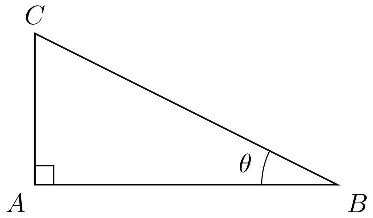
TRIGONOMETRY

A RIGHT-ANGLED TRIANGLE

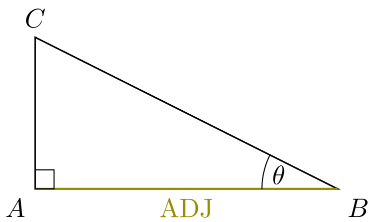
A.1 IDENTIFYING TRIANGLE SIDES

MCQ 1: In the triangle below, identify the adjacent side to the angle θ:



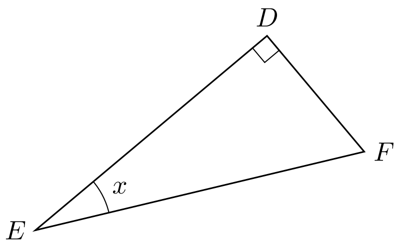
- ☒ $\overline{AB}$
- ☐ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



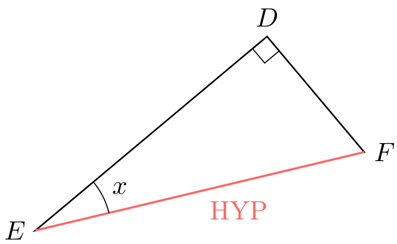
The adjacent side to the angle θ is $\overline{AB}$.

MCQ 2: In the triangle below, identify the hypotenuse relative to the angle x:



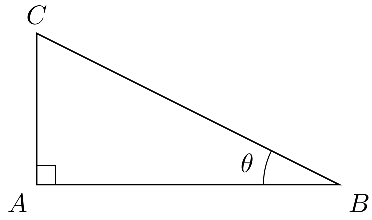
- ☐ $\overline{DE}$
- ☐ $\overline{DF}$
- ☒ $\overline{EF}$

Answer:



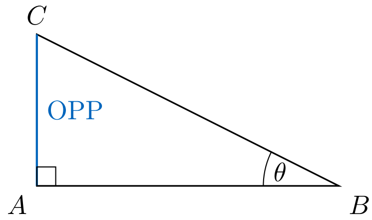
The hypotenuse relative to the angle x is $\overline{EF}$.

MCQ 3: In the triangle below, identify the opposite side to the angle θ:



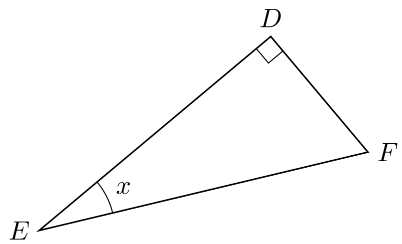
- ☐ $\overline{AB}$
- ☒ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



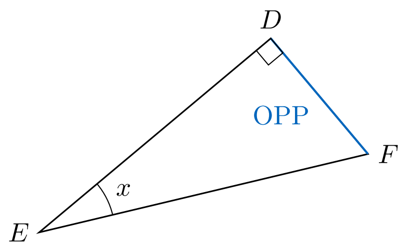
The opposite side to the angle θ is $\overline{AC}$.

MCQ 4: In the triangle below, identify the opposite side to the angle x:



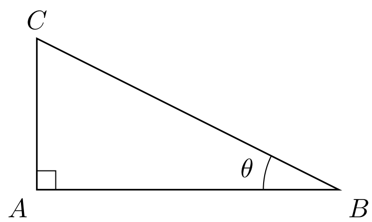
- ☐ $\overline{DE}$
- ☒ $\overline{DF}$
- ☐ $\overline{EF}$

Answer:



The opposite side to the angle x is $\overline{DF}$.

MCQ 5: In the triangle below, identify the hypotenuse relative to the angle θ:

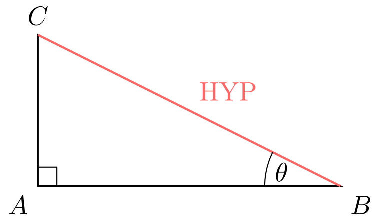


☐ $\overline{AB}$

☐ $\overline{AC}$

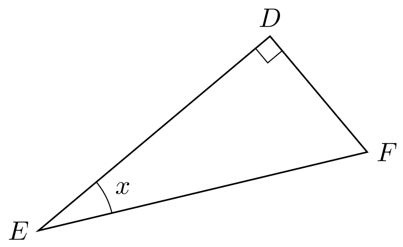
☒ $\overline{BC}$

Answer:



The hypotenuse relative to the angle θ is $\overline{BC}$.

MCQ 6: In the triangle below, identify the adjacent side to the angle x:

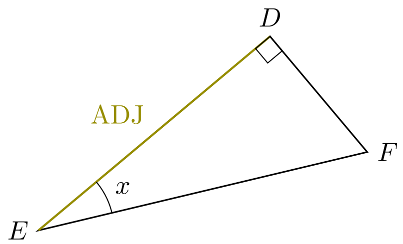


☒ $\overline{DE}$

☐ $\overline{DF}$

☐ $\overline{EF}$

Answer:

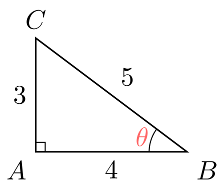


The adjacent side to the angle x is $\overline{DE}$.

B TRIGONOMETRIC FUNCTIONS

B.1 CALCULATING TRIGONOMETRIC RATIOS

Ex 7:



Calculate $\cos(\theta)$.

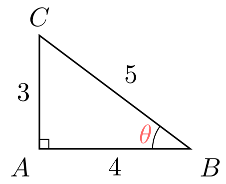
$$\cos(\theta) = \boxed{\frac{4}{5}}$$

Answer: Relative to θ:

- Adjacent side: $AB = 4$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}}\\ &= \frac{4}{5}\end{aligned}$$

Ex 8:



Calculate $\sin(\theta)$.

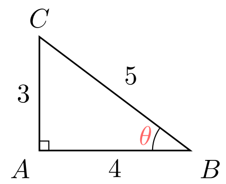
$$\sin(\theta) = \boxed{\frac{3}{5}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}}\\ &= \frac{3}{5}\end{aligned}$$

Ex 9:



Calculate $\tan(\theta)$.

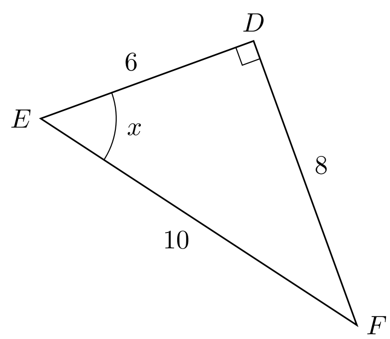
$$\tan(\theta) = \boxed{\frac{3}{4}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Adjacent side: $AB = 4$

$$\begin{aligned}\tan(\theta) &= \frac{\text{OPP}}{\text{ADJ}}\\ &= \frac{3}{4}\end{aligned}$$

Ex 10:



Calculate $\sin(x)$.

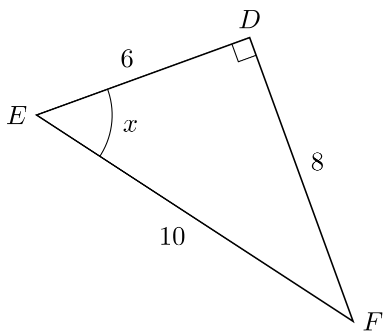
$$\sin(x) = \boxed{\frac{4}{5}}$$

Answer: Relative to x:

- Opposite side: $DF = 8$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\sin(x) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

Ex 11:



Calculate $\tan(x)$.

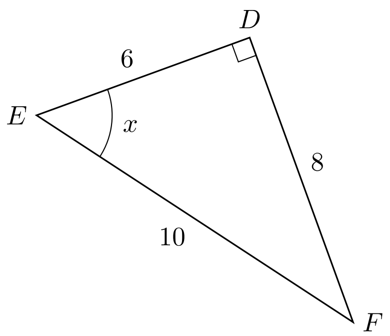
$$\tan(x) = \boxed{\frac{4}{3}}$$

Answer: Relative to x:

- Opposite side: $DF = 8$
- Adjacent side: $DE = 6$

$$\begin{aligned}\tan(x) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{8}{6} \\ &= \frac{4}{3}\end{aligned}$$

Ex 12:



Calculate $\cos(x)$.

$$\cos(x) = \boxed{\frac{3}{5}}$$

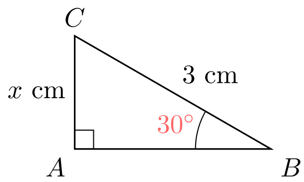
Answer: Relative to x:

- Adjacent side: $DE = 6$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\cos(x) &= \frac{\text{ADJ}}{\text{HYP}} \\ &= \frac{6}{10} \\ &= \frac{3}{5}\end{aligned}$$

B.2 CALCULATING SIDE LENGTHS

Ex 13:



Calculate x.

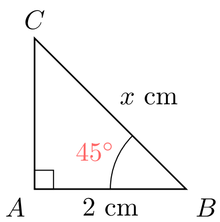
$$x \approx \boxed{1.50} \text{ cm (round to 2 decimal places)}$$

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = x$
- Hypotenuse: $BC = 3$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}} \\ \sin(30°) &= \frac{x}{3} \\ x &= 3 \times \sin(30°) \\ x &= 1.50\,\text{cm}\end{aligned}$$

Ex 14:



Calculate x.

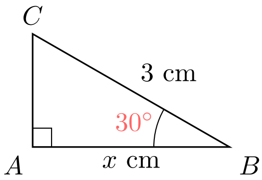
$$x \approx \boxed{2.83} \text{ cm (round to 2 decimal places)}$$

Answer: Relative to $\theta = 45°$:

- Adjacent side: $AB = 2$
- Hypotenuse: $BC = x$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}} \\ \cos(45°) &= \frac{2}{x} \\ x &= \frac{2}{\cos(45°)} \\ x &\approx 2.83\,\text{cm} \quad \text{(round to 2 decimal places)}\end{aligned}$$

Ex 15:



Calculate x.

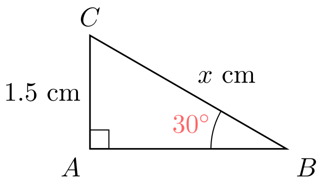
$x \approx$ [2.60] cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Adjacent side: $AB = x$
- Hypotenuse: $BC = 3$

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}}$$
$$\cos(30°) = \frac{x}{3}$$
$$x = 3 \times \cos(30°)$$
$$x \approx 2.60\,\text{cm} \quad \text{(round to 2 decimal places)}$$

Ex 16:



Calculate x.

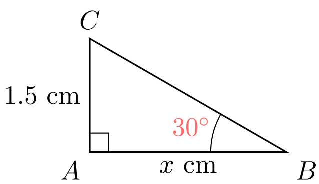
$x \approx$ [3.00] cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Hypotenuse: $BC = x$

$$\sin(\theta) = \frac{\text{OPP}}{\text{HYP}}$$
$$\sin(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\sin(30°)}$$
$$x = 3.00\,\text{cm}$$

Ex 17:



Calculate x.

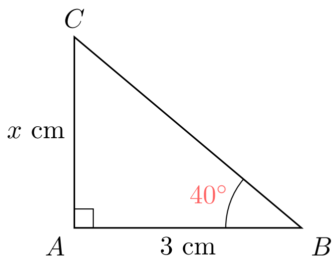
$x \approx$ [2.60] cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Adjacent side: $AB = x$

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\tan(30°)}$$
$$x \approx 2.60\,\text{cm} \quad \text{(round to 2 decimal places)}$$

Ex 18:



Calculate x.

$x \approx$ [2.52] cm (round to 2 decimal places)

Answer: Relative to $\theta = 40°$:

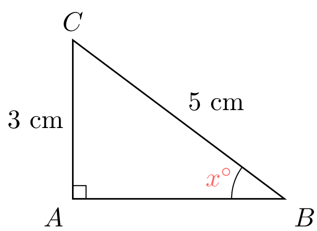
- Opposite side: $AC = x$
- Adjacent side: $AB = 3$

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(40°) = \frac{x}{3}$$
$$x = 3 \times \tan(40°)$$
$$x \approx 2.52\,\text{cm} \quad \text{(round to 2 decimal places)}$$

C INVERSE TRIGONOMETRIC FUNCTIONS

C.1 CALCULATING ANGLES

Ex 19:



Calculate the angle $x°$.

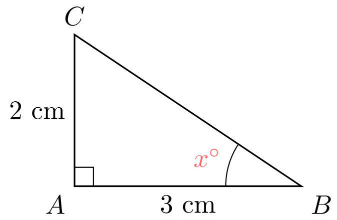
$x° \approx$ [36.9]° (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 5$ cm

$$
\begin{aligned}
x^\circ &= \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right)\\
&= \sin^{-1}\left(\frac{3}{5}\right)\\
&\approx 36.9^\circ \quad \text{(round to 1 decimal place)}
\end{aligned}
$$

Ex 20:



Calculate the angle x°.

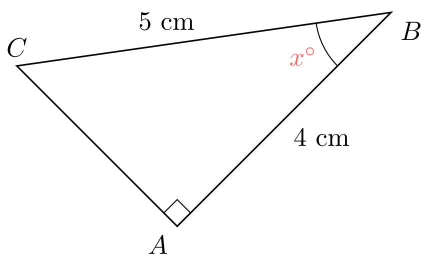
$x^\circ \approx \boxed{33.7}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 2$ cm
- Adjacent side: $AB = 3$ cm

$$
\begin{aligned}
x^\circ &= \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right)\\
&= \tan^{-1}\left(\frac{2}{3}\right)\\
&\approx 33.7^\circ \quad \text{(round to 1 decimal place)}
\end{aligned}
$$

Ex 21:



Calculate the angle x°.

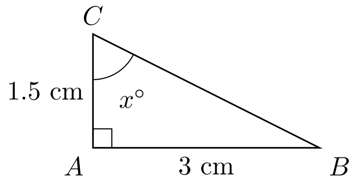
$x^\circ \approx \boxed{36.9}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Adjacent side: $AB = 4$ cm
- Hypotenuse: $BC = 5$ cm

$$
\begin{aligned}
x^\circ &= \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right)\\
&= \cos^{-1}\left(\frac{4}{5}\right)\\
&\approx 36.9^\circ \quad \text{(round to 1 decimal place)}
\end{aligned}
$$

Ex 22:



Calculate the angle x°.

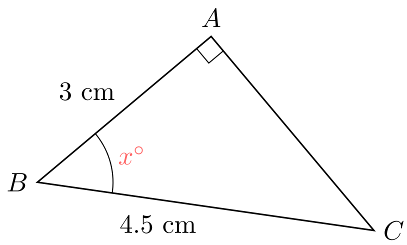
$x^\circ \approx \boxed{63.4}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AB = 3$ cm
- Adjacent side: $AC = 1.5$ cm

$$
\begin{aligned}
x^\circ &= \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right)\\
&= \tan^{-1}\left(\frac{3}{1.5}\right)\\
&\approx 63.4^\circ \quad \text{(round to 1 decimal place)}
\end{aligned}
$$

Ex 23:



Calculate the angle x°.

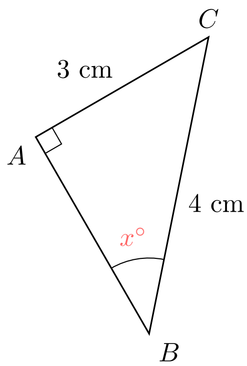
$x^\circ \approx \boxed{48.2}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Adjacent side: $AB = 3$ cm
- Hypotenuse: $BC = 4.5$ cm

$$
\begin{aligned}
x^\circ &= \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right)\\
&= \cos^{-1}\left(\frac{3}{4.5}\right)\\
&\approx 48.2^\circ \quad \text{(round to 1 decimal place)}
\end{aligned}
$$

Ex 24:



Calculate the angle x°.

$x^\circ \approx \boxed{48.6}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 4$ cm

$$\begin{aligned} x^\circ &= \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right) \\ &= \sin^{-1}\left(\frac{3}{4}\right) \\ &\approx 48.6^\circ \quad \text{(round to 1 decimal place)} \end{aligned}$$

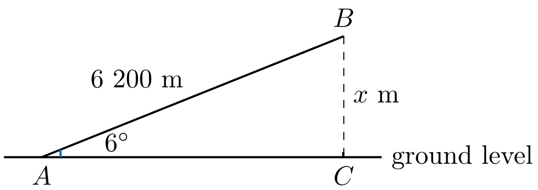
D SOLVING REAL-WORLD TRIGONOMETRY PROBLEMS

D.1 SOLVING REAL-WORLD TRIGONOMETRY PROBLEMS

Ex 25: A cyclist in France rides up a long incline with an average rise of 6°. If he rides for 6 200 m, how far has he climbed vertically?

$\boxed{648}$ m (round to the nearest integer)

Answer:



The cyclist rides 6.2 km (6200 m) up an incline with an angle of 6°. This forms a right triangle ABC, with the right angle at C, hypotenuse $AB = 6200$ m, and the vertical height $BC = x$.
Applying the sine definition:

- Hypotenuse: $AB = 6200$ m
- Opposite side: $BC = x$
- Angle: 6°

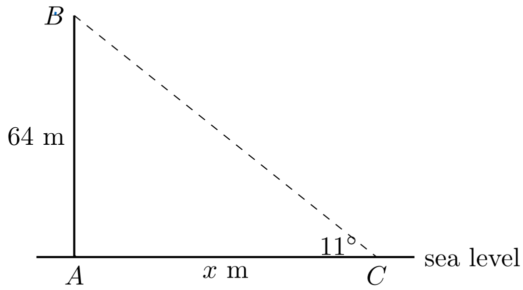
$$\begin{aligned} \sin(6^\circ) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{x}{6200} \\ x &= 6200 \times \sin(6^\circ) \\ &\approx 648\,\text{m} \quad \text{(round to the nearest integer)} \end{aligned}$$

Thus, the cyclist has climbed a vertical height of approximately 648 m.

Ex 26: The lamp in a lighthouse is 64 m above sea level. The angle of depression from the lamp to a fishing boat is 11°. How far horizontally is the boat from the lighthouse?

$\boxed{339}$ m (round to the nearest integer)

Answer:



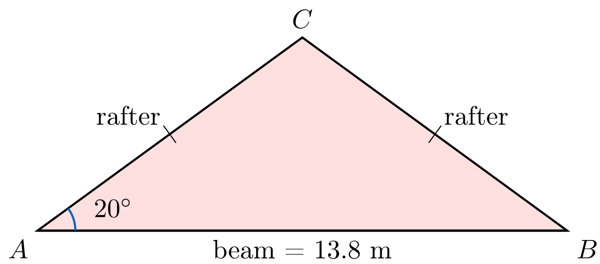
The lighthouse lamp (B) is 64 m above sea level (A). The angle of depression from B to the fishing boat (C) is 11°, which matches the angle of elevation from C to B.
This forms a right triangle ABC with the right angle at A, vertical side $AB = 64$ m, and horizontal side $AC = x$.

- Opposite side (to 11°): $AB = 64$ m
- Adjacent side: $AC = x$
- Angle: 11°

$$\begin{aligned} \tan(11^\circ) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{64}{x} \\ x &= \frac{64}{\tan(11^\circ)} \\ &\approx 339\,\text{m} \quad \text{(round to the nearest integer)} \end{aligned}$$

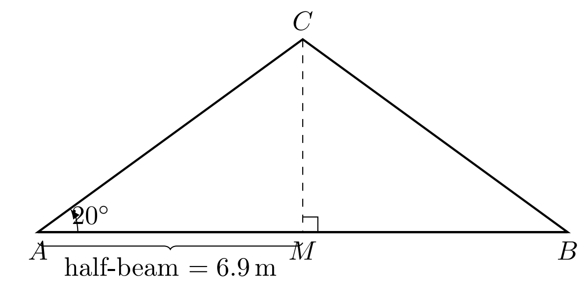
Thus, the horizontal distance from the boat to the lighthouse is approximately 339 m.

Ex 27: For the triangular roof truss illustrated, find the length of a rafter if the beam is 13.8 m and the pitch is 20°.



$\boxed{7.34}$ m (round to 2 decimal places)

Answer: Because the roof truss is isosceles, dropping a perpendicular from the ridge to the midpoint of the beam forms a right triangle whose hypotenuse is the rafter. Applying the cosine definition:

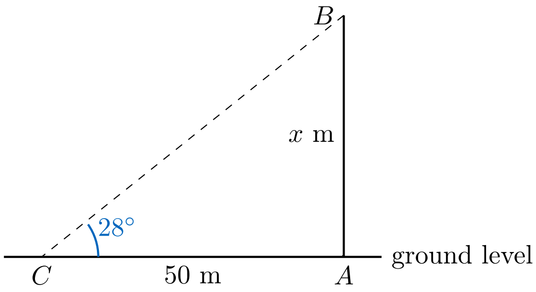


$$\begin{aligned}\cos(20°) &= \frac{\text{adjacent (half-beam)}}{\text{hypotenuse (rafter)}}\\ \text{rafter} &= \frac{\text{half-beam}}{\cos(20°)}\\ &= \frac{13.8/2}{\cos(20°)}\\ &\approx 7.34\,\text{m} \quad \text{(round to 2 decimal places)}\end{aligned}$$

Ex 28: A person standing 50 m from the base of a tower looks up at the top with an angle of elevation of 28°. Find the height of the tower.

[27] m (round to the nearest integer)

Answer:



The tower is vertical from base A to top B. The person at C is 50 m from A, with an angle of elevation of 28° from C to B. This forms a right triangle CAB with the right angle at A, opposite side $AB = x$ (height), adjacent side $CA = 50$ m.

- Opposite side (to 28°): $AB = x$
- Adjacent side: $CA = 50$ m
- Angle: 28°

$$\begin{aligned}\tan(28°) &= \frac{\text{OPP}}{\text{ADJ}}\\ &= \frac{x}{50}\\ x &= 50 \times \tan(28°)\\ &\approx 27\,\text{m} \quad \text{(round to the nearest integer)}\end{aligned}$$

Thus, the height of the tower is approximately 27 m.