightarrow{F_e}$, and the wind force, $\overrightarrow{F_w}$.
2. Calculate the net force vector $\sum \overrightarrow{F}$ acting on the boat.
3. Use Newton's Second Law to find the acceleration vector $\overrightarrow{a}$ of the boat.

A.2 APPLYING THE VECTOR PRODUCT IN PHYSICS

Ex 4: The magnetic force $\overrightarrow{F}$ exerted on a particle with electric charge q moving with velocity $\overrightarrow{v}$ through a magnetic field $\overrightarrow{B}$ is given by the Lorentz force formula:

$$\overrightarrow{F} = q(\overrightarrow{v} \times \overrightarrow{B})$$

A particle with charge $q = 2$ C enters a uniform magnetic field $\overrightarrow{B} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$ T with a velocity of $\overrightarrow{v} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ m/s, as shown below.

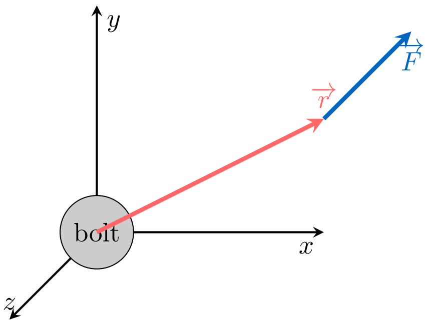


Calculate the magnetic force $\overrightarrow{F}$ acting on the particle. The resulting force will be in Newtons (N).

Ex 5: Torque, or the turning moment of a force, is a measure of how much a force acting on an object causes that object to rotate. The torque vector $\overrightarrow{\tau}$ is calculated using the vector product of the position vector $\overrightarrow{r}$ (from the axis of rotation to the point where the force is applied) and the force vector $\overrightarrow{F}$.

$$\overrightarrow{\tau} = \overrightarrow{r} \times \overrightarrow{F}$$

A force of $\overrightarrow{F} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$ N is applied to a wrench at a position $\overrightarrow{r} = \begin{pmatrix} 0.2 \\ 0.1 \\ 0 \end{pmatrix}$ m relative to the center of a bolt, as shown.

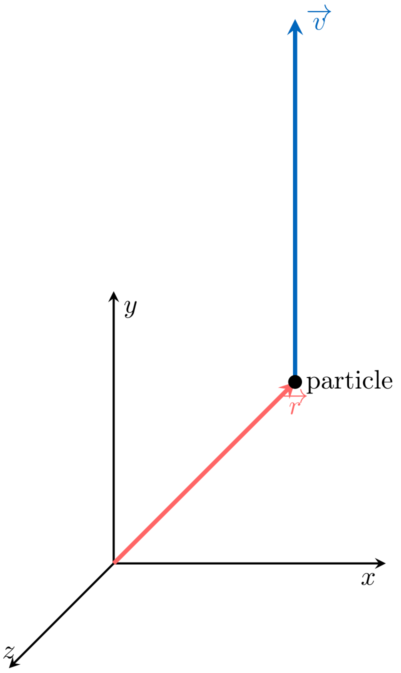


Calculate the torque vector $\overrightarrow{\tau}$ on the bolt. The resulting torque will be in Newton-meters (Nm).

Ex 6: The angular momentum $\overrightarrow{L}$ of a particle relative to a point of origin is a measure of its rotational motion. It is defined by the vector product of the particle's position vector $\overrightarrow{r}$ and its linear momentum vector $\overrightarrow{p}$, where $\overrightarrow{p} = m\overrightarrow{v}$.

$$\overrightarrow{L} = \overrightarrow{r} \times \overrightarrow{p}$$

A particle of mass $m = 3$ kg is at position $\overrightarrow{r} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ m and is moving with a velocity $\overrightarrow{v} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ m/s.

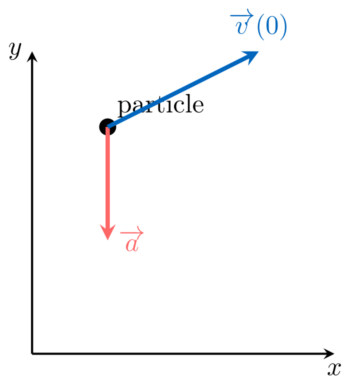


Calculate the angular momentum vector $\vec{L}$ of the particle. The units will be kg·m^2/s.

B VELOCITY AND ACCELERATION WITH CALCULUS

B.1 APPLYING CALCULUS TO VECTOR KINEMATICS

Ex 7: A particle of mass m is launched with an initial velocity of $\vec{v}(0) = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$ m/s. The only force acting on it is gravity, which results in a constant acceleration of $\vec{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ m/s^2.



Find the velocity vector $\vec{v}(t)$ of the particle at time t.

Ex 8: The position vector of a particle at time t seconds is given by:

$$\vec{r}(t) = \begin{pmatrix} 3t^2 - 4t \\ 8t + 1 \end{pmatrix} \text{ m}$$

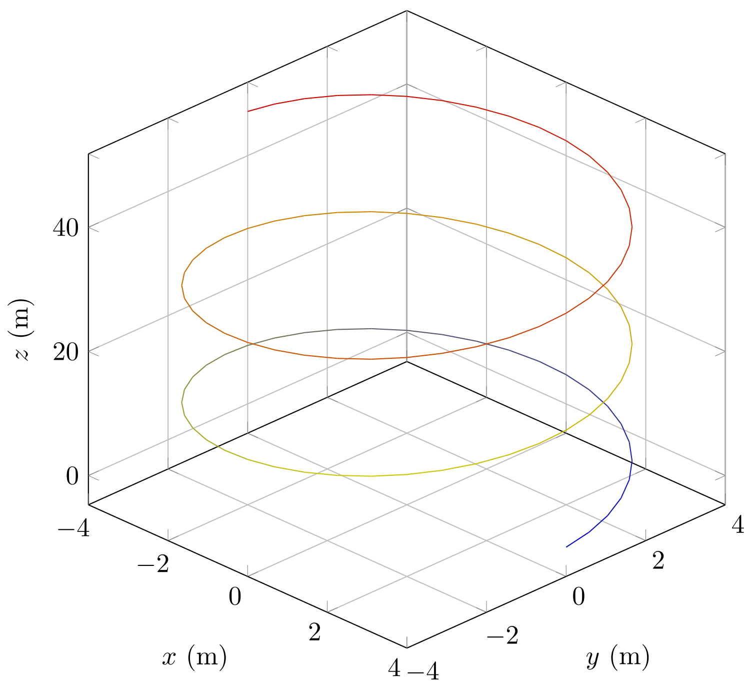
1. Find the velocity vector, $\vec{v}(t)$.
2. Calculate the speed of the particle at time $t = 2$ seconds.

Ex 9: A particle starts from rest at the point $(5, 1)$. Its acceleration vector at time t is given by $\vec{a}(t) = \begin{pmatrix} 3t \\ -2 \end{pmatrix}$ m/s^2.

1. Find the velocity vector, $\vec{v}(t)$.
2. Find the position vector, $\vec{r}(t)$.

Ex 10: The position vector of a particle at time t seconds is given by:

$$\overrightarrow{r}(t) = \begin{pmatrix} 4\cos(t) \\ 4\sin(t) \\ 3t \end{pmatrix} \text{ m}$$



1. Find the velocity vector, $\overrightarrow{v}(t)$.
2. Calculate the speed of the particle at any time t.

C MOTION WITH CONSTANT VELOCITY

C.1 CALCULATING VELOCITY AND SPEED FROM DISPLACEMENT

Ex 11: A helicopter travels at a constant velocity. It is initially at position $A(6, 9, 3)$, and 10 minutes later it is at position $B(3, 10, 2.5)$. Distances are measured in kilometres.

1. Find the velocity vector of the helicopter in km/h.
2. Find the speed of the helicopter in km/h, correct to one decimal place.

Ex 12: A car travels at a constant velocity. At 1:00 PM, its position is $A(15, 20)$. At 1:30 PM, its position is $B(55, -10)$. Distances are measured in kilometres.

1. Find the velocity vector of the car in km/h.
2. Find the speed of the car in km/h.

Ex 13: An airplane travels at a constant velocity. At 8:00 AM, it is at position $A(100, 200, 10)$. At 8:15 AM, it is at position $B(160, 170, 9.5)$. Distances are measured in kilometres.

1. Find the velocity vector of the airplane in km/h.
2. Find the speed of the airplane in km/h, correct to one decimal place.