SOLVING INEQUALITIES

A INEQUALITIES

A.1 RECOGNIZING INEQUALITIES

MCQ 1: Is $2x - 3 = 0$ an inequality?

☐ Yes.

☒ No.

Answer: No, $2x - 3 = 0$ is not an inequality. It is an equation, because it uses the equals sign "=" and asks for which value(s) of x the expression is equal to 0.
An inequality would use signs like "<", ">", "≤", or "≥" to compare expressions rather than asking for equality.

MCQ 2: Is $5x + 1 > 0$ an inequality?

☒ Yes.

☐ No.

Answer: Yes, $5x + 1 > 0$ is an inequality because it uses the "greater than" sign ($>$) and asks for which values of x the expression $5x + 1$ is strictly greater than 0.

MCQ 3: Is $x - 4 \leq 2$ an inequality?

☒ Yes.

☐ No.

Answer: Yes, $x - 4 \leq 2$ is an inequality because it uses the "less than or equal to" sign ($\leq$) and describes the set of values of x for which $x - 4$ is less than or equal to 2.

MCQ 4: Is $3x + 2 = 7$ an inequality?

☐ Yes.

☒ No.

Answer: No, $3x + 2 = 7$ is not an inequality. It is an equation, because it uses the equals sign "=" and asks for the value of x that makes the two sides equal.

B PROPERTIES OF INEQUALITIES

B.1 FINDING THE SOLUTION SET OF INEQUALITIES

MCQ 5: For $2x - 6 \geq 0$, the set of solutions is:

☒ $[3, +\infty)$

☐ $(-\infty, 6]$

☐ $(-\infty, 3]$

☐ $[6, +\infty)$

Answer:

$$\begin{aligned} 2x - 6 &\geq 0 \\ 2x &\geq 6 \quad \text{(add 6)} \\ x &\geq 3 \quad \text{(divide by 2)} \end{aligned}$$

The set of solutions is $[3, +\infty)$, which includes all numbers greater than or equal to 3.

MCQ 6: For $3x - 5 \leq 10$, the set of solutions is:

☐ $\left[\frac{5}{3}, +\infty\right)$

☐ $(-\infty, 3]$

☒ $(-\infty, 5]$

☐ $[5, +\infty)$

Answer:

$$\begin{aligned} 3x - 5 &\leq 10 \\ 3x &\leq 15 \quad \text{(add 5)} \\ x &\leq 5 \quad \text{(divide by 3)} \end{aligned}$$

The set of solutions is $(-\infty, 5]$, which includes all numbers less than or equal to 5.

MCQ 7: For $-2x + 3 \geq 5$, the set of solutions is:

☐ $(-\infty, 1]$

☐ $[1, +\infty)$

☒ $(-\infty, -1]$

☐ $[-1, +\infty)$

Answer:

$$\begin{aligned} -2x + 3 &\geq 5 \\ -2x &\geq 2 \quad \text{(subtract 3)} \\ x &\leq -1 \quad \text{(divide by } -2\text{, reverse the inequality)} \end{aligned}$$

The set of solutions is $(-\infty, -1]$, which includes all numbers less than or equal to -1.

MCQ 8: For $3x - 2 < 4$, the set of solutions is:

☐ $(-\infty, 2]$

☐ $[2, +\infty)$

☒ $(-\infty, 2)$

☐ $(2, +\infty)$

Answer:

$$\begin{aligned} 3x - 2 &< 4 \\ 3x &< 6 \quad \text{(add 2)} \\ x &< 2 \quad \text{(divide by 3)} \end{aligned}$$

The set of solutions is $(-\infty, 2)$, which includes all numbers less than 2 (but not 2 itself).

B.2 SOLVING INEQUALITIES: LEVEL 1

Ex 9: Solve $2x - 6 \geq 0$. Justify your answer.

Answer:

$$\begin{aligned} 2x - 6 &\geq 0 \\ 2x &\geq 6 \quad \text{(add 6 to both sides)} \\ x &\geq 3 \quad \text{(divide by 2)} \end{aligned}$$

The set of solutions is $[3, +\infty)$, which includes all numbers greater than or equal to 3.

Ex 10: Solve $4x + 2 > 6$. Justify your answer.

Answer:

$$\begin{aligned} 4x+2 &> 6 \\ 4x &> 4 \quad \text{(subtract 2 from both sides)} \\ x &> 1 \quad \text{(divide by 4)} \end{aligned}$$

The set of solutions is $(1, +\infty)$, which includes all numbers greater than 1.

Ex 11: Solve $5 - 3x \leq 2$. Justify your answer.

Answer:

$$\begin{aligned} 5-3x &\leq 2 \\ -3x &\leq -3 \quad \text{(subtract 5 from both sides)} \\ x &\geq 1 \quad \text{(divide by -3 and reverse inequality)} \end{aligned}$$

The set of solutions is $[1, +\infty)$, which includes all numbers greater than or equal to 1.

Ex 12: Solve $-x + 4 < 7$. Justify your answer.

Answer:

$$\begin{aligned} -x+4 &< 7 \\ -x &< 3 \quad \text{(subtract 4 from both sides)} \\ x &> -3 \quad \text{(multiply by -1 and reverse inequality)} \end{aligned}$$

The set of solutions is $(-3, +\infty)$, which includes all numbers greater than -3.

B.3 SOLVING INEQUALITIES: LEVEL 2

Ex 13: Solve $2x - 2 > 4x + 1$. Justify your answer.

Answer:

$$\begin{aligned} 2x-2 &> 4x+1 \\ -2x &> 3 \quad \text{(subtract 4x from both sides and add 2)} \\ x &< -\frac{3}{2} \quad \text{(divide by -2 and reverse inequality)} \end{aligned}$$

The set of solutions is $\left(-\infty, -\frac{3}{2}\right)$, which includes all numbers less than $-\frac{3}{2}$.

Ex 14: Solve $3(2x - 1) \leq 5x + 4$. Justify your answer.

Answer:

$$\begin{aligned} 3(2x-1) &\leq 5x+4 \\ 6x-3 &\leq 5x+4 \quad \text{(distribute 3)} \\ x &\leq 7 \quad \text{(subtract 5x from both sides and add 3)} \end{aligned}$$

The set of solutions is $(-\infty, 7]$, which includes all numbers less than or equal to 7.

Ex 15: Solve $-4x + 7 < 2x - 3$. Justify your answer.

Answer:

$$\begin{aligned} -4x+7 &< 2x-3 \\ -6x &< -10 \quad \text{(subtract 2x from both sides and subtract 7)} \\ x &> \frac{5}{3} \quad \text{(divide by -6 and reverse inequality)} \end{aligned}$$

The set of solutions is $\left(\frac{5}{3}, +\infty\right)$, which includes all numbers greater than $\frac{5}{3}$.

Ex 16: Solve $5 - 2(x + 1) \geq 3x$. Justify your answer.

Answer:

$$\begin{aligned} 5-2(x+1) &\geq 3x \\ 5-2x-2 &\geq 3x \quad \text{(distribute -2)} \\ 3-2x &\geq 3x \quad \text{(combine like terms)} \\ 3 &\geq 5x \quad \text{(add 2x to both sides)} \\ \frac{3}{5} &\geq x \quad \text{(divide by 5)} \end{aligned}$$

The set of solutions is $\left(-\infty, \frac{3}{5}\right]$, which includes all numbers less than or equal to $\frac{3}{5}$.

C SOLVING NON-LINEAR INEQUALITIES USING A SIGN TABLE

C.1 COMPLETING TABLES OF SIGNS FOR LINEAR EXPRESSIONS

Ex 17: Complete the table of signs for the expression $x - 2$.

x	$-\infty$		$+\infty$
$x-2$			

Answer:

- $$\begin{aligned} x-2 &> 0 \\ x &> 2 \quad \text{(add 2 to both sides)} \end{aligned}$$
- So $x - 2$ is positive $(+)$ for $x > 2$ and $x - 2$ is negative $(-)$ for $x < 2$, and zero at $x = 2$.
-

x	$-\infty$		2		$+\infty$
$x-2$		$-$	0	$+$	

Ex 18: Complete the table of signs for the expression $3 - x$.

x	$-\infty$	$+\infty$
$3-x$		

Answer:

- $$\begin{aligned} 3-x &> 0 \\ -x &> -3 \quad \text{(subtract 3)} \\ x &< 3 \quad \text{(multiply by } -1 \text{ and reverse the inequality)} \end{aligned}$$
- So $3-x$ is positive $(+)$ for $x < 3$, zero at $x = 3$, and negative $(-)$ for $x > 3$.
-

x	$-\infty$		3		$+\infty$
$3-x$		$+$	0	$-$	

Ex 19: Complete the table of signs for the expression $2x - 4$.

x	$-\infty$		$+\infty$
$2x-4$			

Answer:

- $$\begin{aligned} 2x - 4 &> 0 \\ 2x &> 4 \quad \text{(add 4 to both sides)} \\ x &> 2 \quad \text{(divide by 2)} \end{aligned}$$
- So $2x - 4$ is positive $(+)$ for $x > 2$, zero at $x = 2$, and negative $(-)$ for $x < 2$.
-

x	$-\infty$		2		$+\infty$
$2x-4$		$-$	0	$+$	

Ex 20: Complete the table of signs for the expression $1 - 2x$.

x	$-\infty$		$+\infty$
$1-2x$			

Answer:

- $$\begin{aligned} 1 - 2x &> 0 \\ -2x &> -1 \quad \text{(subtract 1)} \\ x &< \frac{-1}{-2} \quad \text{(divide by } -2\text{, reverse the inequality)} \\ x &< \frac{1}{2} \end{aligned}$$
- So $1 - 2x$ is positive $(+)$ for $x < \frac{1}{2}$, zero at $x = \frac{1}{2}$, and negative $(-)$ for $x > \frac{1}{2}$.
-

x	$-\infty$		$\frac{1}{2}$		$+\infty$
$1-2x$		$+$	0	$-$	

C.2 READING TABLE OF SIGNS

Ex 21:

x	$-\infty$		0		2		$+\infty$
x		$-$	0	$+$		$+$	
$(x-2)$		$-$		$-$	0	$+$	
$x(x-2)$		$+$	0	$-$	0	$+$	

For $x = -1$, the sign of $x(x-2)$ is $\boxed{+}$.

Answer: By reading the table of signs, since $-1 \in (-\infty, 0)$, the sign of $x(x-2)$ is positive.
Checking directly: $(-1)((-1)-2) = (-1) \times (-3) = 3 > 0$.

Ex 22:

x	$-\infty$		-2		2		$+\infty$
$4-2x$		$+$		$+$	0	$-$	
$2+x$		$-$	0	$+$		$+$	
$(4-2x)(2+x)$		$-$	0	$+$	0	$-$	

For $x = 3$, the sign of $(4-2x)(2+x)$ is $\boxed{-}$.

Answer: By reading the table of signs, since $3 \in (2, +\infty)$, the sign of $(4-2x)(2+x)$ is negative.
Checking directly: $(4-2\times 3)\times(2+3) = (4-6)\times 5 = (-2)\times 5 = -10 < 0$.

Ex 23:

x	$-\infty$		1		2		$+\infty$
$(x-2)$		$-$		$-$	0	$+$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

For $x = \frac{3}{2}$, the sign of $(x-2)(x-1)$ is $\boxed{-}$.

Answer: By reading the table of signs, since $\frac{3}{2} \in (1, 2)$, the sign of $(x-2)(x-1)$ is negative.
Checking directly: $(\frac{3}{2}-2)\times(\frac{3}{2}-1) = (-\frac{1}{2})\times(\frac{1}{2}) = -\frac{1}{4} < 0$.

Ex 24:

x	$-\infty$		-1		$\frac{1}{2}$		$+\infty$
$1-2x$		$+$		$+$	0	$-$	
$-1-x$		$+$	0	$-$		$-$	
$(1-2x)(-1-x)$		$+$	0	$-$	0	$+$	

For $x = 0$, the sign of $(1-2x)(-1-x)$ is $\boxed{-}$.

Answer: By reading the table of signs, since $0 \in (-1, \frac{1}{2})$, the sign of $(1-2x)(-1-x)$ is negative.
Checking directly: $(1-2\times 0)\times(-1-0) = (1)\times(-1) = -1 < 0$.

C.3 IDENTIFYING THE SIGN TABLE FOR EXPRESSIONS

MCQ 25: Choose the correct table of signs for the expression $(x-2)(x-1)$.

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x	$-\infty$		1		2		$+\infty$
$(x-2)$		$+$		$+$	0	$-$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$-$	0	$+$	0	$-$	

☒

x	$-\infty$		1		2		$+\infty$
$(x-2)$		$-$		$-$	0	$+$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

☐

x	$-\infty$		-2		-1		$+\infty$
$(x-2)$		$-$		$+$	0	$+$	
$(x-1)$		$-$	0	$-$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

Answer:

- $(x-2) \geqslant 0$ for $x \geqslant 2$
- $(x-1) \geqslant 0$ for $x \geqslant 1$
- The sign of the product $(x-2)(x-1)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		1		2		$+\infty$
$(x-2)$		$-$		$-$	0	$+$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

MCQ 26: Choose the correct table of signs for the expression $(3-x)(x+1)$.

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x	$-\infty$		-1		3		$+\infty$
$(3-x)$		$+$		$+$	0	$-$	
$(x+1)$		$-$	0	$+$		$+$	
$(3-x)(x+1)$		$-$	0	$+$	0	$-$	

☐

x	$-\infty$		-1		3		$+\infty$
$(3-x)$		$-$		$-$	0	$+$	
$(x+1)$		$-$	0	$+$		$+$	
$(3-x)(x+1)$		$+$	0	$-$	0	$+$	

☐

x	$-\infty$		-1		3		$+\infty$
$(3-x)$		$+$		$+$	0	$-$	
$(x+1)$		$+$	0	$-$		$-$	
$(3-x)(x+1)$		$+$	0	$-$	0	$+$	

Answer:

- $(3-x) \geqslant 0$ for $x \leqslant 3$
- $(x+1) \geqslant 0$ for $x \geqslant -1$
- The sign of the product $(3-x)(x+1)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		-1		3		$+\infty$
$(3-x)$		$+$		$+$	0	$-$	
$(x+1)$		$-$	0	$+$		$+$	
$(3-x)(x+1)$		$-$	0	$+$	0	$-$	

MCQ 27: Choose the correct table of signs for the expression $(2-x)(-2-x)$.

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x	$-\infty$		-2		2		$+\infty$
$(2-x)$		$+$		$+$	0	$-$	
$(-2-x)$		$+$	0	$-$		$-$	
$(2-x)(-2-x)$		$+$	0	$-$	0	$+$	

☐

x	$-\infty$		-2		2		$+\infty$
$(2-x)$		$-$		$-$	0	$+$	
$(-2-x)$		$-$	0	$-$		$+$	
$(2-x)(-2-x)$		$+$	0	$-$	0	$+$	

x	$-\infty$		-2		2		$+\infty$
$(2-x)$		$+$		$+$	0	$-$	
$(-2-x)$		$+$	0	$-$		$+$	
$(2-x)(-2-x)$		$-$	0	$+$	0	$-$	

☐

Answer:

- $(2-x) \geqslant 0$ for $x \leqslant 2$
- $(-2-x) \geqslant 0$ for $x \leqslant -2$
- The sign of the product $(2-x)(-2-x)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		-2		2		$+\infty$
$(2-x)$		$+$		$+$	0	$-$	
$(-2-x)$		$+$	0	$-$		$-$	
$(2-x)(-2-x)$		$+$	0	$-$	0	$+$	

MCQ 28: Choose the correct table of signs for the expression $(2x-1)(x+3)$.

x	$-\infty$		-3		$\frac{1}{2}$		$+\infty$
$(2x-1)$		$-$		$+$	0	$+$	
$(x+3)$		$-$	0	$-$		$+$	
$(2x-1)(x+3)$		$+$	0	$-$	0	$+$	

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x	$-\infty$		-3		$\frac{1}{2}$		$+\infty$
$(2x-1)$		$-$		$-$	0	$+$	
$(x+3)$		$-$	0	$+$		$+$	
$(2x-1)(x+3)$		$+$	0	$-$	0	$+$	

☒

x	$-\infty$		-3		$\frac{1}{2}$		$+\infty$
$(2x-1)$		$+$		$-$	0	$-$	
$(x+3)$		$-$	0	$+$		$+$	
$(2x-1)(x+3)$		$+$	0	$-$	0	$+$	

☐

Answer:

- $(2x-1) \geqslant 0$ for $x \geqslant \frac{1}{2}$
- $(x+3) \geqslant 0$ for $x \geqslant -3$
- The sign of the product $(2x-1)(x+3)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		-3		$\frac{1}{2}$		$+\infty$
$(2x-1)$		$-$		$-$	0	$+$	
$(x+3)$		$-$	0	$+$		$+$	
$(2x-1)(x+3)$		$+$	0	$-$	0	$+$	

C.4 COMPLETING TABLES OF SIGNS FOR FACTORIZED QUADRATIC EXPRESSIONS

Ex 29: Complete the table of signs for the expression $(x-2)(x-1)$.

x	$-\infty$	$+\infty$
$(x-2)$		
$(x-1)$		
$(x-2)(x-1)$		

Answer:

- $(x-2) \geqslant 0$ for $x \geqslant 2$
- $(x-1) \geqslant 0$ for $x \geqslant 1$
- The sign of the product $(x-2)(x-1)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		1		2		$+\infty$
$(x-2)$		$-$		$-$	0	$+$	
$(x-1)$		$-$	0	$+$		$+$	
$(x-2)(x-1)$		$+$	0	$-$	0	$+$	

Ex 30: Complete the table of signs for the expression $x(1-x)$.

x	$-\infty$	$+\infty$
x		
$(1-x)$		
$x(1-x)$		

Answer:

- $(1-x) \geqslant 0$ for $x \leqslant 1$
- The sign of the product $x(1-x)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		0		1		$+\infty$
x		$-$	0	$+$		$+$	
$(1-x)$		$+$		$+$	0	$-$	
$x(1-x)$		$-$	0	$+$	0	$-$	

Ex 31: Complete the table of signs for the expression $(2-x)(3-x)$.

x	$-\infty$	$+\infty$
$(2-x)$		
$(3-x)$		
$(2-x)(3-x)$		

Answer:

- $(2-x) \geqslant 0$ for $x \leqslant 2$
- $(3-x) \geqslant 0$ for $x \leqslant 3$
- The sign of the product $(2-x)(3-x)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		2		3		$+\infty$
$(2-x)$		$+$	0	$-$		$-$	
$(3-x)$		$+$		$+$	0	$-$	
$(2-x)(3-x)$		$+$	0	$-$	0	$+$	

Ex 32: Complete the table of signs for the expression $(2-x)(2+x)$.

x	$-\infty$	$+\infty$
$(2-x)$		
$(2+x)$		
$(2-x)(2+x)$		

Answer:

- $(2-x) \geqslant 0$ for $x \leqslant 2$
- $(2+x) \geqslant 0$ for $x \geqslant -2$
- The sign of the product $(2-x)(2+x)$ is determined by multiplying the signs of the two factors in each interval.

x	$-\infty$		-2		2		$+\infty$
$(2-x)$		$+$		$+$	0	$-$	
$(2+x)$		$-$	0	$+$		$+$	
$(2-x)(2+x)$		$-$	0	$+$	0	$-$	