SOLVING EQUATIONS

A WHAT IS AN EQUATION?

A.1 MATH ESCAPE ROOM: LEVEL 1

MCQ 1: For this Math escape room, the code is:

$$\bigcirc + 5 = 9$$

Which code do you enter?

☐ $\bigcirc = 2$

☒ $\bigcirc = 4$

☐ $\bigcirc = 5$

☐ $\bigcirc = 9$

Answer:

- For $\bigcirc = 2$:
$$(2) + 5 = 9$$
$$7 = 9 \quad \text{(False)}$$
- For $\bigcirc = 4$:
$$(4) + 5 = 9$$
$$9 = 9 \quad \text{(True)}$$
- For $\bigcirc = 5$:
$$(5) + 5 = 9$$
$$10 = 9 \quad \text{(False)}$$
- For $\bigcirc = 9$:
$$(9) + 5 = 9$$
$$14 = 9 \quad \text{(False)}$$

Therefore, the correct code to enter is $\bigcirc = 4$.

MCQ 2: For this Math escape room, the code is:

$$\triangle + 10 = 1 + 2 \times 6$$

Which code do you enter?

☒ $\triangle = 3$

☐ $\triangle = 5$

☐ $\triangle = 8$

☐ $\triangle = 10$

Answer:

- For $\triangle = 3$:
$$(3) + 10 = 1 + 2 \times 6$$
$$13 = 1 + 12$$
$$13 = 13 \quad \text{(True)}$$
- For $\triangle = 5$:
$$(5) + 10 = 1 + 2 \times 6$$
$$15 = 1 + 12$$
$$15 = 13 \quad \text{(False)}$$
- For $\triangle = 8$:
$$(8) + 10 = 1 + 2 \times 6$$
$$18 = 1 + 12$$
$$18 = 13 \quad \text{(False)}$$
- For $\triangle = 10$:
$$(10) + 10 = 1 + 2 \times 6$$
$$20 = 1 + 12$$
$$20 = 13 \quad \text{(False)}$$

Therefore, the correct code to enter is $\triangle = 3$.

MCQ 3: For this Math escape room, the code is:

$$\square + 5 = 2 \times 4 + 1$$

Which code do you enter?

☐ $\square = 6$

☐ $\square = 8$

☐ $\square = 5$

☒ $\square = 4$

Answer:

- For $\square = 6$:
$$(6) + 5 = 2 \times 4 + 1$$
$$11 = 8 + 1$$
$$11 = 9 \quad \text{(False)}$$
- For $\square = 8$:
$$(8) + 5 = 2 \times 4 + 1$$
$$13 = 8 + 1$$
$$13 = 9 \quad \text{(False)}$$
- For $\square = 5$:
$$(5) + 5 = 2 \times 4 + 1$$
$$10 = 8 + 1$$
$$10 = 9 \quad \text{(False)}$$
- For $\square = 4$:
$$(4) + 5 = 2 \times 4 + 1$$
$$9 = 8 + 1$$
$$9 = 9 \quad \text{(True)}$$

Therefore, the correct code to enter is $\square = 4$.

MCQ 4: For this Math escape room, the code is:

$$\bigcirc - 4 = 3 \times 2 - 1$$

Which code do you enter?

☐ $\bigcirc = 7$

☐ $\bigcirc = 6$

☐ $\bigcirc = 5$

☒ $\bigcirc = 9$

Answer:

- For $\bigcirc = 7$:

$$\begin{aligned} (7) - 4 &= 3 \times 2 - 1 \\ 3 &= 6 - 1 \\ 3 &= 5 \qquad \text{(False)} \end{aligned}$$

- For $\bigcirc = 6$:

$$\begin{aligned} (6) - 4 &= 3 \times 2 - 1 \\ 2 &= 6 - 1 \\ 2 &= 5 \qquad \text{(False)} \end{aligned}$$

- For $\bigcirc = 5$:

$$\begin{aligned} (5) - 4 &= 3 \times 2 - 1 \\ 1 &= 6 - 1 \\ 1 &= 5 \qquad \text{(False)} \end{aligned}$$

- For $\bigcirc = 9$:

$$\begin{aligned} (9) - 4 &= 3 \times 2 - 1 \\ 5 &= 6 - 1 \\ 5 &= 5 \qquad \text{(True)} \end{aligned}$$

Therefore, the correct code to enter is $\bigcirc = 9$.

A.2 MATH ESCAPE ROOM: LEVEL 2

MCQ 5: For this Math escape room, the code is:

$$2 \times \bigcirc - 2 = \bigcirc + 10$$

Which code do you enter?

☐ $\bigcirc = 8$

☐ $\bigcirc = 10$

☒ $\bigcirc = 12$

☐ $\bigcirc = 14$

Answer:

- For $\bigcirc = 8$:

$$\begin{aligned} 2 \times (8) - 2 &= (8) + 10 \\ 16 - 2 &= 8 + 10 \\ 14 &= 18 \qquad \text{(False)} \end{aligned}$$

- For $\bigcirc = 10$:

$$\begin{aligned} 2 \times (10) - 2 &= (10) + 10 \\ 20 - 2 &= 10 + 10 \\ 18 &= 20 \qquad \text{(False)} \end{aligned}$$

- For $\bigcirc = 12$:

$$\begin{aligned} 2 \times (12) - 2 &= (12) + 10 \\ 24 - 2 &= 12 + 10 \\ 22 &= 22 \qquad \text{(True)} \end{aligned}$$

- For $\bigcirc = 14$:

$$\begin{aligned} 2 \times (14) - 2 &= (14) + 10 \\ 28 - 2 &= 14 + 10 \\ 26 &= 24 \qquad \text{(False)} \end{aligned}$$

Therefore, the correct code to enter is $\bigcirc = 12$.

MCQ 6: For this Math escape room, the code is:

$$3x + 7 = x + 19$$

Which code do you enter?

☐ $x = 2$

☐ $x = 4$

☒ $x = 6$

☐ $x = 8$

Answer:

- For $x = 2$:

$$\begin{aligned} 3 \times (2) + 7 &= (2) + 19 \\ 6 + 7 &= 2 + 19 \\ 13 &= 21 \qquad \text{(False)} \end{aligned}$$

- For $x = 4$:

$$\begin{aligned} 3 \times (4) + 7 &= (4) + 19 \\ 12 + 7 &= 4 + 19 \\ 19 &= 23 \qquad \text{(False)} \end{aligned}$$

- For $x = 6$:

$$\begin{aligned} 3 \times (6) + 7 &= (6) + 19 \\ 18 + 7 &= 6 + 19 \\ 25 &= 25 \qquad \text{(True)} \end{aligned}$$

- For $x = 8$:

$$\begin{aligned} 3 \times (8) + 7 &= (8) + 19 \\ 24 + 7 &= 8 + 19 \\ 31 &= 27 \qquad \text{(False)} \end{aligned}$$

Therefore, the correct code to enter is $x = 6$.

MCQ 7: For this Math escape room, the code is:

$$2x - 2 = x + 10$$

Which code do you enter?

☐ $x = 6$

☐ $x = 8$

☐ $x = 10$

☒ $x = 12$

Answer:

- For $x = 6$:

$$\begin{aligned} 2 \times (6) - 2 &= (6) + 10 \\ 12 - 2 &= 6 + 10 \\ 10 &= 16 \qquad \text{(False)} \end{aligned}$$

- For $x = 8$:

$$\begin{aligned} 2 \times (8) - 2 &= (8) + 10 \\ 16 - 2 &= 8 + 10 \\ 14 &= 18 \qquad \text{(False)} \end{aligned}$$

- For $x = 10$:

$$\begin{aligned} 2 \times (10) - 2 &= (10) + 10 \\ 20 - 2 &= 10 + 10 \\ 18 &= 20 \qquad \text{(False)} \end{aligned}$$

- For $x = 12$:

$$\begin{aligned} 2 \times (12) - 2 &= (12) + 10 \\ 24 - 2 &= 12 + 10 \\ 22 &= 22 \qquad \text{(True)} \end{aligned}$$

Therefore, the correct code to enter is $x = 12$.

MCQ 8: For this Math escape room, the code is:

$$x \times (x - 2) = 24$$

Which code do you enter?

☒ $x = 6$

☐ $x = 7$

☐ $x = 8$

☐ $x = 9$

Answer:

- For $x = 6$:

$$\begin{aligned} (6) \times ((6) - 2) &= 24 \\ 6 \times 4 &= 24 \\ 24 &= 24 \quad \text{(True)} \end{aligned}$$

- For $x = 7$:

$$\begin{aligned} (7) \times ((7) - 2) &= 24 \\ 7 \times 5 &= 24 \\ 35 &= 24 \quad \text{(False)} \end{aligned}$$

- For $x = 8$:

$$\begin{aligned} (8) \times ((8) - 2) &= 24 \\ 8 \times 6 &= 24 \\ 48 &= 24 \quad \text{(False)} \end{aligned}$$

- For $x = 9$:

$$\begin{aligned} (9) \times ((9) - 2) &= 24 \\ 9 \times 7 &= 24 \\ 63 &= 24 \quad \text{(False)} \end{aligned}$$

Therefore, the correct code to enter is $x = 6$.

A.3 MATH ESCAPE ROOM: LEVEL 3

MCQ 9: For this Math escape room, the code is:

$$x^2 - 4 = 0$$

Which code do you enter?

☒ $x = 2$

☐ $x = 3$

☒ $x = -2$

☐ $x = -3$

Answer:

- For $x = 2$:

$$\begin{aligned} (2)^2 - 4 &= 0 \\ 4 - 4 &= 0 \\ 0 &= 0 \quad \text{(True)} \end{aligned}$$

$x = 2$ is a solution.

- For $x = 3$:

$$\begin{aligned} (3)^2 - 4 &= 0 \\ 9 - 4 &= 0 \\ 5 &= 0 \quad \text{(False)} \end{aligned}$$

- For $x = -2$:

$$\begin{aligned} (-2)^2 - 4 &= 0 \\ 4 - 4 &= 0 \\ 0 &= 0 \quad \text{(True)} \end{aligned}$$

$x = -2$ is a solution.

- For $x = -3$:

$$\begin{aligned} (-3)^2 - 4 &= 0 \\ 9 - 4 &= 0 \\ 5 &= 0 \quad \text{(False)} \end{aligned}$$

Therefore, the correct code to enter is $x = -2$ or $x = 2$.

MCQ 10: For this Math escape room, the code is:

$$x^2 - 2x + 1 = 0$$

Which code do you enter?

☐ $x = 0$

☒ $x = 1$

☐ $x = 2$

☐ $x = 3$

Answer:

- For $x = 0$:

$$\begin{aligned} (0)^2 - 2 \times (0) + 1 &= 0 \\ 0 - 0 + 1 &= 0 \\ 1 &= 0 \quad \text{(False)} \end{aligned}$$

- For $x = 1$:

$$\begin{aligned} (1)^2 - 2 \times (1) + 1 &= 0 \\ 1 - 2 + 1 &= 0 \\ 0 &= 0 \quad \text{(True)} \end{aligned}$$

- For $x = 2$:

$$\begin{aligned}(2)^2 - 2 \times (2) + 1 &= 0\\ 4 - 4 + 1 &= 0\\ 1 &= 0 \quad \text{(False)}\end{aligned}$$

- For $x = 3$:

$$\begin{aligned}(3)^2 - 2 \times (3) + 1 &= 0\\ 9 - 6 + 1 &= 0\\ 4 &= 0 \quad \text{(False)}\end{aligned}$$

Therefore, the correct code to enter is $x = 1$.

MCQ 11: For this Math escape room, the code is:

$$\frac{2x+1}{x-1} = 3$$

Which code do you enter?

☐ $x = 2$

☐ $x = 3$

☒ $x = 4$

☐ $x = 5$

Answer:

- For $x = 2$:

$$\begin{aligned}\frac{2 \times (2) + 1}{(2) - 1} &= 3\\ \frac{4+1}{2-1} &= 3\\ \frac{5}{1} &= 3\\ 5 &= 3 \quad \text{(False)}\end{aligned}$$

- For $x = 3$:

$$\begin{aligned}\frac{2 \times (3) + 1}{(3) - 1} &= 3\\ \frac{6+1}{3-1} &= 3\\ \frac{7}{2} &= 3\\ 3.5 &= 3 \quad \text{(False)}\end{aligned}$$

- For $x = 4$:

$$\begin{aligned}\frac{2 \times (4) + 1}{(4) - 1} &= 3\\ \frac{8+1}{4-1} &= 3\\ \frac{9}{3} &= 3\\ 3 &= 3 \quad \text{(True)}\end{aligned}$$

- For $x = 5$:

$$\begin{aligned}\frac{2 \times (5) + 1}{(5) - 1} &= 3\\ \frac{10+1}{5-1} &= 3\\ \frac{11}{4} &= 3\\ 2.75 &= 3 \quad \text{(False)}\end{aligned}$$

Therefore, the correct code to enter is $x = 4$.

B SOLVING BY INSPECTION AND TRIAL-AND-ERROR

B.1 FINDING A SOLUTION: LEVEL 1

Ex 12: Consider the equation $2x + 3 = 11$.
Use the trial-and-error method to find a solution (try $x = 2, 3, \ldots$).

$$x = \boxed{4}$$

Answer:

- Try $x = 2$:

$$\begin{aligned}2 \times (2) + 3 &= 11\\ 4 + 3 &= 11\\ 7 &= 11 \quad \text{(False)}\end{aligned}$$

- Try $x = 3$:

$$\begin{aligned}2 \times (3) + 3 &= 11\\ 6 + 3 &= 11\\ 9 &= 11 \quad \text{(False)}\end{aligned}$$

- Try $x = 4$:

$$\begin{aligned}2 \times (4) + 3 &= 11\\ 8 + 3 &= 11\\ 11 &= 11 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $2x + 3 = 11$ is $x = 4$.

Ex 13: Consider the equation $3x - 5 = 10$.
Use the trial-and-error method to find a solution (try $x = 4, 5, \ldots$).

$$x = \boxed{5}$$

Answer:

- Try $x = 4$:

$$\begin{aligned}3 \times (4) - 5 &= 10\\ 12 - 5 &= 10\\ 7 &= 10 \quad \text{(False)}\end{aligned}$$

- Try $x = 5$:

$$\begin{aligned}3 \times (5) - 5 &= 10\\ 15 - 5 &= 10\\ 10 &= 10 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $3x - 5 = 10$ is $x = 5$.

Ex 14: Consider the equation $x(x - 1) = 6$.
Use the trial-and-error method to find a solution (try $x = 2, 3, \ldots$).

$$x = \boxed{3}$$

Answer:

- Try $x = 2$:

$$\begin{aligned}(2) \times ((2) - 1) &= 6\\ 2 \times 1 &= 6\\ 2 &= 6 \quad \text{(False)}\end{aligned}$$

- Try $x = 3$:

$$\begin{aligned}(3) \times ((3) - 1) &= 6\\ 3 \times 2 &= 6\\ 6 &= 6 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $x(x-1) = 6$ is $x = 3$.

Ex 15: Consider the equation $2x - 3 = 5x - 9$.
Use the trial-and-error method to find a solution (try $x = 0, 1, \ldots$).

$$x = \boxed{2}$$

Answer:

- Try $x = 0$:

$$\begin{aligned}2 \times (0) - 3 &= 5 \times (0) - 9\\ 0 - 3 &= 0 - 9\\ -3 &= -9 \quad \text{(False)}\end{aligned}$$

- Try $x = 1$:

$$\begin{aligned}2 \times (1) - 3 &= 5 \times (1) - 9\\ 2 - 3 &= 5 - 9\\ -1 &= -4 \quad \text{(False)}\end{aligned}$$

- Try $x = 2$:

$$\begin{aligned}2 \times (2) - 3 &= 5 \times (2) - 9\\ 4 - 3 &= 10 - 9\\ 1 &= 1 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $2x - 3 = 5x - 9$ is $x = 2$.

B.2 FINDING A SOLUTION: LEVEL 2

Ex 16: Consider the equation $x^2 - 2x + 1 = 0$.
Use the trial-and-error method to find a solution (try $x = 0, 1, \ldots$).

$$x = \boxed{1}$$

Answer:

- Try $x = 0$:

$$\begin{aligned}(0)^2 - 2 \times (0) + 1 &= 0\\ 0 - 0 + 1 &= 0\\ 1 &= 0 \quad \text{(False)}\end{aligned}$$

- Try $x = 1$:

$$\begin{aligned}(1)^2 - 2 \times (1) + 1 &= 0\\ 1 - 2 + 1 &= 0\\ 0 &= 0 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $x^2 - 2x + 1 = 0$ is $x = 1$.

Ex 17: Consider the equation $x^2 - 9 = 0$.
Use the trial-and-error method to find a solution (try $x = 2, 3, \ldots$).

$$x = \boxed{3}$$

Answer:

- Try $x = 2$:

$$\begin{aligned}(2)^2 - 9 &= 0\\ 4 - 9 &= 0\\ -5 &= 0 \quad \text{(False)}\end{aligned}$$

- Try $x = 3$:

$$\begin{aligned}(3)^2 - 9 &= 0\\ 9 - 9 &= 0\\ 0 &= 0 \quad \text{(True)}\end{aligned}$$

Therefore, a solution to the equation $x^2 - 9 = 0$ is $x = 3$.

Ex 18: Consider the equation $\dfrac{x+2}{x-2} = 2$.
Use the trial-and-error method to find a solution (try $x = 6, 3, 4$).

$$x = \boxed{6}$$

Answer:

- Try $x = 6$:

$$\begin{aligned}\frac{6+2}{6-2} &= 2\\ \frac{8}{4} &= 2\\ 2 &= 2 \quad \text{(True)}\end{aligned}$$

- Try $x = 3$:

$$\begin{aligned}\frac{3+2}{3-2} &= 2\\ \frac{5}{1} &= 2\\ 5 &= 2 \quad \text{(False)}\end{aligned}$$

- Try $x = 4$:

$$\begin{aligned}\frac{4+2}{4-2} &= 2\\ \frac{6}{2} &= 2\\ 3 &= 2 \quad \text{(False)}\end{aligned}$$

Therefore, a solution to the equation $\frac{x+2}{x-2} = 2$ is $x = 6$.

C THE PRINCIPLE OF BALANCE

C.1 SOLVING EQUATIONS BY ADDING OR SUBTRACTING

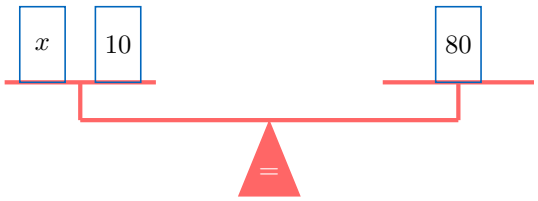
Ex 19: Solve for x:

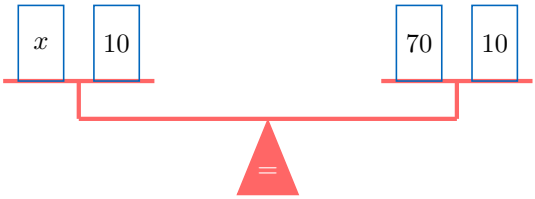
$$x + 10 = 80$$

$$x = \boxed{70}$$

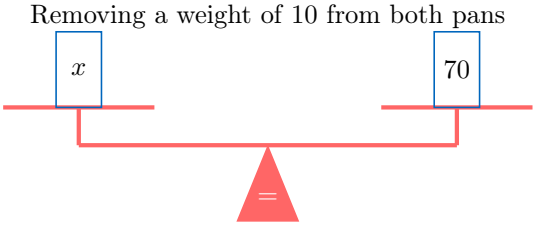
Answer:

- Balance scale:





Removing a weight of 10 from both pans



- Equivalent equations:

$$\begin{aligned} x + 10 &= 80 \\ x + 10 - 10 &= 80 - 10 \qquad (-10) \\ x + 0 &= 70 \\ x &= 70 \end{aligned}$$

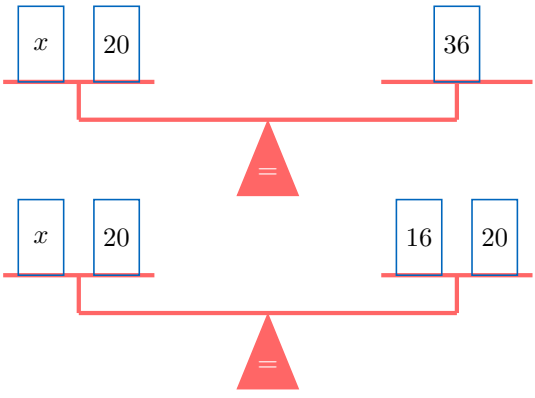
Ex 20: Solve for x:

$$x + 20 = 36$$

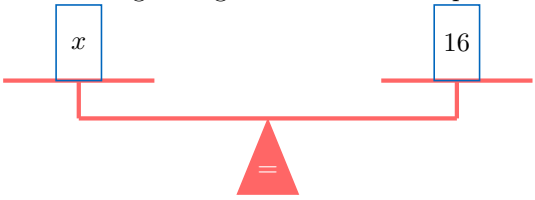
$$x = \boxed{16}$$

Answer:

- Balance scale:



Removing a weight of 20 from both pans



- Equivalent equations:

$$\begin{aligned} x + 20 &= 36 \\ x + 20 - 20 &= 36 - 20 \qquad (-20) \\ x + 0 &= 16 \\ x &= 16 \end{aligned}$$

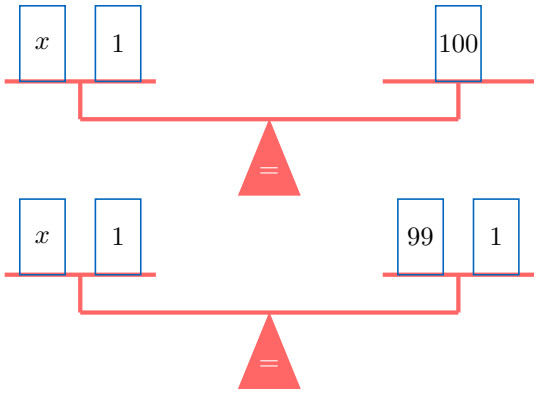
Ex 21: Solve for x:

$$x + 1 = 100$$

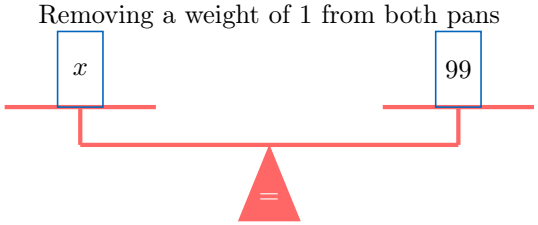
$$x = \boxed{99}$$

Answer:

- Balance scale:



Removing a weight of 1 from both pans



- Equivalent equations:

$$\begin{aligned} x + 1 &= 100 \\ x + 1 - 1 &= 100 - 1 \qquad (-1) \\ x + 0 &= 99 \\ x &= 99 \end{aligned}$$

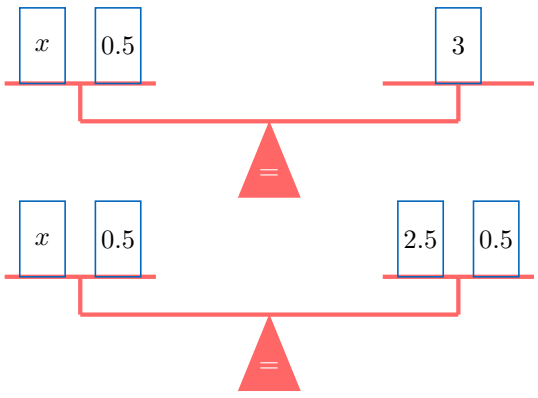
Ex 22: Solve for x:

$$x + 0.5 = 3$$

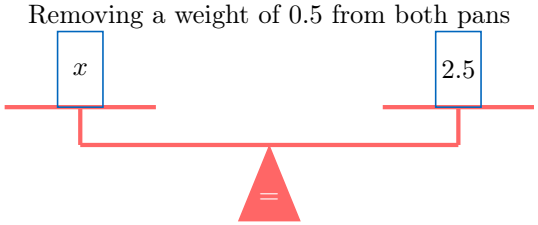
$$x = \boxed{2.5}$$

Answer:

- Balance scale:



Removing a weight of 0.5 from both pans



- Equivalent equations:

$$\begin{aligned} x + 0.5 &= 3 \\ x + 0.5 - 0.5 &= 3 - 0.5 \qquad (-0.5) \\ x + 0 &= 2.5 \\ x &= 2.5 \end{aligned}$$

Ex 23: Solve for x:

$$x - 10 = -20$$

$$x = \boxed{-10}$$

Answer:

$$\begin{aligned} x - 10 &= -20 \\ x - 10 \,{+10} &= -20 \,{+10} \qquad (+10) \\ x + 0 &= -10 \\ x &= -10 \end{aligned}$$

Ex 24: Solve for x:

$$x - 5 = -2$$

$$x = \boxed{3}$$

Answer:

$$\begin{aligned} x - 5 &= -2 \\ x - 5 \,{+5} &= -2 \,{+5} \qquad (+5) \\ x + 0 &= 3 \\ x &= 3 \end{aligned}$$

C.2 SOLVING EQUATIONS BY MULTIPLYING OR DIVIDING

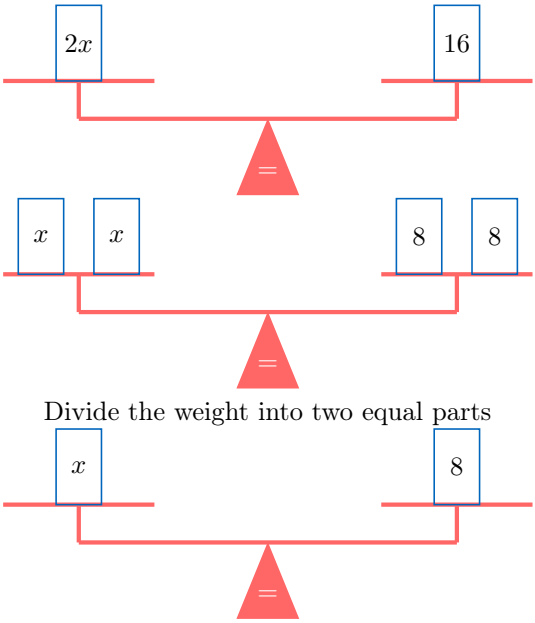
Ex 25: Solve for x:

$$2x = 16$$

$$x = \boxed{8}$$

Answer:

- Balance scale:



- Equivalent equations:

$$\begin{aligned} 2x &= 16 \\ \frac{2x}{2} &= \frac{16}{2} \qquad (\div 2) \\ x &= 8 \end{aligned}$$

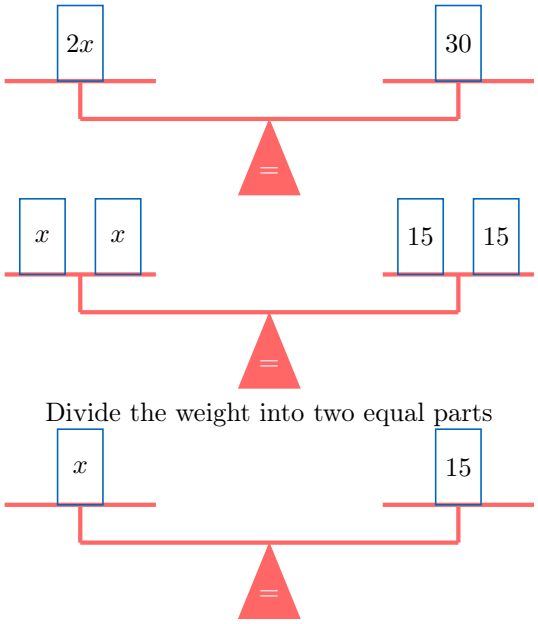
Ex 26: Solve for x:

$$2x = 30$$

$$x = \boxed{15}$$

Answer:

- Balance scale:



Divide the weight into two equal parts

- Equivalent equations:

$$\begin{aligned} 2x &= 30 \\ \frac{2x}{2} &= \frac{30}{2} \qquad (\div 2) \\ x &= 15 \end{aligned}$$

Ex 27: Solve for x:

$$3x = 27$$

$$x = \boxed{9}$$

Answer:

$$\begin{aligned} 3x &= 27 \\ \frac{3x}{3} &= \frac{27}{3} \qquad (\div 3) \\ x &= 9 \end{aligned}$$

Ex 28: Solve for x:

$$\frac{x}{4} = 5$$

$$x = \boxed{20}$$

Answer:

$$\begin{aligned} \frac{x}{4} &= 5 \\ \frac{x}{4} \,{\times 4} &= 5 \,{\times 4} \qquad (\times 4) \\ x &= 20 \end{aligned}$$

Ex 29: Solve for x:

$$\frac{x}{3} = \frac{1}{2}$$

$$x = \boxed{\frac{3}{2}}$$

Answer:

$$\begin{aligned} \frac{x}{3} &= \frac{1}{2} \\ \frac{x}{3} \times 3 &= \frac{1}{2} \times 3 \qquad (\times 3) \\ x &= \frac{3}{2} \end{aligned}$$

D SOLVING BY REVERSING OPERATIONS

D.1 DOING EXPRESSIONS

Ex 30: Build the expression $3 + 5x$

$$\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{+3} \boxed{3 + 5x}$$

Answer: To do the expression $3 + 5x$, follow the steps:

$$\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{+3} \boxed{3 + 5x}$$

Ex 31: Build the expression $2x + 1$

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

Answer: To do the expression $2x + 1$, follow the steps:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

Ex 32: Build the expression $5x - 1$

$$\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{-1} \boxed{5x - 1}$$

Answer: To do the expression $5x - 1$, follow the steps:

$$\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{-1} \boxed{5x - 1}$$

Ex 33: Build the expression $\frac{x}{2} + 3$

$$\boxed{x} \xrightarrow{\div 2} \boxed{x/2} \xrightarrow{+3} \boxed{x/2 + 3}$$

Answer: To do the expression $\frac{x}{2} + 3$, follow the steps:

$$\boxed{x} \xrightarrow{\div 2} \boxed{\frac{x}{2}} \xrightarrow{+3} \boxed{\frac{x}{2} + 3}$$

Ex 34: Build the expression $2 - 5x$

$$\boxed{x} \xrightarrow{\times(-5)} \boxed{-5x} \xrightarrow{+2} \boxed{2 - 5x}$$

Answer: To do the expression $2 - 5x$, follow the steps:

$$\boxed{x} \xrightarrow{\times(-5)} \boxed{-5x} \xrightarrow{+2} \boxed{2 - 5x}$$

Ex 35: Do the expression $2(x - 3)$

$$\boxed{x} \xrightarrow{-3} \boxed{x - 3} \xrightarrow{\times 2} \boxed{2(x - 3)}$$

Answer: To do the expression $2(x - 3)$, follow the steps:

$$\boxed{x} \xrightarrow{-3} \boxed{x - 3} \xrightarrow{\times 2} \boxed{2(x - 3)}$$

Ex 36: Build the expression $\frac{x - 3}{2}$

$$\boxed{x} \xrightarrow{-3} \boxed{x - 3} \xrightarrow{\div 2} \boxed{(x - 3)/2}$$

Answer: To do the expression $\frac{x-3}{2}$, follow the steps:

$$\boxed{x} \xrightarrow{-3} \boxed{x - 3} \xrightarrow{\div 2} \boxed{\frac{x - 3}{2}}$$

D.2 UNDOING EXPRESSIONS

Ex 37: Undo the expression $2x + 1$

$$\boxed{2x + 1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

Answer:

- To do the expression $2x + 1$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

- To undo the expression $2x + 1$, perform inverse operations in the reverse order:

$$\boxed{2x + 1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

Ex 38: Undo the expression $3x - 2$

$$\boxed{3x - 2} \xrightarrow{+2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$$

Answer:

- To do the expression $3x - 2$:

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-2} \boxed{3x - 2}$$

- To undo the expression $3x - 2$, perform inverse operations in the reverse order:

$$\boxed{3x - 2} \xrightarrow{+2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$$

Ex 39: Undo the expression $\frac{x}{3} + 2$

$$\boxed{x/3 + 2} \xrightarrow{-2} \boxed{x/3} \xrightarrow{\times 3} \boxed{x}$$

Answer:

- To do the expression $\frac{x}{3} + 2$:

$$\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{+2} \boxed{\frac{x}{3} + 2}$$

- To undo the expression $\frac{x}{3} + 2$, perform inverse operations in the reverse order:

$$\boxed{\frac{x}{3} + 2} \xrightarrow{-2} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$$

Ex 40: Undo the expression $\dfrac{x+2}{5}$

$$\boxed{\boxed{(x+2)/5}} \xrightarrow{\times 5} \boxed{\boxed{x+2}} \xrightarrow{-2} \boxed{\boxed{x}}$$

Answer:

- To do the expression $\frac{x+2}{5}$:

$$\boxed{x} \xrightarrow{+2} \boxed{x+2} \xrightarrow{\div 5} \boxed{\frac{x+2}{5}}$$

- To undo the expression $\frac{x+2}{5}$, perform inverse operations in the reverse order:

$$\boxed{\frac{x+2}{5}} \xrightarrow{\times 5} \boxed{x+2} \xrightarrow{-2} \boxed{x}$$

D.3 SOLVING LINEAR EQUATIONS: LEVEL 1

Ex 41: Solve for x:

$$2x+1=7$$

$$x = \boxed{3}$$

Answer:

- To **do** the expression $2x+1$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x+1}$$

- To **undo** the expression $2x+1$, do the inverse operations in reverse order:

$$\boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

- So,

$$\begin{aligned} 2x+1 &= 7 \\ 2x+1\color{red}{-1} &= 7\color{red}{-1} \quad (\color{red}{-1}) \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \quad (\div 2) \\ x &= 3 \end{aligned}$$

Ex 42: Solve for x:

$$2x-4=5$$

$$x = \boxed{\frac{9}{2}}$$

Answer:

- To **do** the expression $2x-4$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-4} \boxed{2x-4}$$

- To **undo** the expression $2x-4$, do the inverse operations in reverse order:

$$\boxed{2x-4} \xrightarrow{+4} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

- So,

$$\begin{aligned} 2x-4 &= 5 \\ 2x-4\color{red}{+4} &= 5\color{red}{+4} \quad (\color{red}{+4}) \\ 2x &= 9 \\ \frac{2x}{2} &= \frac{9}{2} \quad (\div 2) \\ x &= \frac{9}{2} \end{aligned}$$

Ex 43: Solve for x:

$$4x-7=9$$

$$x = \boxed{4}$$

Answer:

- To **do** the expression $4x-7$:

$$\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{-7} \boxed{4x-7}$$

- To **undo** the expression $4x-7$, do the inverse operations in reverse order:

$$\boxed{4x-7} \xrightarrow{+7} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$$

- So,

$$\begin{aligned} 4x-7 &= 9 \\ 4x-7\color{red}{+7} &= 9\color{red}{+7} \quad (\color{red}{+7}) \\ 4x &= 16 \\ \frac{4x}{4} &= \frac{16}{4} \quad (\div 4) \\ x &= 4 \end{aligned}$$

Ex 44: Solve for x:

$$3x+10=4$$

$$x = \boxed{-2}$$

Answer:

- To **do** the expression $3x+10$:

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+10} \boxed{3x+10}$$

- To **undo** the expression $3x+10$, do the inverse operations in reverse order:

$$\boxed{3x+10} \xrightarrow{-10} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$$

- So,

$$\begin{aligned} 3x+10 &= 4 \\ 3x+10\color{red}{-10} &= 4\color{red}{-10} \quad (\color{red}{-10}) \\ 3x &= -6 \\ \frac{3x}{3} &= \frac{-6}{3} \quad (\div 3) \\ x &= -2 \end{aligned}$$

D.4 SOLVING LINEAR EQUATIONS: LEVEL 2

Ex 45: Solve for x:

$$\frac{x-17}{3} = 10$$

$$x = \boxed{47}$$

Answer:

- To **do** the expression $\dfrac{x-17}{3}$:

$$\boxed{x} \xrightarrow{-17} \boxed{x-17} \xrightarrow{\div 3} \boxed{\frac{x-17}{3}}$$

- To **undo** the expression $\dfrac{x-17}{3}$, do the inverse operations in reverse order:

$$\boxed{\frac{x-17}{3}} \xrightarrow{\times 3} \boxed{x-17} \xrightarrow{+17} \boxed{x}$$

- So,

$$\begin{aligned} \frac{x-17}{3} &= 10 \\ \frac{x-17}{3}\times 3 &= 10\times 3 \quad (\times 3) \\ x-17 &= 30 \\ x-17+17 &= 30+17 \quad (+17) \\ x &= 47 \end{aligned}$$

Ex 46: Solve for x:

$$\frac{x-2}{6} = \frac{2}{3}$$

$$x = \boxed{6}$$

Answer:

- To **do** the expression $\dfrac{x-2}{6}$:

$$\boxed{x} \xrightarrow{-2} \boxed{x-2} \xrightarrow{\div 6} \boxed{\frac{x-2}{6}}$$

- To **undo** the expression $\dfrac{x-2}{6}$, do the inverse operations in reverse order:

$$\boxed{\frac{x-2}{6}} \xrightarrow{\times 6} \boxed{x-2} \xrightarrow{+2} \boxed{x}$$

- So,

$$\begin{aligned} \frac{x-2}{6} &= \frac{2}{3} \\ \frac{x-2}{6}\times 6 &= \frac{2}{3}\times 6 \quad (\times 6) \\ x-2 &= 4 \\ x-2+2 &= 4+2 \quad (+2) \\ x &= 6 \end{aligned}$$

Ex 47: Solve for x:

$$4(x+2) = 40$$

$$x = \boxed{8}$$

Answer:

- To **do** the expression $4(x+2)$:

$$\boxed{x} \xrightarrow{+2} \boxed{x+2} \xrightarrow{\times 4} \boxed{4(x+2)}$$

- To **undo** the expression $4(x+2)$, do the inverse operations in reverse order:

$$\boxed{4(x+2)} \xrightarrow{\div 4} \boxed{x+2} \xrightarrow{-2} \boxed{x}$$

- So,

$$\begin{aligned} 4(x+2) &= 40 \\ 4(x+2)\div 4 &= 40\div 4 \quad (\div 4) \\ x+2 &= 10 \\ x+2-2 &= 10-2 \quad (-2) \\ x &= 8 \end{aligned}$$

Ex 48: Solve for x:

$$\frac{2x+5}{4} = 3$$

$$x = \boxed{\frac{7}{2}}$$

Answer:

- To **do** the expression $\dfrac{2x+5}{4}$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5} \xrightarrow{\div 4} \boxed{\frac{2x+5}{4}}$$

- To **undo** the expression $\dfrac{2x+5}{4}$, do the inverse operations in reverse order:

$$\boxed{\frac{2x+5}{4}} \xrightarrow{\times 4} \boxed{2x+5} \xrightarrow{-5} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

- So,

$$\begin{aligned} \frac{2x+5}{4} &= 3 \\ \frac{2x+5}{4}\times 4 &= 3\times 4 \quad (\times 4) \\ 2x+5 &= 12 \\ 2x+5-5 &= 12-5 \quad (-5) \\ 2x &= 7 \\ \frac{2x}{2} &= \frac{7}{2} \quad (\div 2) \\ x &= \frac{7}{2} \end{aligned}$$

E SOLVING PRODUCT OF LINEAR FACTORS

E.1 SOLVING EQUATIONS USING THE NULL FACTOR LAW: LEVEL 1

Ex 49: Solve for x:

$$x(x+1)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-1}, \boxed{0}\}$.

Answer:

$$\begin{aligned} x(x+1)&=0\\ x=0 \text{ or } (x+1)&=0 \text{ (null factor law)}\\ x=0 \text{ or } x&=-1 \end{aligned}$$

The set of solutions is $\boxed{\{-1,0\}}$.
Verification:

- for $x=0$, $0(0+1)=0$
- for $x=-1$, $(-1)((-1)+1)=(-1)\times 0=0$

Ex 50: Solve for x:

$$(x+2)(x-1)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-2}, \boxed{1}\}$.

Answer:

$$\begin{aligned} (x+2)(x-1)&=0\\ x+2=0 \text{ or } x-1&=0 \text{ (null factor law)}\\ x=-2 \text{ or } x&=1 \end{aligned}$$

The set of solutions is $\boxed{\{-2,\ 1\}}$.
Verification:

- For $x=-2$: $(-2+2)\times(-2-1)=0\times(-3)=0$
- For $x=1$: $(1+2)\times(1-1)=3\times 0=0$

Ex 51: Solve for x:

$$(x+6)(x-3)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-6}, \boxed{3}\}$.

Answer:

$$\begin{aligned} (x+6)(x-3)&=0\\ x+6=0 \text{ or } x-3&=0 \text{ (null factor law)}\\ x=-6 \text{ or } x&=3 \end{aligned}$$

The set of solutions is $\boxed{\{-6,\ 3\}}$.
Verification:

- For $x=-6$: $(-6+6)\times(-6-3)=0\times(-9)=0$
- For $x=3$: $(3+6)\times(3-3)=9\times 0=0$

Ex 52: Solve for x:

$$(x-1)(x-2)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{1}, \boxed{2}\}$.

Answer:

$$\begin{aligned} (x-1)(x-2)&=0\\ x-1=0 \text{ or } x-2&=0 \text{ (null factor law)}\\ x=1 \text{ or } x&=2 \end{aligned}$$

The set of solutions is $\boxed{\{1,\ 2\}}$.
Verification:

- For $x=1$: $(1-1)\times(1-2)=0\times(-1)=0$
- For $x=2$: $(2-1)\times(2-2)=1\times 0=0$

E.2 SOLVING EQUATIONS USING THE NULL FACTOR LAW: LEVEL 2

Ex 53: Solve for x:

$$(2x+6)(x+2)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-3}, \boxed{-2}\}$.

Answer:

$$\begin{aligned} (2x+6)(x+2)&=0\\ 2x+6=0 \text{ or } x+2&=0 \text{ (null factor law)}\\ 2x=-6 \text{ or } x&=-2\\ x=-3 \text{ or } x&=-2 \end{aligned}$$

The set of solutions is $\boxed{\{-3,\ -2\}}$.
Verification:

- For $x=-3$: $(2\times(-3)+6)((-3)+2)=0\times(-1)=0$
- For $x=-2$: $(2\times(-2)+6)((-2)+2)=2\times 0=0$

Ex 54: Solve for x:

$$(x+2)(2x-1)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-2}, \boxed{\frac{1}{2}}\}$.

Answer:

$$\begin{aligned} (x+2)(2x-1)&=0\\ x+2=0 \text{ or } 2x-1&=0 \text{ (null factor law)}\\ x=-2 \text{ or } 2x&=1\\ x&=\frac{1}{2} \end{aligned}$$

The set of solutions is $\boxed{\left\{-2,\ \frac{1}{2}\right\}}$.
Verification:

- For $x=-2$: $(-2+2)(2\times(-2)-1)=0\times(-5)=0$
- For $x=\frac{1}{2}$: $\left(\frac{1}{2}+2\right)\left(2\times\frac{1}{2}-1\right)=\frac{5}{2}\times 0=0$

Ex 55: Solve for x:

$$\left(\frac{x}{2}-1\right)(2x+2)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-1},\ \boxed{2}\}$.

Answer:

$$\left(\frac{x}{2}-1\right)(2x+2)=0$$
$$\frac{x}{2}-1=0 \text{ or } 2x+2=0 \text{ (null factor law)}$$
$$\frac{x}{2}=1 \text{ or } 2x=-2$$
$$x=2 \text{ or } x=-1$$

The set of solutions is $\boxed{\{-1,\ 2\}}$.
Verification:

- For $x=-1$: $\left(\frac{-1}{2}-1\right)(2\times(-1)+2)=\left(-\frac{1}{2}-1\right)(-2+2)=\left(-\frac{3}{2}\right)\times 0=0$
- For $x=2$: $\left(\frac{2}{2}-1\right)(2\times 2+2)=(1-1)\times(4+2)=0\times 6=0$

Ex 56: Solve for x:

$$x(x-1)(x-2)=0$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{0},\ \boxed{1},\ \boxed{2}\}$.

Answer:

$$x(x-1)(x-2)=0$$
$$x=0 \text{ or } x-1=0 \text{ or } x-2=0 \text{ (null factor law)}$$
$$x=0 \text{ or } x=1 \text{ or } x=2$$

The set of solutions is $\boxed{\{0,\ 1,\ 2\}}$.
Verification:

- For $x=0$: $0\times(0-1)\times(0-2)=0\times(-1)\times(-2)=0$
- For $x=1$: $1\times(1-1)\times(1-2)=1\times 0\times(-1)=0$
- For $x=2$: $2\times(2-1)\times(2-2)=2\times 1\times 0=0$

F SOLVING BASIC QUADRATIC EQUATIONS

F.1 SOLVING QUADRATIC EQUATIONS IN THE FORM $x^2=k$

Ex 57: Solve for x:

$$x^2=9$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-3},\ \boxed{3}\}$.

Answer: We are given $x^2=9$.

- **Applying the proposition** : The solutions of this equation $x^2=9$ are
$$\{-\sqrt{9},\ \sqrt{9}\}=\{-3,\ 3\}$$
- **Formal resolution**: To solve for x:
$$\begin{aligned} x^2&=9\\ x^2-9&=0\\ x^2-3^2&=0\\ (x-3)(x+3)&=0 \quad \text{(difference of squares)}\\ x-3=0 \quad &\text{or} \quad x+3=0\\ x=3 \quad &\text{or} \quad x=-3 \end{aligned}$$

The set of solutions is $\boxed{\{-3,\ 3\}}$.
Verification:

- For $x=-3$: $(-3)^2=9$
- For $x=3$: $(3)^2=9$

Ex 58: Solve for x:

$$x^2=25$$

Give your answers in increasing order:

The set of solutions is $\{\boxed{-5},\ \boxed{5}\}$.

Answer: We are given $x^2=25$.

- **Applying the proposition** : The solutions of this equation $x^2=25$ are
$$\{-\sqrt{25},\ \sqrt{25}\}=\{-5,\ 5\}$$
- **Formal resolution**: To solve for x:
$$\begin{aligned} x^2&=25\\ x^2-25&=0\\ x^2-5^2&=0\\ (x-5)(x+5)&=0 \quad \text{(difference of squares)}\\ x-5=0 \quad &\text{or} \quad x+5=0\\ x=5 \quad &\text{or} \quad x=-5 \end{aligned}$$

The set of solutions is $\boxed{\{-5,\ 5\}}$.
Verification:

- For $x=-5$: $(-5)^2=25$
- For $x=5$: $(5)^2=25$

Ex 59: Solve for x:

$$x^2=10$$

Give your answers in increasing order:

The set of solutions is $\{\ \boxed{-\sqrt{10}},\ \boxed{\sqrt{10}}\ \}$.

Answer: We are given $x^2=10$.

- **Applying the proposition**: The solutions of this equation $x^2=10$ are
$$\left\{-\sqrt{10},\ \sqrt{10}\right\}$$

- **Formal resolution**: To solve for x:

$$\begin{aligned} x^2 &= 10 \\ x^2 - 10 &= 0 \\ x^2 - (\sqrt{10})^2 &= 0 \\ (x - \sqrt{10})(x + \sqrt{10}) &= 0 \quad \text{(difference of squares)} \\ x - \sqrt{10} = 0 \quad &\text{or} \quad x + \sqrt{10} = 0 \\ x = \sqrt{10} \quad &\text{or} \quad x = -\sqrt{10} \end{aligned}$$

The set of solutions is $\boxed{\left\{-\sqrt{10},\ \sqrt{10}\right\}}$.

Verification:

- For $x = -\sqrt{10}$: $\left(-\sqrt{10}\right)^2 = 10$
- For $x = \sqrt{10}$: $(\sqrt{10})^2 = 10$

Ex 60: Solve for x:

$$x^2 - 3 = 0$$

Give your answers in increasing order:

The set of solutions is $\{\ \boxed{-\sqrt{3}},\ \boxed{\sqrt{3}}\ \}$.

Answer: We are given $x^2 - 3 = 0$.

- **Applying the proposition**: The solutions of this equation $x^2 = 3$ are

$$\left\{-\sqrt{3},\ \sqrt{3}\right\}$$

- **Formal resolution**: To solve for x:

$$\begin{aligned} x^2 - 3 &= 0 \\ x^2 - (\sqrt{3})^2 &= 0 \\ (x - \sqrt{3})(x + \sqrt{3}) &= 0 \quad \text{(difference of squares)} \\ x - \sqrt{3} = 0 \quad &\text{or} \quad x + \sqrt{3} = 0 \\ x = \sqrt{3} \quad &\text{or} \quad x = -\sqrt{3} \end{aligned}$$

The set of solutions is $\boxed{\left\{-\sqrt{3},\ \sqrt{3}\right\}}$.

Verification:

- For $x = -\sqrt{3}$: $\left(-\sqrt{3}\right)^2 - 3 = 3 - 3 = 0$
- For $x = \sqrt{3}$: $(\sqrt{3})^2 - 3 = 3 - 3 = 0$