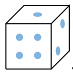
SET THEORY

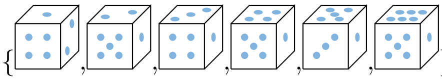
A DEFINITIONS

A.1 SETS

Definition Set

A **set** is a collection of objects, called elements.
We list its elements between curly brackets.

Ex: List all eelments of the set E, which includes all possible results when rolling a standard die .

Answer: $E = \{1, 2, 3, 4, 5, 6\} = \{$$\}$.

Definition Element

- An **element** is an object contained in a set.
- $\in$ means "is an element of" or "belongs to".
- $\notin$ means "is not an element of" or "does not belong to".

Ex: $2 \in \{1, 2, 3, 4, 5, 6\}$ and $7 \notin \{1, 2, 3, 4, 5, 6\}$.

Definition Equal sets

Two sets are **equal** if they have exactly the same elements.

Ex: Determine if the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal.

Answer: Yes, the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal because they contain the same elements: 2, 4, and 6.

Ex: Determine if the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are equal.

Answer: No, the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are not equal because element 3 belongs to $\{1, 2, 3\}$ but not to $\{1, 2, 4\}$.

Definition Empty Set

The **empty set** is a set with no elements. It is written as $\{\}$ or $\emptyset$.

A.2 NATURAL NUMBERS

Definition Natural Numbers

The set of **natural numbers**, denoted $\mathbb{N}$, is the set of counting numbers starting from zero:

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

A.3 SUBSETS

Definition Subset

A set A is a **subset** of a set B if every element in A is also in B. We write this as $A \subseteq B$.

Ex: Is $A \subseteq B$ when $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$?

Answer: Check each element: 2, 4, and 6 from A are all in $B = \{1, 2, 3, 4, 5, 6\}$. Since every element of A is in B, $A \subseteq B$.

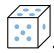
A.4 SET-BUILDER NOTATION

Definition Set-Builder Notation

The **set-builder notation** is a way to describe a set by giving a rule that its elements must follow. It is written like this:

$$\{x \in E \mid \text{condition on } x\}$$

In this notation, x represents an element from a set E, and the symbol $\mid$ (or sometimes :) separates x from the condition it must meet.
It reads: "the set of all x in E such that x satisfies the condition."

Ex: Let $E = \{1, 2, 3, 4, 5, 6\}$ be the set of results from rolling a standard six-sided die .
What are the elements of the set $\{x \in E \mid x \text{ is even}\}$?

Answer: The set $\{x \in E \mid x \text{ is even}\}$ contains all the even numbers in E. Given $E = \{1, 2, 3, 4, 5, 6\}$, the even numbers are 2, 4, and 6, so:

$$\{x \in E \mid x \text{ is even}\} = \{2, 4, 6\}$$

A.5 ORDERED PAIRS/N-TUPLES

Definition Ordered Pair/n-tuple

An **ordered pair**, denoted by (a, b), is a collection of two elements where one is designated as the first element (a) and the other as the second element (b). Two ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.
More generally, an **ordered n-tuple** is a finite sequence of n elements, denoted $(a_1, a_2, \ldots, a_n)$. Two n-tuples $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ are equal if and only if their corresponding elements are equal, i.e., $a_i = b_i$ for all $i = 1, \ldots, n$.

Ex: In a sprint relay race, two runners are paired up. Let L denote Louis and H denote Hugo.
The ordered pair (L, H) means Louis runs first, then passes the baton to Hugo.
The ordered pair (H, L) means Hugo runs first, then passes the baton to Louis.
These are two different orders/teams (ordered pairs).

A.6 CARDINALITY

Definition Cardinality

$n(A)$ denotes the number of elements in the set A.

Ex: $n(\{1, 2, 3, 4, 5, 6\}) = 6$ 

Definition Finite and Infinite Sets

- A **finite set** has a finite number of elements. That is, you can count all its elements and finish counting.
- An **infinite set** has an unlimited number of elements; it is not finite.

Ex:

- $\{1, 2, 3\}$ is finite because it has exactly 3 elements. 
- $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ is infinite because there is no end to counting.

B OPERATIONS

B.1 INTERSECTION AND UNION

Definition Intersection

The **intersection** of two sets A and B, written $A \cap B$, is the set of elements that are in both A and B.

Ex: What is the intersection $\{1, 2, 3\} \cap \{2, 3, 4\}$?

Answer: For the intersection ∩, include all common elements: 2 and 3. So

$$\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

Definition Union

The **union** of two sets A and B, written $A \cup B$, is the set of all elements in A or B (or both).

Ex: What is the union $\{1, 2, 3\} \cup \{2, 3, 4\}$?

Answer: For the union ∪, include all elements from both sets without repeats: 1, 2, 3, 4. So,

$$\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

B.2 COMPLEMENT

Definition Universal set

A **universal set** is the set of all elements considered.

Definition Complement

The **complement** of a set A, denoted A', consists of all elements in universal set U that are not in A. Sets A and A' are said to be **complementary**.

Ex: Given the universe $U = \{1, 2, 3, 4, 5, 6\}$ and the set $A = \{1, 3, 5\}$, find the complement A'.

Answer: Start with the universe $U = \{1, 2, 3, 4, 5, 6\}$.
The set $A = \{1, 3, 5\}$ includes 1, 3, and 5.
The complement A' is all the elements in U that are not in A:

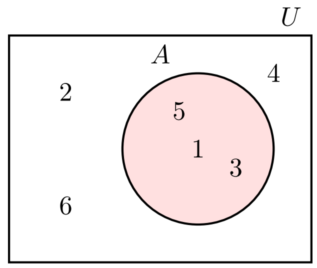
$$A' = \{2, 4, 6\}$$

B.3 VENN DIAGRAMS

Definition Venn Diagram

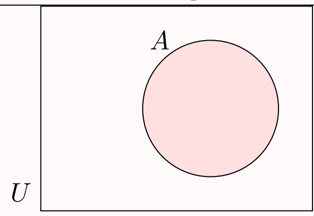
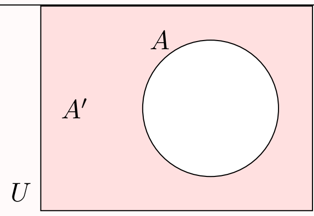
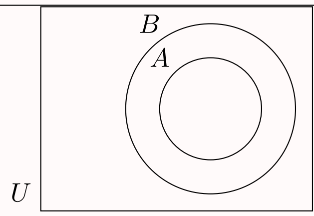
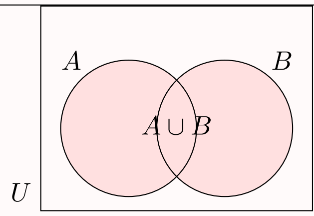
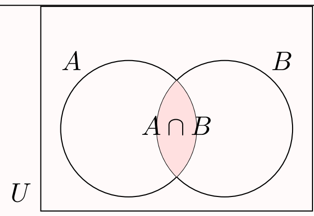
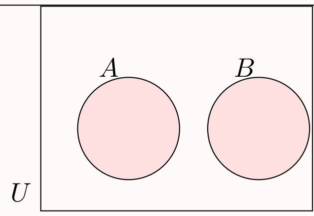
A **Venn diagram** uses a rectangle to show the universal set U and circles to represent other sets within it.

Ex: Here's a Venn diagram for $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$:



Definition **Key Venn Diagram Concepts**

This table shows common set operations and their Venn diagrams:

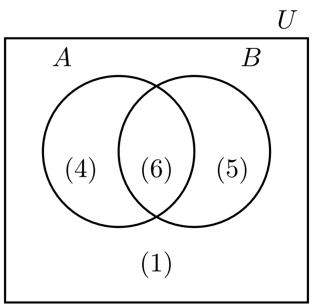
Notation	**Meaning**	**Venn Diagram**
A	Set A	
A'	Complement of A (everything in U not in A)	
$A \subseteq B$	A is a subset of B	
$A \cup B$	Union of A and B (all elements in A or B)	
$A \cap B$	Intersection of A and B (elements in both)	
$A \cap B = \emptyset$	A and B are disjoint (no common elements)	

Venn diagrams help solve problems by showing the number of elements in each region.

Definition **Counting Elements**

In a Venn diagram, we use parentheses around numbers to show how many elements are in each region.

Ex: Consider this Venn diagram:



Here, there are 6 elements in $A \cap B$, 4 in A but not in B, 5 in B but not in A, and 1 outside both. Total: A has $4 + 6 = 10$, B has $6 + 5 = 11$.

C NUMBER SETS

C.1 NUMBER SETS

Number sets are groups of numbers defined by specific properties and form the foundation of mathematics. We begin with the simplest set, the natural numbers ($\mathbb{N}$), used for counting. We then build the integers ($\mathbb{Z}$) by adding negative numbers. Next, we expand to the rational numbers ($\mathbb{Q}$) by allowing fractions, covering all possible divisions between integers. Finally, we reach the real numbers ($\mathbb{R}$) by including irrational numbers, which fills the entire number line. Each set contains the previous ones.

Definition Number Sets

- The **natural numbers**, denoted $\mathbb{N}$, are the counting numbers, starting from zero:

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \dots\}$$

- The **integers**, denoted $\mathbb{Z}$, include all integers (negative, zero, positive):

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

- The **rational numbers**, denoted $\mathbb{Q}$, are all numbers that can be written as a fraction $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

$$\mathbb{Q} = \left\{\frac{p}{q} \mid p, q \in \mathbb{Z},\ q \neq 0\right\}$$

- The **real numbers**, denoted $\mathbb{R}$, include all points on the number line: rational and irrational numbers (such as $\sqrt{2}$ and π):

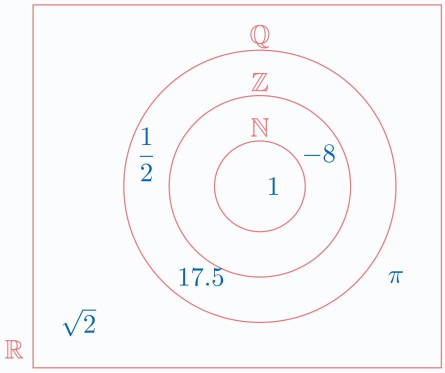
$$\mathbb{R} = \{\text{all rational and irrational numbers}\}$$

Proposition Relationships Between Number Sets

The number sets are nested as follows:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

This means: every natural number is an integer, every integer is a rational number, and every rational number is a real number.



C.2 INTERVALS

Definition Interval

An **interval** is a set of all real numbers between two endpoints, which may or may not be included in the set.

Ex: The set of all real numbers between 0 and 1, including 1 but not 0, is an interval. It is written as $\{x \in \mathbb{R} \mid 0 < x \leq 1\}$.

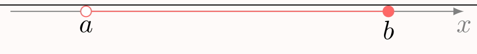
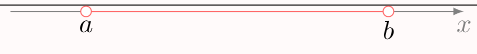
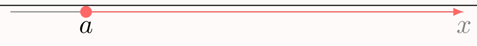
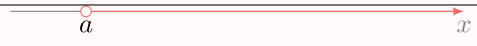
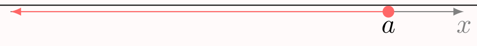
Method Representing Intervals on a Number Line

Intervals are often shown on a number line using these conventions:

1. An *open point* (empty circle) or a parenthesis, means the endpoint is not included.
2. A *closed point* (filled circle) or a bracket, means the endpoint is included.
3. An *arrow* shows that the interval extends indefinitely (to $+\infty$ or $-\infty$).

Ex: The number line representation of $\{x \in \mathbb{R} \mid 0 < x \leq 1\}$ is:

Definition Interval Notation

Interval Notation	Set-builder notation	Number line representation
$[a, b]$	$\{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$	
$[a, b)$	$\{x \in \mathbb{R} \mid a \leqslant x < b\}$	
$(a, b]$	$\{x \in \mathbb{R} \mid a < x \leqslant b\}$	
(a, b)	$\{x \in \mathbb{R} \mid a < x < b\}$	
$[a, +\infty)$	$\{x \in \mathbb{R} \mid a \leqslant x\}$	
$(a, +\infty)$	$\{x \in \mathbb{R} \mid a < x\}$	
$(-\infty, a]$	$\{x \in \mathbb{R} \mid x \leqslant a\}$	
$(-\infty, a)$	$\{x \in \mathbb{R} \mid x < a\}$