SQUARE ROOTS
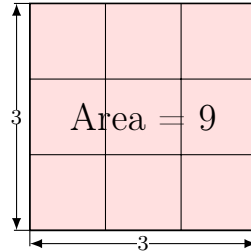
A WHAT ARE SQUARE ROOTS?
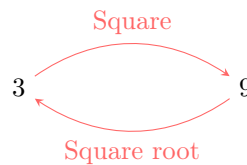
Discover:

- When we **square** a number, we multiply it by itself. For example, 3 squared is 3×3, which we write as 3^2.
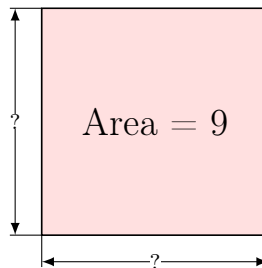

$3^2 = 9$. The area of a square with side length 3 is 9 square units.

- The **square root** is the inverse operation: it undoes squaring.

It finds the side length of a square, given its area. If a square has an area of 9, what is its side length?

The side length is $\sqrt{9} = 3$, because $3 \times 3 = 9$.

Definition Square root

The **square root** of a non-negative number a (that is, $a \geq 0$), written as $\sqrt{a}$, is the **non-negative number** that, when multiplied by itself, gives a.
$$\left(\sqrt{a}\right)^2 = a$$

Note

- The square root symbol $\sqrt{}$ always asks for the **positive** root. For example, $\sqrt{25} = 5$. It is a common mistake to think that $\sqrt{25}$ is ± 5.
  While it's true that both $5^2 = 25$ and $(-5)^2 = 25$, the symbol $\sqrt{25}$ refers only to the positive solution, which is 5.

- **Why can't we take the square root of a negative number (in the real numbers)?**
  Consider $\sqrt{-9}$. To find this value, we need a number that, when multiplied by itself, gives -9.

  - A positive number squared is positive ($3 \times 3 = 9$).
  - A negative number squared is also positive ($-3 \times -3 = 9$).

  No real number, when squared, can result in a negative number. Therefore, we cannot find the square root of a negative number in the set of real numbers.

> **Definition Perfect Squares**
>
> A **perfect square** is an integer that is the square of another integer. The square root of a perfect square is an integer.

Ex: The first few perfect squares are:
$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots$$

Their square roots are:
$$\sqrt{1} = 1, \quad \sqrt{4} = 2, \quad \sqrt{9} = 3, \quad \sqrt{16} = 4, \quad \ldots$$

B CALCULATING SQUARE ROOTS

While the square roots of perfect squares are easy to find, most numbers are not perfect squares. We can estimate their square roots or use a calculator for a more precise value.

> **Method Use a calculator**
>
> On most calculators, you can find a square root using the $\boxed{\sqrt{\ }}$ button.

Ex: Use a calculator to find $\sqrt{10}$, rounded to 2 decimal places.

Answer: Entering $\sqrt{10}$ into a calculator gives approximately $3.162277\ldots$
Rounded to 2 decimal places, $\sqrt{10} \approx 3.16$.