SOLVING QUADRATIC EQUATIONS

A QUADRATIC EQUATION

A.1 IDENTIFYING COEFFICIENTS OF QUADRATIC EQUATIONS: LEVEL 1

Ex 1: For the equation $5x^2 - 2x - 3 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{5},\ b = \boxed{-2} \text{ and } c = \boxed{-3}$$

Answer:

$$\begin{aligned} 5x^2 - 2x \quad & -3 = 0 \\ 5x^2 + (-2)x \quad & +(-3) = 0 \end{aligned}$$

We identify

$$a = 5, \quad b = -2, \quad c = -3$$

Ex 2: For the equation $x^2 + 2x + 1 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{2} \text{ and } c = \boxed{1}$$

Answer:

$$\begin{aligned} x^2 + 2x + 1 = 0 \\ 1x^2 + 2x + 1 = 0 \end{aligned}$$

We identify

$$a = 1, \quad b = 2, \quad c = 1$$

Ex 3: For the equation $-x^2 + 2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{-1},\ b = \boxed{0} \text{ and } c = \boxed{2}$$

Answer:

$$\begin{aligned} -x^2 \quad & +2 = 0 \\ (-1)x^2 + 0x \quad & +2 = 0 \end{aligned}$$

We identify

$$a = -1, \quad b = 0, \quad c = 2$$

Ex 4: For the equation $-x^2 + 2x = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{-1},\ b = \boxed{2} \text{ and } c = \boxed{0}$$

Answer: Write the equation in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} -x^2 + 2x = 0 \\ -x^2 + 2x + 0 = 0 \end{aligned}$$

We identify:

$$a = -1, \quad b = 2, \quad c = 0$$

A.2 IDENTIFYING COEFFICIENTS OF QUADRATIC EQUATIONS: LEVEL 2

Ex 5: For the equation $x^2 - x + 3 = 1$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{-1} \text{ and } c = \boxed{2}$$

Answer: First, rewrite the equation in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} x^2 - x + 3 = 1 \\ x^2 - x + 3 - 1 = 0 \\ x^2 - x + 2 = 0 \end{aligned}$$

We identify:

$$a = 1, \quad b = -1, \quad c = 2$$

Ex 6: For the equation $(x+1)^2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{2} \text{ and } c = \boxed{1}$$

Answer: First, expand the equation and write it in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} (x+1)^2 = 0 \\ x^2 + 2x + 1 = 0 \end{aligned}$$

We identify:

$$a = 1, \quad b = 2, \quad c = 1$$

Ex 7: For the equation $(x-2)^2 + 2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{-4} \text{ and } c = \boxed{6}$$

Answer: First, expand the equation and write it in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} (x-2)^2 + 2 = 0 \\ x^2 - 4x + 4 + 2 = 0 \\ x^2 - 4x + 6 = 0 \end{aligned}$$

We identify:

$$a = 1, \quad b = -4, \quad c = 6$$

Ex 8: For the equation $x(x-2) = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{-2} \text{ and } c = \boxed{0}$$

Answer: Expand and write in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} x(x-2) = 0 \\ x^2 - 2x = 0 \end{aligned}$$

We identify:

$$a = 1, \quad b = -2, \quad c = 0$$

Ex 9: For the equation $(x-2)(x+1) = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$$a = \boxed{1},\ b = \boxed{-1} \text{ and } c = \boxed{-2}$$

Answer: Expand and write in the form $ax^2 + bx + c = 0$:

$$\begin{aligned} (x-2)(x+1) = 0 \\ x^2 + x - 2x - 2 = 0 \\ x^2 - x - 2 = 0 \end{aligned}$$

We identify:

$$a = 1, \quad b = -1, \quad c = -2$$

A.3 RECOGNIZING QUADRATIC EQUATIONS

MCQ 10: Is the equation $2x^2-3x+2=0$ a quadratic equation?

☒ Yes.

☐ No.

Answer: The equation $2x^2-3x+2=0$ is a quadratic equation because it has the form $ax^2+bx+c=0$, where $a=2$, $b=-3$, and $c=2$, with a non-zero x^2 term.
The correct choice is: Yes.

MCQ 11: Is the equation $2x-3=0$ a quadratic equation?

☐ Yes.

☒ No.

Answer: The equation $2x-3=0$ is not a quadratic equation because it lacks an x^2 term; it is a linear equation of the form $ax+b=0$.
The correct choice is: No.

MCQ 12: Is the equation $2x^2-3x+\frac{1}{x}=0$ a quadratic equation?

☐ Yes.

☒ No.

Answer: The equation $2x^2-3x+\frac{1}{x}=0$ is not a quadratic equation because it contains a term $\frac{1}{x}$, which makes it a rational equation rather than a polynomial equation of the form $ax^2+bx+c=0$.
The correct choice is: No.

MCQ 13: Is the equation $(x-1)(x+2)=0$ a quadratic equation?

☒ Yes.

☐ No.

Answer: The equation $(x-1)(x+2)=0$ is a quadratic equation because, when expanded, it becomes $x^2+x-2=0$, which is in the form $ax^2+bx+c=0$ with $a=1$, $b=1$, and $c=-2$, and contains a non-zero x^2 term.
The correct choice is: Yes.

A.4 VERIFYING ROOTS OF QUADRATIC EQUATIONS

MCQ 14: Is 1 a root of the equation $x^2-2x+1=0$?

☒ Yes.

☐ No.

Answer: Substitute $x=1$ into the equation: $1^2-2(1)+1=1-2+1=0$. Since it equals zero, 1 is a root.
The correct choice is: Yes.

MCQ 15: Is 1 a root of the equation $x^2+2x+1=0$?

☐ Yes.

☒ No.

Answer: Substitute $x=1$ into the equation: $1^2+2(1)+1=1+2+1=4\neq 0$. Since it does not equal zero, 1 is not a root.
The correct choice is: No.

MCQ 16: Is 2 a root of the equation $(x-1)(x-2)=0$?

☒ Yes.

☐ No.

Answer: Substitute $x=2$ into the equation: $(2-1)(2-2)=1\cdot 0=0$. Since the product equals zero, 2 is a root.
The correct choice is: Yes.

MCQ 17: Is 5 a root of the equation $(x-2)^2-8=0$?

☐ Yes.

☒ No.

Answer: Substitute $x=5$ into the equation: $(5-2)^2-8=3^2-8=9-8=1\neq 0$. Since the result is not zero, 5 is not a root.
The correct choice is: No.

B SOLVING BY FACTORIZATION

B.1 FINDING SOLUTION SETS OF FACTORED QUADRATIC EQUATIONS

MCQ 18: For the equation $(x-1)(x+2)=0$, the set of solutions is

☒ $S=\{-2,1\}$

☐ $S=\{-1,2\}$

☐ $S=\{2\}$

☐ $S=\{1\}$

Answer:

$$\begin{aligned}(x-1)(x+2)&=0\\ x-1=0 \quad \text{or} \quad x+2&=0 \quad \text{(null factor law)}\\ x=1 \quad \text{or} \quad x&=-2 \quad \text{(solve each equation)}\end{aligned}$$

The set of solutions is $\{-2,1\}$.

MCQ 19: For the equation $x(x-\sqrt{2})=0$, the set of solutions is

☐ $S=\{0,-\sqrt{2}\}$

☐ $S=\{-\sqrt{2}\}$

☐ $S=\{0\}$

☒ $S=\{0,\sqrt{2}\}$

Answer:

$$\begin{aligned}x(x-\sqrt{2})&=0\\ x=0 \quad \text{or} \quad x-\sqrt{2}&=0 \quad \text{(null factor law)}\\ x=0 \quad \text{or} \quad x&=\sqrt{2} \quad \text{(solve each equation)}\end{aligned}$$

The set of solutions is $\{0,\sqrt{2}\}$.

MCQ 20: For the equation $(x-1)^2=0$, the set of solutions is

☒ $S=\{1\}$

☐ $S=\{-1\}$

☐ $S=\{1,-1\}$

☐ $S=\{0\}$

Answer:

$$\begin{aligned}(x-1)^2 &= 0\\(x-1)(x-1) &= 0\\x-1=0 \quad &\text{or} \quad x-1=0 \qquad \text{(null factor law)}\\x &= 1 \quad \text{(solve the equation)}\end{aligned}$$

Since $(x-1)^2 = 0$ has a double root, the set of solutions is $\{1\}$.

MCQ 21: For the equation $(2x-1)(x+1) = 0$, the set of solutions is

- ☐ $S = \{-1, 2\}$
- ☒ $S = \left\{\frac{1}{2}, -1\right\}$
- ☐ $S = \{2\}$
- ☐ $S = \{-1\}$

Answer:

$$\begin{aligned}(2x-1)(x+1) &= 0\\2x-1=0 \quad &\text{or} \quad x+1=0 \quad \text{(null factor law)}\\2x=1 \quad &\text{or} \quad x=-1 \quad \text{(solve each equation)}\\x=\frac{1}{2} \quad &\text{or} \quad x=-1\end{aligned}$$

The set of solutions is $\left\{\frac{1}{2}, -1\right\}$.

B.2 SOLVING FACTORED QUADRATIC EQUATIONS

Ex 22: Solve the equation $(x-1)(x+2) = 0$. Justify your answer.

Answer:

$$\begin{aligned}(x-1)(x+2) &= 0\\x-1=0 \quad &\text{or} \quad x+2=0 \quad \text{(null factor law)}\\x=1 \quad &\text{or} \quad x=-2\end{aligned}$$

The solutions are 1 and -2.

Ex 23: Solve the equation $(x+1)(x-1) = 0$. Justify your answer.

Answer:

$$\begin{aligned}(x+1)(x-1) &= 0\\x+1=0 \quad &\text{or} \quad x-1=0 \quad \text{(null factor law)}\\x=-1 \quad &\text{or} \quad x=1\end{aligned}$$

The solutions are -1 and 1.

Ex 24: Solve the equation $((x-2)+3)((x-2)-3) = 0$. Justify your answer.

Answer:

$$\begin{aligned}((x-2)+3)((x-2)-3) &= 0\\x-2+3=0 \quad &\text{or} \quad x-2-3=0 \quad \text{(null factor law)}\\x+1=0 \quad &\text{or} \quad x-5=0\\x=-1 \quad &\text{or} \quad x=5\end{aligned}$$

The solutions are -1 and 5.

Ex 25: Solve the equation $(x+\sqrt{2})(x-\sqrt{2}) = 0$. Justify your answer.

Answer:

$$\begin{aligned}(x+\sqrt{2})(x-\sqrt{2}) &= 0\\x+\sqrt{2}=0 \quad &\text{or} \quad x-\sqrt{2}=0 \quad \text{(null factor law)}\\x=-\sqrt{2} \quad &\text{or} \quad x=\sqrt{2}\end{aligned}$$

The solutions are $x = -\sqrt{2}$ and $x = \sqrt{2}$.

C FACTORIZATION TECHNIQUES FOR SPECIAL FORMS OF EQUATIONS

C.1 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS IN THE FORM $ax^2 + bx$

MCQ 26: For the equation $x^2 + x = 0$, the set of solutions is

- ☐ $S = \{-1, 0, 1\}$
- ☐ $S = \{1\}$
- ☐ $S = \{0\}$
- ☒ $S = \{0, -1\}$

Answer:

$$\begin{aligned}x^2+x &= 0\\x(x+1) &= 0 \quad \text{(factorize)}\\x=0 \quad &\text{or} \quad x+1=0 \quad \text{(null factor law)}\\x=0 \quad &\text{or} \quad x=-1 \quad \text{(solve each equation)}\end{aligned}$$

The set of solutions is $\{-1, 0\}$.

MCQ 27: For the equation $x^2 - 2x = 0$, the set of solutions is

- ☐ $S = \{-2, 0\}$
- ☐ $S = \{2\}$
- ☐ $S = \{0\}$
- ☒ $S = \{0, 2\}$

Answer:

$$\begin{aligned}x^2-2x &= 0\\x(x-2) &= 0 \quad \text{(factorize)}\\x=0 \quad &\text{or} \quad x-2=0 \quad \text{(null factor law)}\\x=0 \quad &\text{or} \quad x=2 \quad \text{(solve each equation)}\end{aligned}$$

The set of solutions is $\{0, 2\}$.

MCQ 28: For the equation $2x^2 + x = 0$, the set of solutions is

- ☒ $S = \left\{-\frac{1}{2}, 0\right\}$
- ☐ $S = \{-2, 0\}$
- ☐ $S = \{2, 0\}$
- ☐ $S = \{0, 1\}$

Answer:

$$\begin{aligned}2x^2+x &= 0\\x(2x+1) &= 0 \quad \text{(factorize)}\\x=0 \quad &\text{or} \quad 2x+1=0 \quad \text{(null factor law)}\\x=0 \quad &\text{or} \quad 2x=-1\\x=0 \quad &\text{or} \quad x=-\frac{1}{2} \quad \text{(solve each equation)}\end{aligned}$$

The set of solutions is $S = \left\{0, -\frac{1}{2}\right\}$.

MCQ 29: For the equation $3x^2 = x$, the set of solutions is

- ☐ $S = \{-3,\ 0\}$
- ☒ $S = \left\{0,\ \frac{1}{3}\right\}$
- ☐ $S = \{0,\ 3\}$
- ☐ $S = \{0,\ 1\}$

Answer:

$$\begin{aligned} 3x^2 &= x \\ 3x^2 - x &= 0 \\ x(3x-1) &= 0 && \text{(factorize)} \\ x = 0 \quad \text{or} \quad 3x - 1 &= 0 && \text{(null factor law)} \\ x = 0 \quad \text{or} \quad 3x &= 1 \\ x = 0 \quad \text{or} \quad x &= \frac{1}{3} && \text{(solve each equation)} \end{aligned}$$

The set of solutions is $S = \left\{0,\ \frac{1}{3}\right\}$.

C.2 SOLVING QUADRATIC EQUATIONS IN THE FORM $ax^2 + bx$

Ex 30: Solve the equation $x^2 + x = 0$. Justify your answer.

Answer:

$$\begin{aligned} x^2 + x &= 0 \\ x(x+1) &= 0 && \text{(factorize)} \\ x = 0 \quad \text{or} \quad x + 1 &= 0 && \text{(null factor law)} \\ x = 0 \quad \text{or} \quad x &= -1 && \text{(solve each equation)} \end{aligned}$$

The set of solutions is $\{-1, 0\}$.

Ex 31: Solve the equation $x^2 - 2x = 0$. Justify your answer.

Answer:

$$\begin{aligned} x^2 - 2x &= 0 \\ x(x-2) &= 0 && \text{(factorize)} \\ x = 0 \quad \text{or} \quad x - 2 &= 0 && \text{(null factor law)} \\ x = 0 \quad \text{or} \quad x &= 2 && \text{(solve each equation)} \end{aligned}$$

The set of solutions is $\{0, 2\}$.

Ex 32: Solve the equation $2x^2 - x = 0$. Justify your answer.

Answer:

$$\begin{aligned} 2x^2 - x &= 0 \\ x(2x-1) &= 0 && \text{(factorize)} \\ x = 0 \quad \text{or} \quad 2x - 1 &= 0 && \text{(null factor law)} \\ x = 0 \quad \text{or} \quad 2x &= 1 && \text{(solve each equation)} \\ x = 0 \quad \text{or} \quad x &= \frac{1}{2} \end{aligned}$$

The set of solutions is $\left\{0, \frac{1}{2}\right\}$.

Ex 33: Solve the equation $2x^2 = 4x$. Justify your answer.

Answer:

$$\begin{aligned} 2x^2 &= 4x \\ 2x^2 - 4x &= 0 && \text{(put all terms on one side)} \\ 2x(x-2) &= 0 && \text{(factorize)} \\ 2x = 0 \quad \text{or} \quad x - 2 &= 0 && \text{(null factor law)} \\ x = 0 \quad \text{or} \quad x &= 2 && \text{(solve each equation)} \end{aligned}$$

The set of solutions is $\{0,\ 2\}$.

C.3 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS IN THE FORM OF A DIFFERENCE OF SQUARES

MCQ 34: For the equation $x^2 - 4 = 0$, the set of solutions is

- ☐ $S = \{-4, 4\}$
- ☐ $S = \{2\}$
- ☐ $S = \{-1, 1\}$
- ☒ $S = \{-2, 2\}$

Answer:

- **Method 1**

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 - 2^2 &= 0 \\ (x-2)(x+2) &= 0 && \text{(difference of squares)} \\ x - 2 = 0 \quad \text{or} \quad x + 2 &= 0 && \text{(null factor law)} \\ x = 2 \quad \text{or} \quad x &= -2 && \text{(solve each equation)} \end{aligned}$$

- **Method 2**

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x = \sqrt{4} \quad &\text{or} \quad x = -\sqrt{4} \\ x = 2 \quad &\text{or} \quad x = -2 \end{aligned}$$

The set of solutions is $S = \{-2, 2\}$.

MCQ 35: For the equation $x^2 = -2$, the set of solutions is

- ☐ $S = \{-2, 2\}$
- ☐ $S = \{2\}$
- ☒ $S = \{\}$
- ☐ $S = \{-\sqrt{2}, \sqrt{2}\}$

Answer:

$$x^2 = -2$$

There is **no real solution** since the square of a real number cannot be negative.
The set of solutions is $\emptyset$.

MCQ 36: For the equation $x^2 - 2 = 0$, the set of solutions is

- ☒ $S = \{-\sqrt{2},\ \sqrt{2}\}$
- ☐ $S = \{-2, 2\}$
- ☐ $S = \{2\}$
- ☐ $S = \{-1, 1\}$

Answer:

- **Method 1**

$$\begin{aligned} x^2 - 2 &= 0 \\ x^2 - \left(\sqrt{2}\right)^2 &= 0 \\ (x-\sqrt{2})(x+\sqrt{2}) &= 0 && \text{(difference of squares)} \\ x - \sqrt{2} = 0 \quad \text{or} \quad x + \sqrt{2} &= 0 && \text{(null factor law)} \\ x = \sqrt{2} \quad \text{or} \quad x &= -\sqrt{2} && \text{(solve each equation)} \end{aligned}$$

- **Method 2**

$$\begin{aligned} x^2 - 2 &= 0 \\ x^2 &= 2 \\ x = \sqrt{2} \quad &\text{or} \quad x = -\sqrt{2} \end{aligned}$$

The set of solutions is $S = \left\{-\sqrt{2},\, \sqrt{2}\right\}$.

MCQ 37: For the equation $(x-1)^2-9=0$, the set of solutions is

- ☒ $S = \{-2, 4\}$
- ☐ $S = \{-3, 3\}$
- ☐ $S = \{2, 4\}$
- ☐ $S = \{-1, 1\}$

Answer:

- **Method 1**

$$\begin{aligned} (x-1)^2 - 9 &= 0 \\ (x-1)^2 - 3^2 &= 0 \\ [(x-1)-3]\,[(x-1)+3] &= 0 \\ (x-1)-3 = 0 \quad &\text{or} \quad (x-1)+3 = 0 \\ x-4 = 0 \quad &\text{or} \quad x+2 = 0 \\ x = 4 \quad &\text{or} \quad x = -2 \end{aligned}$$

- **Method 2**

$$\begin{aligned} (x-1)^2 - 9 &= 0 \\ (x-1)^2 &= 9 \\ x-1 = \sqrt{9} \quad &\text{or} \quad x-1 = -\sqrt{9} \\ x-1 = 3 \quad &\text{or} \quad x-1 = -3 \\ x = 4 \quad &\text{or} \quad x = -2 \end{aligned}$$

The set of solutions is $S = \{-2, 4\}$.

MCQ 38: For the equation $(x-1)^2-2=0$, the set of solutions is

- ☒ $S = \left\{1-\sqrt{2},\, 1+\sqrt{2}\right\}$
- ☐ $S = \{-2, 4\}$
- ☐ $S = \left\{-\sqrt{2}, \sqrt{2}\right\}$
- ☐ $S = \{-1, 1\}$

Answer:

- **Method 1**

$$\begin{aligned} (x-1)^2 - 2 &= 0 \\ (x-1)^2 - (\sqrt{2})^2 &= 0 \\ [(x-1)-\sqrt{2}][(x-1)+\sqrt{2}] &= 0 \\ x-1-\sqrt{2} = 0 \quad &\text{or} \quad x-1+\sqrt{2} = 0 \\ x = 1+\sqrt{2} \quad &\text{or} \quad x = 1-\sqrt{2} \end{aligned}$$

- **Method 2**

$$\begin{aligned} (x-1)^2 - 2 &= 0 \\ (x-1)^2 &= 2 \\ x-1 = \sqrt{2} \quad &\text{or} \quad x-1 = -\sqrt{2} \\ x = 1+\sqrt{2} \quad &\text{or} \quad x = 1-\sqrt{2} \end{aligned}$$

The set of solutions is $S = \left\{1-\sqrt{2},\, 1+\sqrt{2}\right\}$.

C.4 SOLVING QUADRATIC EQUATIONS IN THE FORM OF A DIFFERENCE OF SQUARES

Ex 39: Solve the equation $x^2-4=0$. Justify your answer.

Answer:

- **Method 1**

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 - 2^2 &= 0 \\ (x-2)(x+2) &= 0 && \text{(difference of squares)} \\ x-2 = 0 \quad \text{or} \quad x+2 &= 0 && \text{(null factor law)} \\ x = 2 \quad \text{or} \quad x &= -2 && \text{(solve each equation)} \end{aligned}$$

- **Method 2**

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x = \sqrt{4} \quad &\text{or} \quad x = -\sqrt{4} \\ x = 2 \quad &\text{or} \quad x = -2 \end{aligned}$$

The set of solutions is $S = \{-2, 2\}$.

Ex 40: Solve the equation $x^2=-2$. Justify your answer.

Answer:

$$x^2 = -2$$

There is **no real solution** since the square of a real number cannot be negative.
The set of solutions is $\emptyset$.

Ex 41: Solve the equation $x^2-2=0$. Justify your answer.

Answer:

- **Method 1**

$$\begin{aligned} x^2 - 2 &= 0 \\ x^2 - \left(\sqrt{2}\right)^2 &= 0 \\ (x-\sqrt{2})(x+\sqrt{2}) &= 0 && \text{(difference of squares)} \\ x-\sqrt{2} = 0 \quad \text{or} \quad x+\sqrt{2} &= 0 && \text{(null factor law)} \\ x = \sqrt{2} \quad \text{or} \quad x &= -\sqrt{2} && \text{(solve each equation)} \end{aligned}$$

- **Method 2**

$$\begin{aligned} x^2 - 2 &= 0 \\ x^2 &= 2 \\ x = \sqrt{2} \quad &\text{or} \quad x = -\sqrt{2} \end{aligned}$$

The set of solutions is $S = \left\{-\sqrt{2},\, \sqrt{2}\right\}$.

Ex 42: Solve the equation $(x-1)^2-9=0$. Justify your answer.

Answer:

- **Method 1**

$$\begin{aligned} (x-1)^2 - 9 &= 0 \\ (x-1)^2 - 3^2 &= 0 \\ [(x-1)-3]\,[(x-1)+3] &= 0 \\ (x-1)-3 = 0 \quad &\text{or} \quad (x-1)+3 = 0 \\ x-4 = 0 \quad &\text{or} \quad x+2 = 0 \\ x = 4 \quad &\text{or} \quad x = -2 \end{aligned}$$

- **Method 2**

$$\begin{aligned} (x-1)^2-9&=0\\ (x-1)^2&=9\\ x-1=\sqrt{9} \quad &\text{or} \quad x-1=-\sqrt{9}\\ x-1=3 \quad &\text{or} \quad x-1=-3\\ x=4 \quad &\text{or} \quad x=-2 \end{aligned}$$

The set of solutions is $S=\{-2,4\}$.

Ex 43: Solve the equation $(x-1)^2-2=0$. Justify your answer.

Answer:

- **Method 1**

$$\begin{aligned} (x-1)^2-2&=0\\ (x-1)^2-(\sqrt{2})^2&=0\\ [(x-1)-\sqrt{2}][(x-1)+\sqrt{2}]&=0\\ x-1-\sqrt{2}=0 \quad &\text{or} \quad x-1+\sqrt{2}=0\\ x=1+\sqrt{2} \quad &\text{or} \quad x=1-\sqrt{2} \end{aligned}$$

- **Method 2**

$$\begin{aligned} (x-1)^2-2&=0\\ (x-1)^2&=2\\ x-1=\sqrt{2} \quad &\text{or} \quad x-1=-\sqrt{2}\\ x=1+\sqrt{2} \quad &\text{or} \quad x=1-\sqrt{2} \end{aligned}$$

The set of solutions is $S=\left\{1-\sqrt{2},\,1+\sqrt{2}\right\}$.

D THE GENERAL METHOD: COMPLETING THE SQUARE

D.1 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS

MCQ 44: For the equation $x^2+2x-3=0$, the set of solutions is

☒ $S=\{-3,1\}$

☐ $S=\{3,1\}$

☐ $S=\{-1,3\}$

☐ $S=\{-1,1\}$

Answer: We know that $(x+1)^2=x^2+2x+1$. So

$$\begin{aligned} x^2+2x-3&=0\\ x^2+2x+1-4&=0 &&\text{(rewrite } -3 \text{ as } 1-4)\\ (x+1)^2-4&=0 &&\text{(complete the square)}\\ (x+1)^2-2^2&=0 &&\text{(difference of squares)}\\ [(x+1)-2][(x+1)+2]&=0 &&\text{(factorize)}\\ (x+1)-2=0 \text{ or } (x+1)+2&=0 &&\text{(null factor law)}\\ x=1 \text{ or } x&=-3 &&\text{(solve).} \end{aligned}$$

The set of solutions is $S=\{-3,\,1\}$.

MCQ 45: For the equation $x^2+6x+5=0$, the set of solutions is

☒ $S=\{-5,-1\}$

☐ $S=\{-5,1\}$

☐ $S=\{-1,5\}$

☐ $S=\{1,5\}$

Answer: We know that $(x+3)^2=x^2+6x+9$. So

$$\begin{aligned} x^2+6x+5&=0\\ x^2+6x+9-4&=0 &&\text{(rewrite 5 as } 9-4)\\ (x+3)^2-4&=0 &&\text{(complete the square)}\\ (x+3)^2-2^2&=0 &&\text{(difference of squares)}\\ [(x+3)-2][(x+3)+2]&=0 &&\text{(factorize)}\\ (x+1)(x+5)&=0 &&\text{(simplify)}\\ x+1=0 \text{ or } x+5&=0 &&\text{(null factor law)}\\ x=-1 \text{ or } x&=-5 &&\text{(solve)} \end{aligned}$$

The set of solutions is $S=\{-5,\,-1\}$.

MCQ 46: For the equation $x^2+10x+24=0$, the set of solutions is

☐ $S=\{2,4\}$

☐ $S=\{2,6\}$

☐ $S=\{4,6\}$

☒ $S=\{-4,-6\}$

Answer: We know that $(x+5)^2=x^2+10x+25$. So

$$\begin{aligned} x^2+10x+24&=0\\ x^2+10x+25-1&=0 &&\text{(rewrite 24 as } 25-1)\\ (x+5)^2-1&=0 &&\text{(complete the square)}\\ (x+5)^2-1^2&=0 &&\text{(difference of squares)}\\ [(x+5)-1][(x+5)+1]&=0 &&\text{(factorize)}\\ (x+5)-1=0 \text{ or } (x+5)+1&=0 &&\text{(null factor law)}\\ x=-4 \text{ or } x&=-6 &&\text{(solve).} \end{aligned}$$

MCQ 47: For the equation $x^2-2x-1=0$, the set of solutions is

☒ $S=\left\{1-\sqrt{2},\,1+\sqrt{2}\right\}$

☐ $S=\{-1,2\}$

☐ $S=\{-1,1\}$

☐ $S=\{0,2\}$

Answer: We know that $(x-1)^2=x^2-2x+1$. So

$$\begin{aligned} x^2-2x-1&=0\\ x^2-2x+1-2&=0 &&\text{(rewrite } -1 \text{ as } 1-2)\\ (x-1)^2-2&=0 &&\text{(complete the square)}\\ (x-1)^2-(\sqrt{2})^2&=0 &&\text{(difference of squares)}\\ [(x-1)-\sqrt{2}][(x-1)+\sqrt{2}]&=0 &&\text{(factorize)}\\ x-1-\sqrt{2}=0 \text{ or } x-1+\sqrt{2}&=0 &&\text{(null factor law)}\\ x=1+\sqrt{2} \text{ or } x&=1-\sqrt{2} &&\text{(solve).} \end{aligned}$$

The set of solutions is $S=\left\{1-\sqrt{2},\,1+\sqrt{2}\right\}$.

D.2 SOLVING QUADRATIC EQUATIONS

Ex 48: Solve the equation $x^2+2x-3=0$. Justify your answer.

Answer: We know that $(x+1)^2 = x^2+2x+1$. So

$$\begin{aligned} x^2+2x-3&=0 \\ x^2+2x+1-4&=0 && \text{(rewrite } -3 \text{ as } 1-4) \\ (x+1)^2-4&=0 && \text{(complete the square)} \\ (x+1)^2-2^2&=0 && \text{(difference of squares)} \\ [(x+1)-2][(x+1)+2]&=0 && \text{(factorize)} \\ (x+1)-2=0 \text{ or } (x+1)+2&=0 && \text{(null factor law)} \\ x=1 \text{ or } x&=-3 && \text{(solve).} \end{aligned}$$

The set of solutions is $S=\{-3, 1\}$.

Ex 49: Solve the equation $x^2+6x+5=0$. Justify your answer.

Answer: We know that $(x+3)^2 = x^2+6x+9$. So

$$\begin{aligned} x^2+6x+5&=0 \\ x^2+6x+9-4&=0 && \text{(rewrite 5 as } 9-4) \\ (x+3)^2-4&=0 && \text{(complete the square)} \\ (x+3)^2-2^2&=0 && \text{(difference of squares)} \\ [(x+3)-2][(x+3)+2]&=0 && \text{(factorize)} \\ (x+1)(x+5)&=0 && \text{(simplify)} \\ x+1=0 \text{ or } x+5&=0 && \text{(null factor law)} \\ x=-1 \text{ or } x&=-5 && \text{(solve)} \end{aligned}$$

The set of solutions is $S=\{-5, -1\}$.

Ex 50: Solve the equation $x^2+10x+24=0$. Justify your answer.

Answer: We know that $(x+5)^2 = x^2+10x+25$. So

$$\begin{aligned} x^2+10x+24&=0 \\ x^2+10x+25-1&=0 && \text{(rewrite 24 as } 25-1) \\ (x+5)^2-1&=0 && \text{(complete the square)} \\ (x+5)^2-1^2&=0 && \text{(difference of squares)} \\ [(x+5)-1][(x+5)+1]&=0 && \text{(factorize)} \\ (x+5)-1=0 \text{ or } (x+5)+1&=0 && \text{(null factor law)} \\ x=-4 \text{ or } x&=-6 && \text{(solve).} \end{aligned}$$

Ex 51: Solve the equation $x^2-2x-1=0$. Justify your answer.

Answer: We know that $(x-1)^2 = x^2-2x+1$. So

$$\begin{aligned} x^2-2x-1&=0 \\ x^2-2x+1-2&=0 && \text{(rewrite } -1 \text{ as } 1-2) \\ (x-1)^2-2&=0 && \text{(complete the square)} \\ (x-1)^2-(\sqrt{2})^2&=0 && \text{(difference of squares)} \\ [(x-1)-\sqrt{2}][(x-1)+\sqrt{2}]&=0 && \text{(factorize)} \\ x-1-\sqrt{2}=0 \text{ or } x-1+\sqrt{2}&=0 && \text{(null factor law)} \\ x=1+\sqrt{2} \text{ or } x=1&-\sqrt{2} && \text{(solve).} \end{aligned}$$

The set of solutions is $S=\left\{1-\sqrt{2}, 1+\sqrt{2}\right\}$.

E QUADRATIC FORMULA

E.1 CALCULATING THE DISCRIMINANT

Ex 52: For the equation $5x^2-2x-3=0$, calculate the discriminant:

$$\Delta = \boxed{64}$$

Answer: Given the quadratic equation $ax^2+bx+c=0$, we identify:

$$a=5, \quad b=-2, \quad c=-3$$

The discriminant is given by:

$$\begin{aligned} \Delta &= b^2-4ac \\ &= (-2)^2-4\times5\times(-3) \\ &= 4+60 \\ &= 64 \end{aligned}$$

Ex 53: For the equation $x^2+6x+5=0$, calculate the discriminant:

$$\Delta = \boxed{16}$$

Answer: Given the quadratic equation $ax^2+bx+c=0$, we identify:

$$a=1, \quad b=6, \quad c=5$$

The discriminant is given by:

$$\begin{aligned} \Delta &= b^2-4ac \\ &= 6^2-4\times1\times5 \\ &= 36-20 \\ &= 16 \end{aligned}$$

Ex 54: For the equation $2x^2-x+3=0$, calculate the discriminant:

$$\Delta = \boxed{-23}$$

Answer: Given the quadratic equation $ax^2+bx+c=0$, we identify:

$$a=2, \quad b=-1, \quad c=3$$

The discriminant is given by:

$$\begin{aligned} \Delta &= b^2-4ac \\ &= (-1)^2-4\times2\times3 \\ &= 1-24 \\ &= -23 \end{aligned}$$

Ex 55: For the equation $-2x^2+8=0$, calculate the discriminant:

$$\Delta = \boxed{64}$$

Answer: Given the quadratic equation $ax^2+bx+c=0$, we identify:

$$a=-2, \quad b=0, \quad c=8$$

The discriminant is given by:

$$\begin{aligned} \Delta &= b^2-4ac \\ &= 0^2-4\times(-2)\times8 \\ &= 0+64 \\ &= 64 \end{aligned}$$

E.2 SOLVING QUADRATIC EQUATIONS: STEP BY STEP

Ex 56: Consider the quadratic equation $x^2 + 2x - 3 = 0$.

1. Find the discriminant.

$$\Delta = \boxed{16}$$

2. Hence, state the nature of the roots of the equation. $\boxed{\textbf{As } \Delta > 0\textbf{, there are 2 distinct roots.}}$

3. The solutions of the equation are $\boxed{-3}$ and $\boxed{1}$ (order from lowest to highest).

Answer: $x^2 + 2x - 3 = 0$ has $a = 1$, $b = 2$, $c = -3$.

1. $\Delta = b^2 - 4ac$
$= (2)^2 - 4(1)(-3)$
$= 4 + 12$
$= 16$

2. As $\Delta > 0$, there are 2 distinct roots.

3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
$x = \dfrac{-2 - \sqrt{16}}{2 \cdot 1}$ or $x = \dfrac{-2 + \sqrt{16}}{2 \cdot 1}$
$x = \dfrac{-2 - 4}{2}$ or $x = \dfrac{-2 + 4}{2}$
$x = -3$ or $x = 1$

Ex 57: Consider the quadratic equation $x^2 - 2x - 1 = 0$.

1. Find the discriminant.

$$\Delta = \boxed{8}$$

2. Hence, state the nature of the roots of the equation. $\boxed{\textbf{As } \Delta > 0\textbf{, there are 2 distinct roots.}}$

3. The solutions of the equation are $\boxed{1 - \sqrt{2}}$ and $\boxed{1 + \sqrt{2}}$ (order from lowest to highest).

Answer: $x^2 - 2x - 1 = 0$ has $a = 1$, $b = -2$, $c = -1$.

1. $\Delta = b^2 - 4ac$
$= (-2)^2 - 4(1)(-1)$
$= 4 + 4$
$= 8$

2. As $\Delta > 0$, there are 2 distinct roots.

3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
$x = \dfrac{-(-2) - \sqrt{8}}{2 \cdot 1}$ or $x = \dfrac{-(-2) + \sqrt{8}}{2 \cdot 1}$
$x = \dfrac{2 - \sqrt{4 \cdot 2}}{2}$ or $x = \dfrac{2 + \sqrt{4 \cdot 2}}{2}$
$x = \dfrac{2 - 2\sqrt{2}}{2}$ or $x = \dfrac{2 + 2\sqrt{2}}{2}$
$x = 1 - \sqrt{2}$ or $x = 1 + \sqrt{2}$

Ex 58: Consider the quadratic equation $2x^2 - 3x + 1 = 0$.

1. Find the discriminant.

$$\Delta = \boxed{1}$$

2. Hence, state the nature of the roots of the equation. $\boxed{\textbf{As } \Delta > 0\textbf{, there are 2 distinct roots.}}$

3. The solutions of the equation are $\boxed{\dfrac{1}{2}}$ and $\boxed{1}$ (order from lowest to highest).

Answer: $2x^2 - 3x + 1 = 0$ has $a = 2$, $b = -3$, $c = 1$.

1. $\Delta = b^2 - 4ac$
$= (-3)^2 - 4(2)(1)$
$= 9 - 8$
$= 1$

2. As $\Delta > 0$, there are 2 distinct roots.

3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
$x = \dfrac{-(-3) - \sqrt{1}}{2 \cdot 2}$ or $x = \dfrac{-(-3) + \sqrt{1}}{2 \cdot 2}$
$x = \dfrac{3 - 1}{4}$ or $x = \dfrac{3 + 1}{4}$
$x = \dfrac{2}{4}$ or $x = \dfrac{4}{4}$
$x = \dfrac{1}{2}$ or $x = 1$

Ex 59: Consider the quadratic equation $2x^2 - 4x + 2 = 0$.

1. Find the discriminant.

$$\Delta = \boxed{0}$$

2. Hence, state the nature of the roots of the equation. $\boxed{\textbf{As } \Delta = 0\textbf{, there is 1 double root.}}$

3. The solution of the equation is $\boxed{1}$.

Answer: $2x^2 - 4x + 2 = 0$ has $a = 2$, $b = -4$, $c = 2$.

1. $\Delta = b^2 - 4ac$
$= (-4)^2 - 4(2)(2)$
$= 16 - 16$
$= 0$

2. As $\Delta = 0$, there is 1 double root.

3. $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$
$x = \dfrac{-(-4) \pm \sqrt{0}}{2 \cdot 2}$
$x = \dfrac{4}{4}$
$x = 1$

E.3 SOLVING QUADRATIC EQUATIONS

Ex 60: Solve the quadratic equation $x^2 + 2x - 3 = 0$.

Answer: $x^2 + 2x - 3 = 0$ has $a = 1$, $b = 2$, $c = -3$.

1. $\Delta = b^2 - 4ac$
   $= (2)^2 - 4(1)(-3)$
   $= 4 + 12$
   $= 16$
2. As $\Delta > 0$, there are 2 distinct roots.
3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
   $x = \dfrac{-2 - \sqrt{16}}{2 \cdot 1}$ or $x = \dfrac{-2 + \sqrt{16}}{2 \cdot 1}$
   $x = \dfrac{-2 - 4}{2}$ or $x = \dfrac{-2 + 4}{2}$
   $x = -3$ or $x = 1$

Ex 61: Solve the quadratic equation $x^2 + 2x - 2 = 0$.

Answer: $x^2 + 2x - 2 = 0$ has $a = 1$, $b = 2$, $c = -2$.

1. $\Delta = b^2 - 4ac$
   $= (2)^2 - 4(1)(-2)$
   $= 4 + 8$
   $= 12$
2. As $\Delta > 0$, there are 2 distinct roots.
3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
   $x = \dfrac{-2 - \sqrt{12}}{2 \cdot 1}$ or $x = \dfrac{-2 + \sqrt{12}}{2 \cdot 1}$
   $x = \dfrac{-2 - 2\sqrt{3}}{2}$ or $x = \dfrac{-2 + 2\sqrt{3}}{2}$
   $x = -1 - \sqrt{3}$ or $x = -1 + \sqrt{3}$

Ex 62: Solve the quadratic equation $x^2 - 2x + 6 = 0$.

Answer: $x^2 - 2x + 6 = 0$ has $a = 1$, $b = -2$, $c = 6$.

1. $\Delta = b^2 - 4ac$
   $= (-2)^2 - 4(1)(6)$
   $= 4 - 24$
   $= -20$
2. As $\Delta < 0$, there are no real roots.
3. No real solutions.

Ex 63: Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Answer: $x^2 - 6x + 9 = 0$ has $a = 1$, $b = -6$, $c = 9$.

1. $\Delta = b^2 - 4ac$
   $= (-6)^2 - 4(1)(9)$
   $= 36 - 36$
   $= 0$
2. As $\Delta = 0$, there is 1 double root.
3. $x = \dfrac{-b}{2a}$
   $x = \dfrac{-(-6)}{2 \cdot 1}$
   $x = \dfrac{6}{2}$
   $x = 3$

E.4 SOLVING QUADRATIC EQUATIONS

Ex 64: Solve the quadratic equation $2x^2 - 5x + 2 = 0$.

Answer: $2x^2 - 5x + 2 = 0$ has $a = 2$, $b = -5$, $c = 2$.

1. $\Delta = b^2 - 4ac$
   $= (-5)^2 - 4 \times 2 \times 2$
   $= 25 - 16$
   $= 9$
2. As $\Delta > 0$, there are 2 distinct roots.
3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
   $x = \dfrac{-(-5) - \sqrt{9}}{2 \times 2}$ or $x = \dfrac{-(-5) + \sqrt{9}}{2 \times 2}$
   $x = \dfrac{5 - 3}{4}$ or $x = \dfrac{5 + 3}{4}$
   $x = \dfrac{2}{4}$ or $x = \dfrac{8}{4}$
   $x = \dfrac{1}{2}$ or $x = 2$

Ex 65: Solve the quadratic equation $x^2 + 2x - 2 = 0$.

Answer: $x^2 + 2x - 2 = 0$ has $a = 1$, $b = 2$, $c = -2$.

1. $\Delta = b^2 - 4ac$
   $= (2)^2 - 4 \times 1 \times (-2)$
   $= 4 + 8$
   $= 12$
2. As $\Delta > 0$, there are 2 distinct roots.
3. $x = \dfrac{-b - \sqrt{\Delta}}{2a}$ or $x = \dfrac{-b + \sqrt{\Delta}}{2a}$
   $x = \dfrac{-2 - \sqrt{12}}{2 \times 1}$ or $x = \dfrac{-2 + \sqrt{12}}{2 \times 1}$
   $x = \dfrac{-2 - 2\sqrt{3}}{2}$ or $x = \dfrac{-2 + 2\sqrt{3}}{2}$
   $x = -1 - \sqrt{3}$ or $x = -1 + \sqrt{3}$

Ex 66: Solve the quadratic equation $x^2 - 8x + 15 = 0$.

Answer: $x^2 - 8x + 15 = 0$ has $a = 1$, $b = -8$, $c = 15$.

1. $\Delta = b^2 - 4ac$
   $= (-8)^2 - 4 \times 1 \times 15$
   $= 64 - 60$
   $= 4$
2. As $\Delta > 0$, there are 2 distinct roots.

3. $x = \dfrac{-b-\sqrt{\Delta}}{2a}$ or $x = \dfrac{-b+\sqrt{\Delta}}{2a}$
   $x = \dfrac{-(-8)-\sqrt{4}}{2\times 1}$ or $x = \dfrac{-(-8)+\sqrt{4}}{2\times 1}$
   $x = \dfrac{8-2}{2}$ or $x = \dfrac{8+2}{2}$
   $x = \dfrac{6}{2}$ or $x = \dfrac{10}{2}$
   $x = 3$ or $x = 5$

Ex 67: Solve the quadratic equation $x^2+6x+5=0$.

Answer: $x^2+6x+5=0$ has $a=1$, $b=6$, $c=5$.

1. $\Delta = b^2-4ac$
   $= (6)^2 - 4\times 1\times 5$
   $= 36-20$
   $= 16$
2. As $\Delta > 0$, there are 2 distinct roots.
3. $x = \dfrac{-b-\sqrt{\Delta}}{2a}$ or $x = \dfrac{-b+\sqrt{\Delta}}{2a}$
   $x = \dfrac{-6-\sqrt{16}}{2\times 1}$ or $x = \dfrac{-6+\sqrt{16}}{2\times 1}$
   $x = \dfrac{-6-4}{2}$ or $x = \dfrac{-6+4}{2}$
   $x = \dfrac{-10}{2}$ or $x = \dfrac{-2}{2}$
   $x = -5$ or $x = -1$