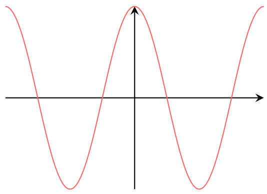
TRIGONOMETRIC FUNCTIONS

A PERIODIC FUNCTION

A.1 IDENTIFYING PERIODIC BEHAVIOUR FROM A GRAPH

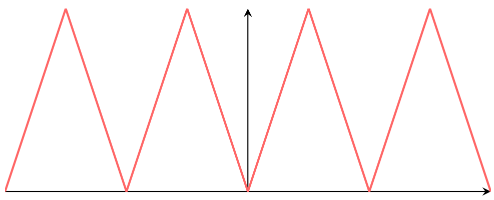
MCQ 1: Is the function shown in the graph below periodic?



☐ Yes

☐ No

MCQ 2: Is the function shown in the graph below periodic?

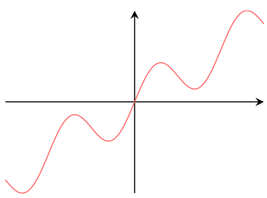


☐ Yes

☐ No

MCQ 3: Is the function shown in the graph below periodic?

$$f(x) = \sin(x) + \frac{x}{3}$$

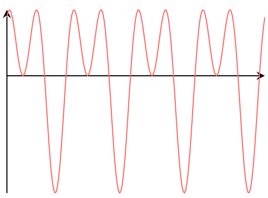


☐ Yes

☐ No

MCQ 4: Is the function shown in the graph below periodic?

$$g(x) = \sin(2x) + \cos(4x)$$

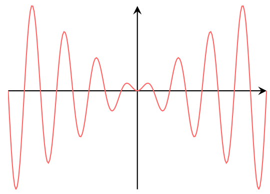


☐ Yes

☐ No

MCQ 5: Is the function shown in the graph below periodic?

$$h(x) = x\sin(2x)$$

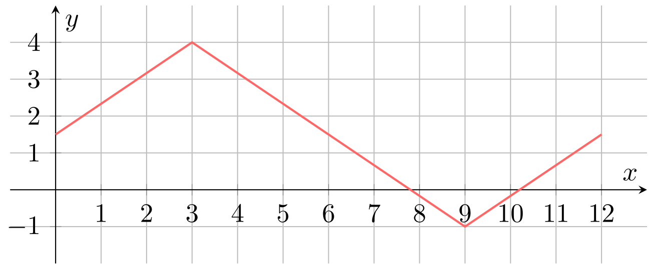


☐ Yes

☐ No

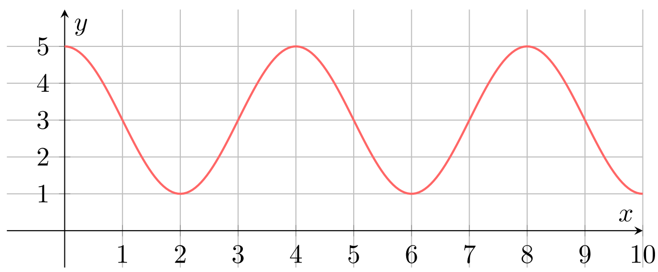
A.2 IDENTIFYING PROPERTIES OF PERIODIC FUNCTIONS

Ex 6: For the periodic function shown below, find:



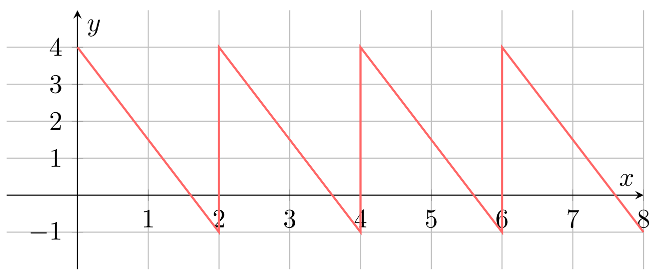
1. The period is ☐
2. The equation of the principal axis is $y =$ ☐
3. The amplitude is ☐

Ex 7: For the periodic function shown below, find:



1. The period is ☐
2. The equation of the principal axis is $y =$ ☐
3. The amplitude is ☐

Ex 8: For the periodic function shown below, find:



1. The period is ☐
2. The equation of the principal axis is $y =$ ☐
3. The amplitude is ☐

B SINE AND COSINE FUNCTION

B.1 COMPLETING TABLES OF VALUES

Ex 9: For $f(x) = \sin(x)$, complete the table of values for the multiples of $\frac{\pi}{8}$ (rounded to 2 decimal places):

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$\sin(x)$					

Ex 10: Complete the table of values for the multiples of $\frac{\pi}{6}$ (rounded to 2 decimal places):

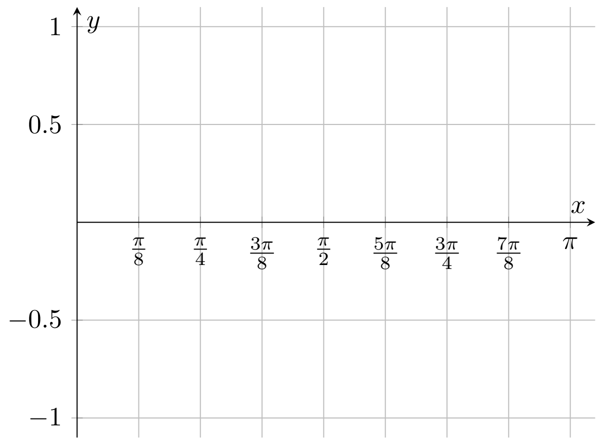
x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
$\cos(x)$						

B.2 PLOTTING GRAPHS

Ex 11: Here is a table of values for the function $f(x) = \sin(x)$:

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π
$\sin(x)$	0	0.38	0.71	0.92	1.00	0.92	0.71	0.38	0

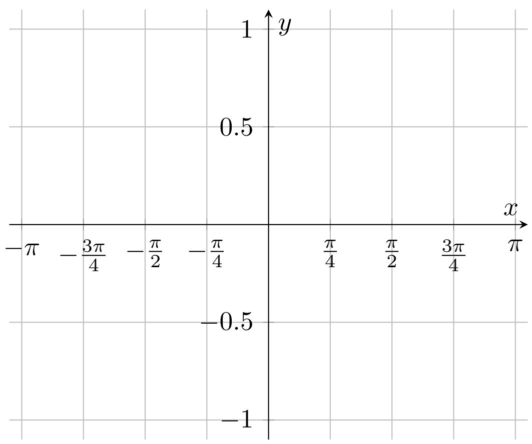
Plot the graph of the function.



Ex 12: Here is a table of values for the function $f(x) = \sin(x)$:

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin(x)$	0	−0.71	−1.00	−0.71	0	0.71	1	0.71

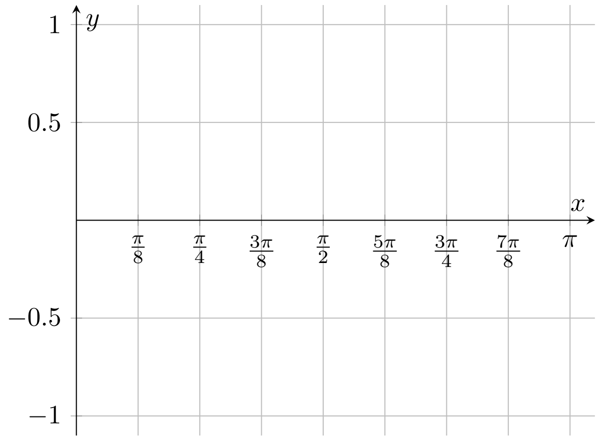
Plot the graph of the function on the interval $[-\pi\,;\,\pi]$:



Ex 13: Here is a table of values for the function $f(x) = \cos(x)$:

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$
$\cos(x)$	1	0.92	0.71	0.38	0	−0.38	−0.71	−0.92

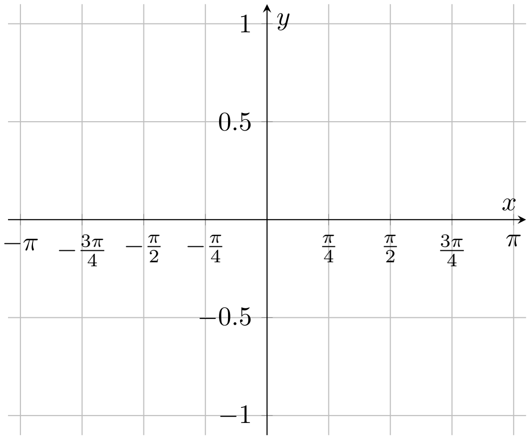
Plot the graph of the function.



Ex 14: Here is a table of values for the function $f(x) = \cos(x)$:

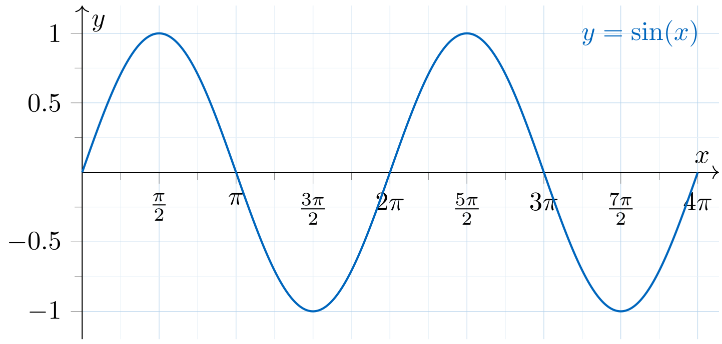
x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\cos(x)$	−1	−0.71	0	0.71	1	0.71	0	−0.71

Plot the graph of the function on the interval $[-\pi\,;\,\pi]$:



B.3 READING GRAPHS

Ex 15: Below is the graph of the function $y = \sin(x)$, for $0 \leq x \leq 4\pi$.



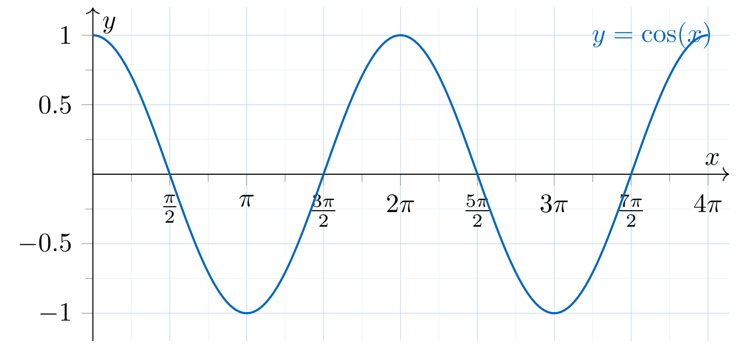
1. Find the y**-intercept** of the graph.

   $(0,\ \square)$

2. Use the graph to determine the values of x in the interval $0 \leq x \leq 4\pi$ such that $\sin(x) = 1$:

$\square$, $\square$

Ex 16: Below is the graph of the function $y = \cos(x)$, for $0 \leq x \leq 4\pi$.



1. Find the y-**intercept** of the graph.

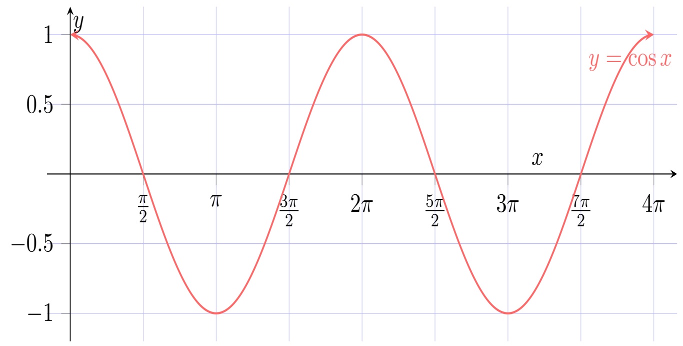
$(0, \square\ )$

2. Use the graph to determine the values of x in the interval $0 \leq x \leq 4\pi$ such that $\cos(x) = 0$:

$\square$, $\square$, $\square$, $\square$

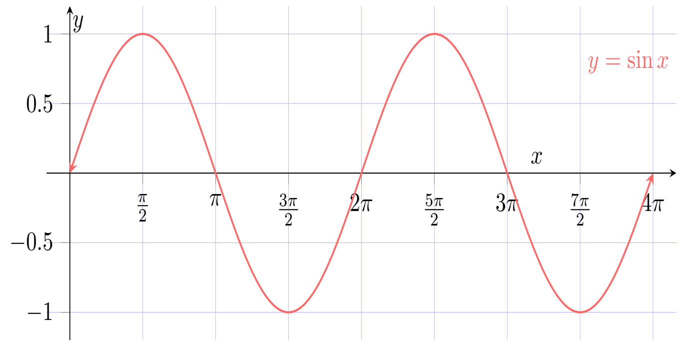
B.4 READING KEY FEATURES FROM A GRAPH

Ex 17: Below is an accurate graph of the function $y = \cos(x)$, for $0 \leq x \leq 4\pi$.



1. Find the y-intercept of the graph.
2. Find the values of x on $0 \leq x \leq 4\pi$ for which:
   (a) $\cos x = 1$
   (b) $\cos x = 0$
3. Find the intervals on $0 \leq x \leq 4\pi$ where $\cos x$ is:
   (a) non negative.
   (b) non positive
4. Find the range of the function.

Ex 18: Below is an accurate graph of the function $y = \sin(x)$, for $0 \leq x \leq 4\pi$.



1. Find the y-intercept of the graph.
2. Find the values of x on $0 \leq x \leq 4\pi$ for which:
   (a) $\sin x = 1$
   (b) $\sin x = 0$
3. Find the intervals on $0 \leq x \leq 4\pi$ where $\sin x$ is:
   (a) non-negative
   (b) non-positive.
4. Find the range of the function.

C GENERAL SINE AND COSINE FUNCTIONS

C.1 IDENTIFYING PROPERTIES FROM AN EQUATION

Ex 19: For the function $y = 4\cos(x) - 2$, state:

1. The amplitude. ☐
2. The period. ☐
3. The phase shift. ☐
4. The principal axis. $y =$ ☐

Ex 20: For the function $y = 2\cos(3x) + 1$, state:

1. The amplitude. ☐
2. The period. ☐
3. The phase shift. ☐
4. The principal axis. $y =$ ☐

Ex 21: For the function $y = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$, state:

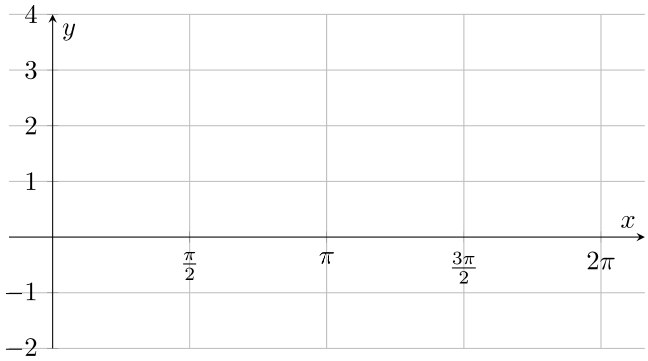
1. The amplitude. ☐
2. The period. ☐
3. The phase shift. ☐
4. The principal axis. $y =$ ☐

Ex 22: For the function $y = -5\sin(3x + \pi) + 7$, state:

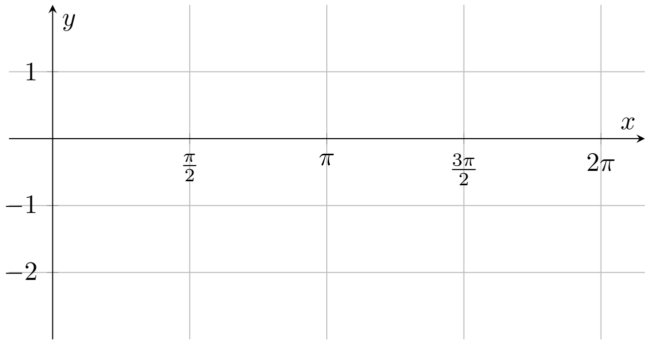
1. The amplitude. ☐
2. The period. ☐
3. The phase shift. ☐
4. The principal axis. $y =$ ☐

C.2 SKETCHING TRANSFORMED FUNCTIONS

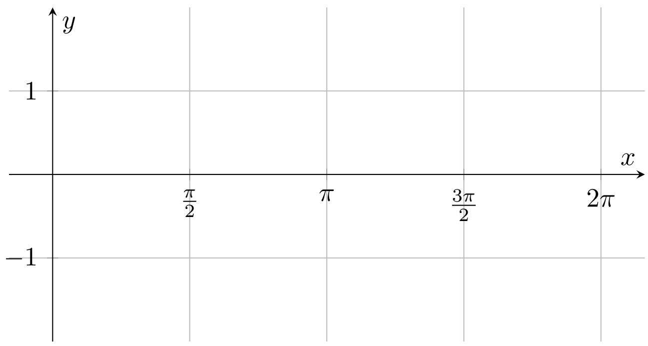
Ex 23: Sketch the graph of $y = 2\cos(x) + 1$ for $0 \leq x \leq 2\pi$.



Ex 24: Sketch the graph of $y = \sin(2x) - 1$ for $0 \leq x \leq 2\pi$.

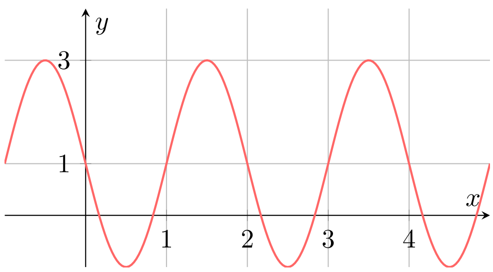


Ex 25: Sketch the graph of $y = \cos\left(x - \frac{\pi}{2}\right)$ for $0 \leq x \leq 2\pi$.



C.3 FINDING THE EQUATION FROM A GRAPH

MCQ 26: Which of the following equations best describes the graph shown below?



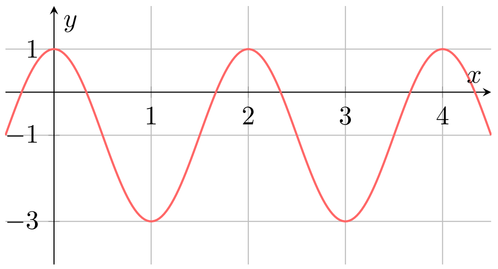
☐ $y = 2\sin(\pi(x - 1)) + 1$

☐ $y = 2\sin(2\pi(x - 1)) + 1$

☐ $y = 3\sin(\pi(x - 1)) - 1$

☐ $y = \sin(\pi(x + 1)) + 2$

MCQ 27: Which of the following equations best describes the graph shown below?

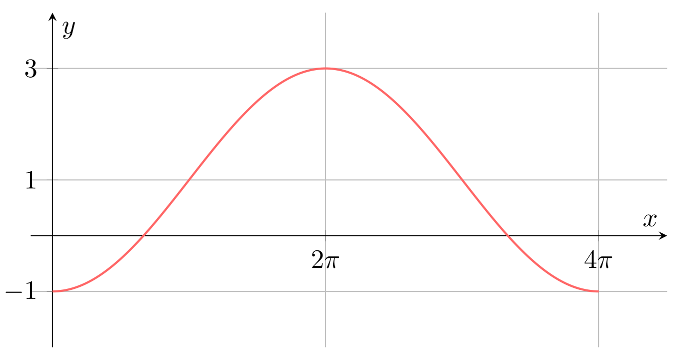


- ☐ $y = 2\cos(\pi x) + 1$
- ☐ $y = \cos(2\pi x) - 1$
- ☐ $y = 2\cos(\pi x) - 1$
- ☐ $y = 2\cos(x) - 1$

MCQ 28: Which of the following equations best describes the graph shown below?

- ☐ $y = 2\sin(\frac{\pi}{2}x) + 2$
- ☐ $y = -2\sin(\pi x) + 2$
- ☐ $y = -4\sin(\frac{\pi}{2}x) + 2$
- ☐ $y = -2\sin(\frac{\pi}{2}x) + 2$

MCQ 29: Which of the following equations best describes the graph shown below?



- ☐ $y = 2\cos(2x) + 1$
- ☐ $y = -2\cos(x) + 1$
- ☐ $y = 2\cos(0.5x) - 1$
- ☐ $y = -2\cos(0.5x) + 1$

D TANGENT FUNCTION

D.1 GRAPHING THE TANGENT FUNCTION FROM VALUES

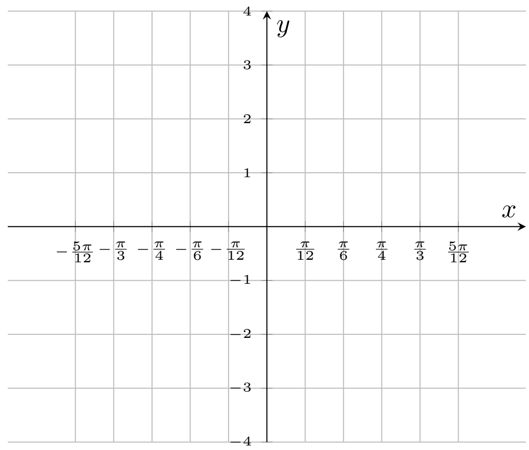
Ex 30: For $f(x) = \tan(x)$, complete the table of values (rounded to 2 decimal places).

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan(x)$				

Ex 31: Here is a table of values for the function $f(x) = \tan(x)$ (rounded to 2 decimal places):

x	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$
$\tan(x)$	-3.73	-1.73	-1.00	-0.58	-0.27	0	0.27	0.58	1.00	1.73	3.73

Plot the graph of the function on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.



E RECIPROCAL TRIGONOMETRIC FUNCTIONS

E.1 FINDING DOMAINS AND ASYMPTOTES

MCQ 32: The function $y = \sec(x)$ is undefined for which of the following values?

- ☐ $x = 0$
- ☐ $x = \pi$
- ☐ $x = \frac{\pi}{2}$
- ☐ $x = \frac{\pi}{4}$

MCQ 33: The function $y = \csc(x)$ is undefined for which of the following values?

- ☐ $x = \frac{\pi}{2}$
- ☐ $x = \frac{3\pi}{2}$
- ☐ $x = \frac{\pi}{4}$
- ☐ $x = \pi$

MCQ 34: The function $y = \cot(x)$ has the same vertical asymptotes as which other function?

- ☐ $y = \sin(x)$
- ☐ $y = \csc(x)$
- ☐ $y = \cos(x)$
- ☐ $y = \sec(x)$

E.2 SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

Ex 35: Express the function $f(x) = \frac{1}{\csc(x)}$ in terms of a primary trigonometric function.

$$f(x) = \square$$

Ex 36: Express the function $f(x) = \tan(x) \cdot \sec(x)$ in terms of sine and cosine.

$$f(x) = \square$$

Ex 37: Express $\sec^2(x)$ in terms of $\tan^2(x)$.

$$\sec^2(x) = \square$$

MCQ 38: The expression $\sin(x) \cdot \cot(x)$ simplifies to:

- ☐ $\sin^2(x)$
- ☐ $\cos^2(x)$
- ☐ $\cos(x)$
- ☐ 1

E.3 EVALUATING RECIPROCAL FUNCTIONS

Ex 39: 🖩 Find the exact value of $\cot(\frac{\pi}{6})$.

$$\cot(\tfrac{\pi}{6}) = \square$$

Ex 40: 🖩 Find the exact value of $\sec(\pi)$.

$$\sec(\pi) = \square$$

Ex 41: 🖩 Find the exact value of $\csc(\frac{3\pi}{2})$.

$$\csc(\tfrac{3\pi}{2}) = \square$$

Ex 42: 🖩 Find the exact value of $\sec\left(\frac{5\pi}{4}\right)$.

$$\sec\left(\tfrac{5\pi}{4}\right) = \square$$

F INVERSE TRIGONOMETRIC FUNCTIONS

F.1 EVALUATING INVERSE TRIGONOMETRIC FUNCTIONS AT SPECIAL ANGLES

Ex 43: Find the angle in radians:

$$\cos^{-1}(1) = \square$$

Ex 44: Find the angle in radians:

$$\sin^{-1}(1) = \square$$

Ex 45: Find the angle in radians:

$$\sin^{-1}\left(\tfrac{1}{2}\right) = \square$$

Ex 46: Find the angle in radians:

$$\cos^{-1}\left(\tfrac{1}{2}\right) = \square$$

Ex 47: Find the angle in radians:

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \square$$

Ex 48: Find the angle in radians:

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \square$$

Ex 49: Find the angle in radians:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \square$$

Ex 50: Find the angle in radians:

$$\tan^{-1}(1) = \square$$

Ex 51: Find the angle in radians:

$$\tan^{-1}\left(\sqrt{3}\right) = \square$$

Ex 52: Find the angle in radians:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \square$$

F.2 SIMPLIFYING EXPRESSIONS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Ex 53: Simplify:

$$\arccos\left(\cos\left(-\tfrac{\pi}{4}\right)\right) = \square$$

Ex 54: Simplify:

$$\arccos\left(\sin\left(\tfrac{2\pi}{3}\right)\right) = \square$$

Ex 55: Simplify:

$$\arctan\left(\cos\left(4\pi\right)\right) = \square$$

Ex 56: Simplify:

$$\arccos\left(\sin\left(\tfrac{\pi}{3}\right)\right) = \square$$

Ex 57: Simplify:

$$\arcsin\left(\cos\left(\tfrac{\pi}{6}\right)\right) = \square$$

Ex 58: Simplify:

$$\arctan\left(-\tan\left(\tfrac{\pi}{6}\right)\right) = \square$$

G SOLVING TRIGONOMETRIC EQUATIONS

G.1 SOLVING BASIC TRIGONOMETRIC EQUATIONS

Ex 59: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$x = \boxed{} < x = \boxed{}$$

Ex 60: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$\sin x = \frac{1}{2}$$

$$x = \boxed{} < x = \boxed{}$$

Ex 61: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \boxed{} < x = \boxed{}$$

Ex 62: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$2\cos x = 1$$

$$x = \boxed{} < x = \boxed{}$$

G.2 SOLVING EQUATIONS OF QUADRATIC FORM

Ex 63: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$\sin^2 x = \frac{1}{2}$$

Ex 64: Solve for x on the domain $0 \leq x \leq 2\pi$:

$$\sin^2 x = \frac{3}{4}$$

G.3 SOLVING EQUATIONS WITH TRANSFORMED ARGUMENTS

Ex 65: Consider the solution of trigonometric equations.

1. Find all solutions to the equation $\sin(x) = \frac{\sqrt{2}}{2}$ on the domain $0 \leq x \leq 2\pi$.
2. Hence, find all solutions to the equation $\sin(2x) = \frac{\sqrt{2}}{2}$ on the domain $0 \leq x \leq \pi$.

Ex 66: Consider the solution of trigonometric equations.

1. Find all solutions to the equation $\cos(x) = -\frac{1}{2}$ on the domain $0 \leq x \leq 2\pi$.
2. Hence, find all solutions to the equation $\cos(x - \frac{\pi}{3}) = -\frac{1}{2}$ on the domain $0 \leq x \leq 2\pi$.

Ex 67: Solve for x on the domain $0 \leq x < 2\pi$:

$$\cos\left(x - \frac{\pi}{5}\right) = 0$$

Ex 68: Consider the solution of trigonometric equations.

1. Find all solutions to the equation $\tan(x) = 1$ on the domain $0 \leq x \leq \pi$.

2. Hence, find all solutions to the equation $\tan(2x) = 1$ on the domain $0 \leq x \leq 2\pi$.

Ex 69: Consider the solution of trigonometric equations.

1. Find all solutions to the equation $\cos(x) = \frac{1}{2}$ on the domain $0 \leq x \leq 2\pi$.

2. Hence, find all solutions to the equation $\cos(\frac{x}{2}) = \frac{1}{2}$ on the domain $0 \leq x \leq 4\pi$.

H MODELING PERIODIC DATA WITH A SINE FUNCTION

H.1 MODELING REAL-WORLD PHENOMENA

Ex 70: The horizontal displacement, D cm, of the bob of a pendulum from its central position is modelled by a sine function of time, t seconds. The bob is released from its maximum displacement of 10 cm at $t = 0.25$ seconds. It swings

to a minimum displacement of -10 cm and first returns to its maximum displacement at $t = 1.25$ seconds.
Find a sine function of the form $D(t) = a\sin(b(t-c)) + d$ to model this motion.

Ex 71: The height H (in metres) of a rider on a Ferris wheel after t seconds is recorded. The wheel rotates at a constant speed. The maximum height is 25 metres and the minimum height is 1 metre. The wheel completes one full revolution every 20 seconds. At $t = 0$, the rider is at the bottom of the wheel.
Find a cosine function of the form $H(t) = a\cos(b(t-c)) + d$ to model the rider's height.

Ex 72: The depth of water, D metres, in a harbour can be modelled by a sinusoidal function of time, t hours after midnight. The depth has a maximum of 14m at 3:00 am and a minimum of 2m at 9:00 am.
Find a cosine function of the form $D(t) = a\cos(b(t-c)) + d$ to model the water depth.