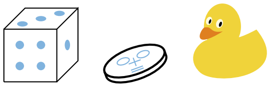
SET THEORY

A DEFINITIONS

A.1 SETS

A.1.1 LISTING THE ELEMENTS

MCQ 1: List the elements of the set A, which includes all objects shown in this figure:



Choose one answer:

- ☐ $A =$ die, coin, duck
- ☐ $A = \{\text{duck, coin}\}$
- ☒ $A = \{\text{die, duck, coin}\}$

Answer:

- The figure shows three objects: a die, a coin, and a duck.
- A set is written with curly brackets $\{\}$ around its elements.
- So, $A = \{\text{die, duck, coin}\}$.

MCQ 2: List the elements of the set A, which includes all objects in this figure:

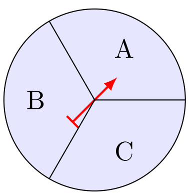
Choose one answer:

- ☐ $A =$ apple, cherry, lemon, orange
- ☐ $A = \{\text{apple, cherry}\}$
- ☒ $A = \{\text{apple, cherry, lemon, orange}\}$
- ☐ $A = \{\text{apple, cherry, lemon, orange, apple}\}$

Answer:

- The figure shows four objects: an apple, a cherry, a lemon, and an orange.
- A set uses curly brackets $\{\}$ and lists each element only once—no repeats!
- So, $A = \{\text{apple, cherry, lemon, orange}\}$.

MCQ 3: List the elements of the set A, which includes all possible results the spinner can land on:



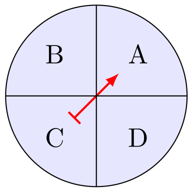
Choose one answer:

- ☒ $A = \{A, B, C\}$
- ☐ $A = \{A, B\}$
- ☐ $A = \{A, C\}$

Answer:

- The spinner has three sections: A, B, and C.
- So, $A = \{A, B, C\}$.

MCQ 4: List the elements of the set A, which includes all possible results the spinner can land on:



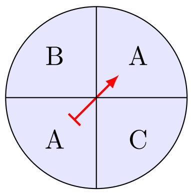
Choose two correct answers:

- ☒ $A = \{A, B, C, D\}$
- ☐ $A = \{A, B, C\}$
- ☐ $A = \{A, B\}$
- ☒ $A = \{D, B, C, A\}$

Answer:

- The spinner has four sections: A, B, C, and D.
- In a set, the order doesn't matter, so both $A = \{A, B, C, D\}$ and $A = \{D, B, C, A\}$ are correct.

MCQ 5: List the elements of the set A, which includes all possible results the spinner can land on:



Choose one answer:

- ☐ $A = \{A, B, A, C\}$
- ☐ $A = \{A, B\}$
- ☐ $A = \{A, C\}$
- ☒ $A = \{A, B, C\}$

Answer:

- The spinner has four sections labeled A, B, A, and C.
- In a set, we list each element only once, even if it appears more than once in the figure.
- So, $A = \{A, B, C\}$.

MCQ 6: Let A be the set of all possible combinations of two children in a family, where B means boy and G means girl (e.g., BG is a boy then a girl). List the elements of A.
Choose one answer:

- ☒ $A = \{BB, BG, GB, GG\}$
- ☐ $A = \{BB, GG\}$
- ☐ $A = \{B, G\}$

Answer:

- For two children, the possible combinations are: both boys (BB), boy then girl (BG), girl then boy (GB), and both girls (GG).
- So, $A = \{BB, BG, GB, GG\}$.

A.1.2 LISTING THE ELEMENTS IN ARITHMETIC

MCQ 7: What is the set A of all factors of 6?
Choose one answer:

- ☒ $A = \{1, 2, 3, 6\}$
- ☐ $A = \{0, 6, 12, 18, 24, \ldots\}$
- ☐ $A = \{0, 6, 12, 18, 24\}$
- ☐ $A = \{2, 3\}$

Answer: The factors of 6 are the numbers that divide 6 exactly: 1 ($1 \times 6 = 6$), 2 ($2 \times 3 = 6$), 3 ($3 \times 2 = 6$), and 6 ($6 \times 1 = 6$). So, $A = \{1, 2, 3, 6\}$.

MCQ 8: What is the set A of all prime numbers between 1 and 10?
Choose one answer:

- ☐ $A = \{1, 2, 3, 5, 7\}$
- ☐ $A = \{2, 4, 6, 8, 10\}$
- ☐ $A = \{3, 5, 7, 9\}$
- ☒ $A = \{2, 3, 5, 7\}$

Answer: Prime numbers have exactly two factors: 1 and themselves. Between 1 and 10, the primes are 2 (factors: 1, 2), 3 (1, 3), 5 (1, 5), and 7 (1, 7). 1 is not prime (only one factor), and 4, 6, 8, 9, 10 have more than two factors. So, $A = \{2, 3, 5, 7\}$.

MCQ 9: What is the set A of all factors of 8?
Choose one answer:

- ☒ $A = \{1, 2, 4, 8\}$
- ☐ $A = \{0, 8, 16, 24, 32, \ldots\}$
- ☐ $A = \{2, 4, 6\}$
- ☐ $A = \{1, 3, 5, 7\}$

Answer: The factors of 8 are numbers that divide 8 exactly: 1 ($1 \times 8 = 8$), 2 ($2 \times 4 = 8$), 4 ($4 \times 2 = 8$), and 8 ($8 \times 1 = 8$). So, $A = \{1, 2, 4, 8\}$.

MCQ 10: What is the set A of all prime numbers between 10 and 20?
Choose one answer:

- ☐ $A = \{11, 13, 15, 17\}$
- ☐ $A = \{10, 12, 14, 16, 18\}$
- ☐ $A = \{13, 15, 17, 19\}$
- ☒ $A = \{11, 13, 17, 19\}$

Answer: Prime numbers have exactly two factors: 1 and themselves. Between 10 and 20, the primes are 11 (factors: 1, 11), 13 (factors: 1, 13), 17 (factors: 1, 17), and 19 (factors: 1, 19). 10, 12, 14, 15, 16, and 18 have more than two factors. So, $A = \{11, 13, 17, 19\}$.

A.1.3 CHECKING MEMBERSHIP

Ex 11: $2 \boxed{\in} \{1, 2, 3, 4, 5, 6\}$

Answer:

- 2 is in the set $\{1, 2, 3, 4, 5, 6\}$.
- So, $2 \in \{1, 2, 3, 4, 5, 6\}$.

Ex 12: $7 \boxed{\notin} \{1, 2, 3, 4, 5, 6\}$

Answer:

- 7 is not in the set $\{1, 2, 3, 4, 5, 6\}$.
- So, $7 \notin \{1, 2, 3, 4, 5, 6\}$.

Ex 13: $d \boxed{\in} \{a, b, c, d\}$

Answer:

- d is in the set $\{a, b, c, d\}$.
- So, $d \in \{a, b, c, d\}$.

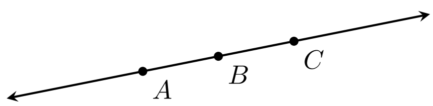
Ex 14: $z \boxed{\notin} \{a, b, c, d\}$

Answer:

- z is not in the set $\{a, b, c, d\}$.
- So, $z \notin \{a, b, c, d\}$.

A.1.4 CHECKING MEMBERSHIP IN GEOMETRY

Ex 15:

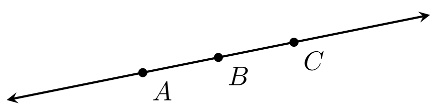


$C \boxed{\in} \overleftrightarrow{AB}$

Answer:

- Point C lies on the line $\overleftrightarrow{AB}$.
- So, $C \in \overleftrightarrow{AB}$.

Ex 16:

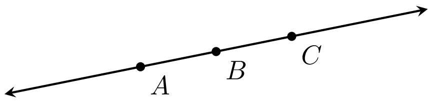


$C \boxed{\notin} \overline{BA}$

Answer:

- The segment $\overline{BA}$ goes from B to A, and C is outside this part.
- So, $C \notin \overline{BA}$.

Ex 17:

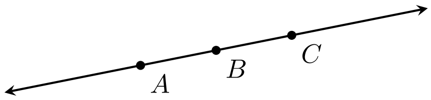


$B \boxed{\in} \overline{AC}$

Answer:

- The segment $\overline{AC}$ goes from A to C, and B is a point between A and C.
- So, $B \in \overline{AC}$.

Ex 18:



$A \boxed{\in} \overleftrightarrow{BC}$

Answer:

- Point A lies on the line $\overleftrightarrow{BC}$, which extends through points B and C in both directions.
- So, $A \in \overleftrightarrow{BC}$.

A.1.5 CHECKING SET EQUALITY

MCQ 19: Is this statement true or false?
$\{a, b, c\} = \{b, a, c\}$
Choose one answer:

☒ True

☐ False

Answer:

- Both sets have the same elements: a, b, and c. The order doesn't matter in sets.
- So, the statement is **True**.

MCQ 20: Is this statement true or false?
$\{a, b, c, d\} = \{a, b, c, d, e\}$
Choose one answer:

☐ True

☒ False

Answer:

- The set $\{a, b, c, d\}$ has 4 elements, but $\{a, b, c, d, e\}$ has 5 because it includes e.
- They're not the same, so the statement is **False**.

MCQ 21: Is this statement true or false?
$\{1, 2, 3\} = \{2, 1, 3\}$
Choose one answer:

☒ True

☐ False

Answer:

- Both sets have the same elements: 1, 2, and 3. The order doesn't matter.
- So, the statement is **True**.

MCQ 22: Is this statement true or false?
$\{1, 2, 3, 4\} = \{1, 2, 3, 4, 5\}$
Choose one answer:

☐ True

☒ False

Answer:

- The set $\{1, 2, 3, 4\}$ has 4 elements, but $\{1, 2, 3, 4, 5\}$ has 5 because it includes 5.
- They're not the same, so the statement is **False**.

A.2 NATURAL NUMBERS

A.2.1 CHECKING MEMBERSHIP

Ex 23: $2 \boxed{\in} \mathbb{N}$

Answer:

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.
- So, $2 \in \mathbb{N}$.

Ex 24: $-2 \boxed{\notin} \mathbb{N}$

Answer:

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.
- Negative numbers like -2 are **not** included.
- So, $-2 \notin \mathbb{N}$.

Ex 25: $\dfrac{1}{2} \boxed{\notin} \mathbb{N}$

Answer:

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ (whole numbers).
- Fractions like $\dfrac{1}{2}$ are **not** included.
- So, $\dfrac{1}{2} \notin \mathbb{N}$.

Ex 26: $10^{10^{1000}} \boxed{\in} \mathbb{N}$

Answer:

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ (whole numbers).
- While extremely large, $10^{10^{1000}}$ is still a finite integer.
- So, $10^{10^{1000}} \in \mathbb{N}$.

Ex 27: $0 \boxed{\in} \mathbb{N}$

Answer:

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ in modern mathematics.
- Some older definitions exclude 0, but current standards include it.
- Therefore, $0 \in \mathbb{N}$.

A.3 SUBSETS

A.3.1 CHECKING SUBSETS

MCQ 28: Given $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$, is $A \subseteq B$?

☒ Yes

☐ No

Answer: Check each element of A: 1, 3, and 5 are all in $B = \{1, 2, 3, 4, 5\}$. Since every element of A is in B, $A \subseteq B$.
So, the answer is **Yes**.

MCQ 29: Given $A = \{4, 9\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$, is $A \subseteq B$?

☐ Yes

☒ No

Answer: Check each element of A: 4 is in $B = \{1, 2, 3, 4, 5, 6, 7\}$, but 9 is not. Since not every element of A is in B, $A \not\subseteq B$.
So, the answer is **No**.

MCQ 30: Given $A = \{7, 8\}$ and $B = \{6, 7, 8, 9, 10\}$, is $A \subseteq B$?

☒ Yes

☐ No

Answer: Check each element of A: 7 and 8 are both in $B = \{6, 7, 8, 9, 10\}$. Since every element of A is in B, $A \subseteq B$.
So, the answer is **Yes**.

MCQ 31: Given $A = \{2, 7, 10\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, is $A \subseteq B$?

☐ Yes

☒ No

Answer: Check each element of A: 2 is in $B = \{1, 2, 3, 4, 5, 6\}$, but 7 and 10 are not. Since not every element of A is in B, $A \not\subseteq B$.
So, the answer is **No**.

A.4 SET-BUILDER NOTATION

A.4.1 CHECKING MEMBERSHIP

MCQ 32: Does "triangle" belong to the set $\{x \mid x \text{ is a polygon}\}$?

☒ Yes

☐ No

Answer: A triangle belongs to $\{x \mid x \text{ is a polygon}\}$ because a triangle is a polygon (a closed figure with three sides). Thus, "triangle" $\in \{x \mid x \text{ is a polygon}\}$.

MCQ 33: Does "January" belong to the set $\{x \mid x \text{ is a day of the week}\}$?

☐ Yes

☒ No

Answer: January does not belong to $\{x \mid x \text{ is a day of the week}\}$ because January is a month, not a day like "Monday" or "Tuesday". Thus, "January" $\notin \{x \mid x \text{ is a day of the week}\}$.

MCQ 34: Does "red" belong to the set $\{x \mid x \text{ is a color in the rainbow}\}$?

☒ Yes

☐ No

Answer: Red belongs to $\{x \mid x \text{ is a color in the rainbow}\}$ because red is one of the seven colors in the rainbow (red, orange, yellow, green, blue, indigo, violet). Thus, "red" $\in \{x \mid x \text{ is a color in the rainbow}\}$.

MCQ 35: Does 9 belong to the set $\{n \in \mathbb{N} \mid n \text{ is a prime number}\}$?

☐ Yes

☒ No

Answer: 9 does not belong to $\{n \in \mathbb{N} \mid n \text{ is a prime number}\}$ because a prime number has exactly two distinct positive factors (1 and itself), and 9 has three: $1, 3, 9$. Thus, $9 \notin \{n \in \mathbb{N} \mid n \text{ is a prime number}\}$.

A.4.2 LISTING THE ELEMENTS

MCQ 36: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is a factor of } 6\}$$

☒ $A = \{1, 2, 3, 6\}$

☐ $A = \{0, 6, 12, 18, 24, \dots\}$

☐ $A = \{0, 6, 12, 18, 24\}$

☐ $A = \{2, 3\}$

Answer: The factors of 6 are numbers that divide 6 evenly: $1, 2, 3, 6$. So $A = \{1, 2, 3, 6\}$.

MCQ 37: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 5\}$$

☐ $A = \{1, 2, 3, 5\}$

☐ $A = \{0, 5, 10, 15, 20\}$

☐ $A = \{2, 3\}$

☒ $A = \{0, 5, 10, 15, 20, \ldots\}$

Answer: Multiples of 5 are numbers of the form $5k$ where $k \in \mathbb{N}$. So $A = \{0, 5, 10, 15, 20, \ldots\}$.

MCQ 38: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 6\}$$

☐ $A = \{1, 2, 3, 6\}$

☒ $A = \{0, 6, 12, 18, 24, \ldots\}$

☐ $A = \{0, 6, 12, 18, 24\}$

☐ $A = \{2, 3\}$

Answer: Multiples of 6 are numbers of the form $6k$ where $k \in \mathbb{N}$. Thus $A = \{0, 6, 12, 18, 24, \ldots\}$.

MCQ 39: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is a factor of } 20\}$$

☐ $A = \{0, 20, 40, 60, \ldots\}$

☐ $A = \{0, 10, 20, 30\}$

☒ $A = \{1, 2, 4, 5, 10, 20\}$

☐ $A = \{2, 5\}$

Answer: The factors of 20 are $1, 2, 4, 5, 10, 20$ (since they divide 20 exactly). So $A = \{1, 2, 4, 5, 10, 20\}$.

MCQ 40: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is a prime number less than } 20\}$$

☒ $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

☐ $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

☐ $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

☐ $A = \{2, 3, 5, 7\}$

Answer: Prime numbers less than 20 are $2, 3, 5, 7, 11, 13, 17, 19$. So $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$.

MCQ 41: List the elements in the set

$$A = \{n \in \mathbb{N} \mid n \text{ is an even number}\}$$

☐ $A = \{1, 3, 5, 7, 9, \ldots\}$

☒ $A = \{0, 2, 4, 6, 8, \ldots\}$

☐ $A = \{0, 2, 4, 6, 8\}$

☐ $A = \{2, 4\}$

Answer: Even numbers are multiples of 2: $A = \{0, 2, 4, 6, 8, \ldots\}$.

A.4.3 WRITING IN SET-BUILDER FORM

MCQ 42: Given the set

$$A = \{0, 2, 4, 6, 8, \ldots\}$$

Choose correct answers:

☒ $A = \{n \in \mathbb{N} \mid n \text{ is an even number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is an odd number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is a prime number}\}$

☒ $A = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 2\}$

Answer: Both $\{n \in \mathbb{N} \mid n \text{ is an even number}\}$ and $\{n \in \mathbb{N} \mid n \text{ is a multiple of } 2\}$ represent $A = \{0, 2, 4, 6, 8, \ldots\}$. The other forms do not.

MCQ 43: Given the set

$$A = \{1, 2, 4, 8\}$$

Choose correct answers:

☐ $A = \{n \in \mathbb{N} \mid n \text{ is an even number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is an odd number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is a prime number}\}$

☒ $A = \{n \in \mathbb{N} \mid n \text{ is a factor of } 8\}$

Answer: Only $\{n \in \mathbb{N} \mid n \text{ is a factor of } 8\}$ matches $A = \{1, 2, 4, 8\}$.

MCQ 44: Given the set

$$A = \{1, 3, 5, 7, \ldots\}$$

Choose correct answers:

☐ $A = \{n \in \mathbb{N} \mid n \text{ is an even number}\}$

☒ $A = \{n \in \mathbb{N} \mid n \text{ is an odd number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is a prime number}\}$

☐ $A = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 2\}$

Answer: A is the set of all odd natural numbers: $\{n \in \mathbb{N} \mid n \text{ is an odd number}\}$.

A.4.4 CHECKING SUBSETS

MCQ 45: Given

$$A = \{n \in \mathbb{N} \mid n \text{ is a prime number greater than } 2\}$$
$$B = \{n \in \mathbb{N} \mid n \text{ is an odd number}\}$$

Is $A \subseteq B$?

☒ Yes

☐ No

Answer: Yes, all prime numbers greater than 2 are odd. So $A \subseteq B$.

MCQ 46: Given

$$A = \{x \mid x \text{ is a person who owns a driver's license}\}$$
$$B = \{x \mid x \text{ is a person who owns a car}\}$$

Is $A \subseteq B$?

☐ Yes

☒ No

Answer: No, not everyone with a driver's license owns a car. So $A \not\subseteq B$.

MCQ 47: Given

$$A = \{n \in \mathbb{N} \mid n \text{ is divisible by } 9\}$$
$$B = \{n \in \mathbb{N} \mid n \text{ is divisible by } 3\}$$

Is $A \subseteq B$?

☒ Yes

☐ No

Answer: Yes, if a number is divisible by 9, it is also divisible by 3. Thus, $A \subseteq B$.

MCQ 48: Given

$$A = \{x \mid x \text{ is a person who is a vegetarian}\}$$
$$B = \{x \mid x \text{ is a person who does not eat meat}\}$$

Is $A \subseteq B$?

☒ Yes

☐ No

Answer: Yes, vegetarians do not eat meat, so $A \subseteq B$.

MCQ 49: Given

$$A = \{n \in \mathbb{N} \mid n \text{ is divisible by } 4\}$$
$$B = \{n \in \mathbb{N} \mid n \text{ is divisible by } 2\}$$

Is $A \subseteq B$?

☒ Yes

☐ No

Answer: Yes, if a number is divisible by 4, it is also divisible by 2. Thus, $A \subseteq B$.

A.5 ORDERED PAIRS/N-TUPLES

A.5.1 COMPARING PAIRS AND SETS

MCQ 50: A teacher picks one student to present on Monday and another to present on Tuesday from Louis and Hugo. The pair $(Louis, Hugo)$ means Louis presents on Monday and Hugo on Tuesday. Is this the same as $(Hugo, Louis)$?
Choose one answer:

☐ True

☒ False

Answer: The pair $(Louis, Hugo)$ means Louis presents on Monday and Hugo on Tuesday. The pair $(Hugo, Louis)$ means Hugo presents on Monday and Louis on Tuesday. Since the days are different, the order matters and the two pairs are not the same. So, the answer is **False**.

MCQ 51: A teacher selects Louis and Hugo for a presentation. The set $\{Louis, Hugo\}$ shows both are chosen. Does $\{Louis, Hugo\}$ equal $\{Hugo, Louis\}$?
Choose one answer:

☒ True

☐ False

Answer: The set $\{Louis, Hugo\}$ means Louis and Hugo are chosen for the presentation. The set $\{Hugo, Louis\}$ means the same: Hugo and Louis are chosen. In sets, order does not matter, so $\{Louis, Hugo\} = \{Hugo, Louis\}$. The answer is **True**.

MCQ 52: A club picks two helpers, Zoe and Eli, for an event. The set $\{Zoe, Eli\}$ shows both are chosen. Does $\{Zoe, Eli\}$ equal $\{Eli, Zoe\}$?
Choose one answer:

☒ True

☐ False

Answer: The set $\{Zoe, Eli\}$ means Zoe and Eli are chosen as helpers. The set $\{Eli, Zoe\}$ means the same: Eli and Zoe are chosen. In sets, order does not matter, so $\{Zoe, Eli\} = \{Eli, Zoe\}$. The answer is **True**.

MCQ 53: A coach assigns two players, Mia and Sam, to shoot baskets: one goes first, the other second. The pair (Mia, Sam) means Mia shoots first and Sam second. Is this the same as (Sam, Mia)?
Choose one answer:

☐ True

☒ False

Answer: The pair (Mia, Sam) means Mia shoots first and Sam shoots second. The pair (Sam, Mia) means Sam shoots first and Mia second. Since the order of shooting changes, the two pairs are different. So, the answer is **False**.

A.5.2 CHOOSING BETWEEN ORDERED PAIRS AND SETS

MCQ 54: A teacher picks one student to present on Monday and another to present on Tuesday. This week, Louis presents on Monday and Hugo presents on Tuesday. The teacher wants to write this selection on the board.
Choose the correct way to write this:

☒ $(Louis, Hugo)$

☐ $\{Louis, Hugo\}$

Answer: The correct way to write this is $(Louis, Hugo)$ because the order is important: Louis presents on Monday and Hugo on Tuesday. $(Louis, Hugo)$ is an **ordered pair**. $\{Louis, Hugo\}$ would only indicate the two names, without showing who presents on which day.

MCQ 55: A teacher picks two students to do a presentation together. This week, Louis and Hugo are chosen. The teacher wants to write this selection on the board.
Choose the correct way to write this:

☐ $(Louis, Hugo)$

☒ $\{Louis, Hugo\}$

Answer: The correct way to write this is $\{Louis, Hugo\}$ because the order does not matter: Louis and Hugo are simply both chosen for the presentation. $\{Louis, Hugo\}$ is a **set**. $(Louis, Hugo)$ would mean that order is important, which is not the case here.

MCQ 56: A coach chooses one player to start the basketball game and another to substitute in the second half. Mia starts the game and Zoe comes in later. The coach wants to write this decision on the board.
Choose the correct way to write this:

☒ (Mia, Zoe)

☐ $\{Mia, Zoe\}$

Answer: The correct way to write this is (Mia, Zoe) because the order is important: Mia starts and Zoe substitutes. (Mia, Zoe) is an **ordered pair**. $\{Mia, Zoe\}$ would just mean both were chosen, without order.

MCQ 57: A school committee selects two parents to organize the end-of-year party. This year, Mr. Dupont and Ms. Lee are chosen. The committee writes their names on the announcement.
Choose the correct way to write this:

☐ $(Mr.\ Dupont, Ms.\ Lee)$

☒ $\{Mr.\ Dupont, Ms.\ Lee\}$

Answer: The correct way to write this is $\{Mr.\ Dupont, Ms.\ Lee\}$ because the order does not matter: both are selected equally for the task. $\{Mr.\ Dupont, Ms.\ Lee\}$ is a **set**. $(Mr.\ Dupont, Ms.\ Lee)$ would mean the order matters, which is not true here.

A.6 CARDINALITY

A.6.1 COUNTING

Ex 58: $n(\{1,2,3\}) = \boxed{3}$

Answer:

- The set $\{1,2,3\}$ has three elements: 1, 2, and 3.
- So, $n(\{1,2,3\}) = 3$.

Ex 59: $n(\{a,b,c,d,e\}) = \boxed{5}$

Answer:

- The set $\{a,b,c,d,e\}$ has five elements: a, b, c, d, and e.
- So, $n(\{a,b,c,d,e\}) = 5$.

Ex 60: $n(\{\text{apple, cherry, lemon, orange}\}) = \boxed{4}$

Answer:

- The set $\{\text{apple, cherry, lemon, orange}\}$ has four elements: apple, cherry, lemon, and orange.
- So, $n(\{\text{apple, cherry, lemon, orange}\}) = 4$.

Ex 61: Let $A = \{\text{die, duck, coin}\}$. Find the number of elements in A.

$$n(A) = \boxed{3}$$

Answer:

- The set $A = \{\text{die, duck, coin}\}$ has three elements: die, duck, and coin.
- So, $n(A) = 3$.

Ex 62: Let $A = \{1,2,3,4,5\}$. Find the number of elements in A.

$$n(A) = \boxed{5}$$

Answer:

- The set $A = \{1,2,3,4,5\}$ has five elements: 1, 2, 3, 4, and 5.
- So, $n(A) = 5$.

A.6.2 COUNTING WAYS

Ex 63: Three friends run a sprint race. How many different podiums (1st, 2nd, 3rd) are possible?

$\boxed{6}$ podiums

Answer: Let the friends be A, B, and C. A podium lists the 1st, 2nd, and 3rd place as an ordered triple: $(1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}})$.
Each friend can be in only one position, so we count all possible orderings:

- (A,B,C) — A 1st, B 2nd, C 3rd
- (A,C,B) — A 1st, C 2nd, B 3rd
- (B,A,C) — B 1st, A 2nd, C 3rd
- (B,C,A) — B 1st, C 2nd, A 3rd
- (C,A,B) — C 1st, A 2nd, B 3rd
- (C,B,A) — C 1st, B 2nd, A 3rd

So the answer is 6 podiums.

Ex 64: You pick 2 flavors from 3 ice cream options (chocolate, vanilla, and strawberry). Order doesn't matter. How many different ice creams can you make?

$\boxed{3}$ ice creams

Answer: Let the flavors be C (chocolate), V (vanilla), and S (strawberry). Since order doesn't matter, $\{C,V\}$ is the same as $\{V,C\}$. Here are all combinations:

- $\{C,V\}$ — chocolate and vanilla
- $\{C,S\}$ — chocolate and strawberry
- $\{V,S\}$ — vanilla and strawberry

So the answer is 3 ice creams.

Ex 65: Three students enter an art contest. How many different ways can the judges award 1st, 2nd, and 3rd place prizes?

$\boxed{6}$ ways

Answer: Let the students be A, B, and C. The judges must pick who is 1st, 2nd, and 3rd, so order matters. Here are all possible ways:

- (A, B, C) — A 1st, B 2nd, C 3rd
- (A, C, B) — A 1st, C 2nd, B 3rd
- (B, A, C) — B 1st, A 2nd, C 3rd
- (B, C, A) — B 1st, C 2nd, A 3rd
- (C, A, B) — C 1st, A 2nd, B 3rd
- (C, B, A) — C 1st, B 2nd, A 3rd

So there are 6 possible ways.

Ex 66: You choose 2 toppings from 3 pizza options (pepperoni, cheese, olives). Order doesn't matter. How many different pizzas can you make?

$$\boxed{3} \text{ pizzas}$$

Answer: Let the toppings be P (pepperoni), C (cheese), and O (olives). Since order doesn't matter, $\{P, C\}$ is the same as $\{C, P\}$. All possible combinations are:

- $\{P, C\}$ — pepperoni and cheese
- $\{P, O\}$ — pepperoni and olives
- $\{C, O\}$ — cheese and olives

So the answer is 3 pizzas.

A.6.3 CLASSIFYING SETS AS FINITE OR INFINITE SETS

MCQ 67: Is the set $A = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 10\}$ finite or infinite?

☐ Finite

☒ Infinite

Answer:

- Multiples of 10 are numbers of the form $10k$ where $k \in \mathbb{N}$:

$$\begin{aligned} 10 \times 0 &= 0 \\ 10 \times 1 &= 10 \\ 10 \times 2 &= 20 \\ 10 \times 3 &= 30 \\ &\vdots \end{aligned}$$

- $A = \{0, 10, 20, 30, 40, \dots\}$
- There is no largest multiple of 10 in $\mathbb{N}$, so the set A is **infinite**.

MCQ 68: Is the set $A = \{x \mid x \text{ is a distinct letter in the word 'BANANA'}\}$ finite or infinite?

☒ Finite

☐ Infinite

Answer:

- The distinct letters in "BANANA" are B, A, N.
- So $A = \{B, A, N\}$, which has $n(A) = 3$ elements.
- Therefore, the set A is **finite**.

MCQ 69: Is the set $A = \{n \in \mathbb{N} \mid n \text{ is an even number}\}$ finite or infinite?

☐ Finite

☒ Infinite

Answer:

- Even numbers are of the form $2k$ where $k \in \mathbb{N}$:

$$\begin{aligned} 2 \times 0 &= 0 \\ 2 \times 1 &= 2 \\ 2 \times 2 &= 4 \\ &\vdots \end{aligned}$$

- $A = \{0, 2, 4, 6, 8, \dots\}$
- There are infinitely many even numbers in $\mathbb{N}$, so A is **infinite**.

MCQ 70: Is the set $A = \{n \in \mathbb{N} \mid n \text{ is a factor of } 1000\}$ finite or infinite?

☒ Finite

☐ Infinite

Answer:

- The set A consists of all natural numbers dividing 1000 without remainder.
- $1000 = 2^3 \times 5^3$ has $4 \times 4 = 16$ positive factors: $1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000$.
- So A has 16 elements, and is **finite**.

A.6.4 COUNTING IN SET-BUILDER

Ex 71: Dr. Tariel has two sons, Hugo and Louis. Find the number of elements in the set $A = \{x \mid x \text{ is a son of Dr Vincent}\}$.

$$n(A) = \boxed{2}$$

Answer:

- Dr. Tariel's sons are Hugo and Louis.
- $A = \{\text{Hugo, Louis}\}$.
- Therefore, the number of elements is $n(A) = 2$.

Ex 72: Let $A = \{x \mid x \text{ is a day of the week}\}$.

$$n(A) = \boxed{7}$$

Answer:

- The days of the week are: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday.
- Therefore, the number of elements is $n(A) = 7$.

Ex 73: Let $A = \{n \in \mathbb{N} \mid n \text{ is a factor of } 18\}$.

$$n(A) = \boxed{6}$$

Answer:

- The factors of 18 are 1, 2, 3, 6, 9, 18 because these numbers divide 18 exactly.
- $A = \{1, 2, 3, 6, 9, 18\}$.
- Therefore, the number of elements is $n(A) = 6$.

Ex 74: Let $A = \{n \in \mathbb{N} \mid n \text{ is a prime number less than } 20\}$.

$$n(A) = \boxed{8}$$

Answer:

- The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
- $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$.
- Therefore, the number of elements is $n(A) = 8$.

Ex 75: Let $A = \{n \in \mathbb{N} \mid n \text{ is a two-digit number which contains the digit } 4\}$.

$$n(A) = \boxed{18}$$

Answer:

- The two-digit positive numbers containing the digit 4 are:

$$A = \{14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84, 94\}$$

- There are 18 such numbers.
- Therefore, the number of elements is $n(A) = 18$.

B OPERATIONS

B.1 INTERSECTION AND UNION

B.1.1 FINDING THE INTERSECTION/UNION: LEVEL 1

Ex 76:

$$\{1,2,3\} \cap \{2,3,4\} = \boxed{\{2,3\}}$$

Answer: For the intersection $\cap$, include all common elements: 2, 3. So,

$$\{1,2,3\} \cap \{2,3,4\} = \{2,3\}$$

Ex 77:

$$\{1,2\} \cup \{2,3,4\} = \boxed{\{1,2,3,4\}}$$

Answer: For the union $\cup$, include all elements from both sets without repeats: 1, 2, 3, 4. So,

$$\{1,2\} \cup \{2,3,4\} = \{1,2,3,4\}$$

Ex 78:

$$\{5,6,7\} \cap \{6,8,9\} = \boxed{\{6\}}$$

Answer: For the intersection $\cap$, include all common elements: 6. So,

$$\{5,6,7\} \cap \{6,8,9\} = \{6\}$$

Ex 79:

$$\{a,b\} \cup \{b,c,d\} = \boxed{\{a,b,c,d\}}$$

Answer: For the union $\cup$, include all elements from both sets without repeats: a, b, c, d. So,

$$\{a,b\} \cup \{b,c,d\} = \{a,b,c,d\}$$

Ex 80:

$$\{1,2,3\} \cap \{4,5,6\} = \boxed{\emptyset}$$

Answer: For the intersection $\cap$, include all common elements: there are none. So,

$$\{1,2,3\} \cap \{4,5,6\} = \emptyset$$

Ex 81:

$$\{3,4,5\} \cap \{5,4,3\} = \boxed{\{3,4,5\}}$$

Answer: For the intersection $\cap$, include all common elements: 3, 4, 5. So,

$$\{3,4,5\} \cap \{5,4,3\} = \{3,4,5\}$$

Ex 82:

$$\{5,6,7\} \cup \emptyset = \boxed{\{5,6,7\}}$$

Answer: For the union $\cup$, include all elements from both sets without repeats: 5, 6, 7. The empty set, $\emptyset$, adds nothing. So,

$$\{5,6,7\} \cup \emptyset = \{5,6,7\}$$

Ex 83:

$$\{a,b,c\} \cap \emptyset = \boxed{\emptyset}$$

Answer: For the intersection $\cap$, include all common elements: there are none, since the empty set has no elements. So,

$$\{a,b,c\} \cap \emptyset = \emptyset$$

B.1.2 FINDING THE INTERSECTION/UNION: LEVEL 2

Ex 84: Given the sets:

- $A = \{2,4,6,8\}$
- $B = \{4,5,6\}$
- $C = \{6,7,9\}$

Find the intersection $A \cap B \cap C$.

$$A \cap B \cap C = \boxed{\{6\}}$$

Answer:

- $A \cap B = \{4, 6\}$
- $A \cap B \cap C = \{4, 6\} \cap \{6, 7, 9\} = \{6\}$
- Therefore, $A \cap B \cap C = \{6\}$.

Ex 85: Given the sets:

- $A = \{2, 4, 6, 8, 9\}$
- $B = \{4, 5, 6\}$
- $C = \{6, 7, 9\}$

Find the set $A \cap (B \cup C)$.

$$A \cap (B \cup C) = \boxed{\{4, 6, 9\}}$$

Answer:

- $B \cup C = \{4, 5, 6, 7, 9\}$
- $A \cap (B \cup C) = \{2, 4, 6, 8, 9\} \cap \{4, 5, 6, 7, 9\} = \{4, 6, 9\}$
- Therefore, $A \cap (B \cup C) = \{4, 6, 9\}$.

Ex 86: Given the sets:

- $A = \{2, 4, 6, 8, 9\}$
- $B = \{4, 5, 6\}$
- $C = \{6, 7, 9\}$

Find the set $(A \cup B) \cap C$.

$$(A \cup B) \cap C = \boxed{\{6, 9\}}$$

Answer:

- $A \cup B = \{2, 4, 5, 6, 8, 9\}$
- $(A \cup B) \cap C = \{2, 4, 5, 6, 8, 9\} \cap \{6, 7, 9\} = \{6, 9\}$
- Therefore, $(A \cup B) \cap C = \{6, 9\}$.

Ex 87: Given the sets:

- $A = \{2, 4, 6, 8, 9\}$
- $B = \{4, 5, 6\}$
- $C = \{6, 7, 9\}$

Find the set $A \cup B \cup C$.

$$A \cup B \cup C = \boxed{\{2, 4, 5, 6, 7, 8, 9\}}$$

Answer:

- $A \cup B \cup C = \{2, 4, 6, 8, 9\} \cup \{4, 5, 6\} \cup \{6, 7, 9\} = \{2, 4, 5, 6, 7, 8, 9\}$
- Therefore, $A \cup B \cup C = \{2, 4, 5, 6, 7, 8, 9\}$.

B.2 COMPLEMENT

B.2.1 FINDING THE COMPLEMENT

MCQ 88: You are given the universe $U = \{1, 2, 3, 4, 5, 6\}$ and the set $A = \{1, 3, 5\}$. What is the complement A'?
Choose one answer:

- ☒ $A' = \{2, 4, 6\}$
- ☐ $A' = \{1, 2, 4, 6\}$
- ☐ $A' = \{1, 2, 3, 4, 5, 6\}$
- ☐ $A' = \{3, 5\}$

Answer:

- The universe $U = \{1, 2, 3, 4, 5, 6\}$ has all the elements.
- The set $A = \{1, 3, 5\}$ has 1, 3, and 5.
- The complement A' is everything in U that's not in A: 2, 4, and 6.
- So, $A' = \{2, 4, 6\}$.

MCQ 89: You are given the universe $U = \{a, b, c, d, e, f\}$ and the set $B = \{a, c, e\}$. What is the complement B'?.
Choose one answer:

- ☐ $B' = \{a, b, d, f\}$
- ☐ $B' = \{a, b, c, d, e, f\}$
- ☐ $B' = \{c, e\}$
- ☒ $B' = \{b, d, f\}$

Answer:

- The universe $U = \{a, b, c, d, e, f\}$ has all the elements.
- The set $B = \{a, c, e\}$ has a, c, and e.
- The complement B' is everything in U not in B: b, d, and f.
- So, $B' = \{b, d, f\}$.

MCQ 90: You are given the universe $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and the set $C = \{2, 4, 6, 8\}$. What is the complement C'?.
Find the complement of C.
Choose one answer:

- ☐ $C' = \{1, 2, 3, 5, 7\}$
- ☒ $C' = \{1, 3, 5, 7\}$
- ☐ $C' = \{2, 4, 6, 8\}$
- ☐ $C' = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Answer:

- The universe $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ has all the elements.
- The set $C = \{2, 4, 6, 8\}$ has 2, 4, 6, and 8.

- The complement C' is everything in U not in C: 1, 3, 5, and 7.
- So, $C' = \{1, 3, 5, 7\}$.

MCQ 91: The universe $U = \{BB, BG, GB, GG\}$ lists all two-child family combinations (B = boy, G = girl; e.g., BG = boy then girl). The set $A = \{BB\}$ includes only families with two boys. What is A'?
Choose one answer:

☒ $A' = \{BG, GB, GG\}$

☐ $A' = \{BB, BG\}$

☐ $A' = \{BG, GB\}$

☐ $A' = \{BB, GG\}$

Answer:

- $U = \{BB, BG, GB, GG\}$ covers all combinations.
- $A = \{BB\}$ is just families with two boys.
- A' is all in U except A: BG, GB, GG.
- So, $A' = \{BG, GB, GG\}$.

MCQ 92: The universe $U = \{BB, BG, GB, GG\}$ lists all two-child family combinations (B = boy, G = girl; e.g., BG = boy then girl). The set $A = \{BG, GB\}$ includes families with one boy and one girl. What is A'?
Choose one answer:

☐ $A' = \{BG, GB, GG\}$

☐ $A' = \{BB, BG\}$

☐ $A' = \{BG, GB\}$

☒ $A' = \{BB, GG\}$

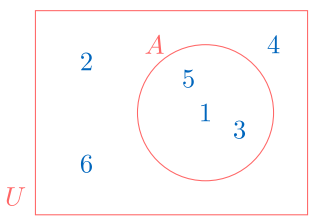
Answer:

- $U = \{BB, BG, GB, GG\}$ covers all combinations.
- $A = \{BG, GB\}$ is families with one boy and one girl.
- A' is all in U except A: BB, GG.
- So, $A' = \{BB, GG\}$.

B.3 VENN DIAGRAMS

B.3.1 IDENTIFYING ELEMENTS USING VENN DIAGRAMS

MCQ 93: For this Venn diagram:



Find A.

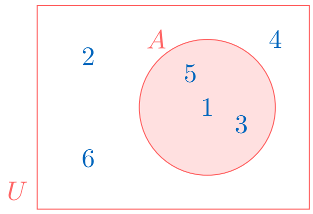
☐ $A = \{2, 4, 6\}$

☒ $A = \{1, 3, 5\}$

☐ $A = \{1, 2, 3, 4, 5, 6\}$

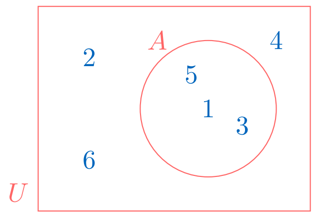
Answer: The set A includes the elements that are inside the circle labeled A in the Venn diagram.

- The elements inside the circle A are 1, 3, and 5.



- Therefore, $A = \{1, 3, 5\}$.

MCQ 94: For this Venn diagram:



Find A'.

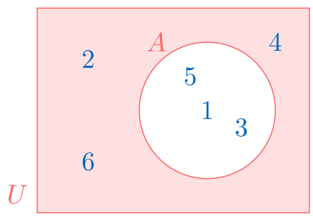
☒ $A' = \{2, 4, 6\}$

☐ $A' = \{1, 3, 5\}$

☐ $A' = \{1, 2, 3, 4, 5, 6\}$

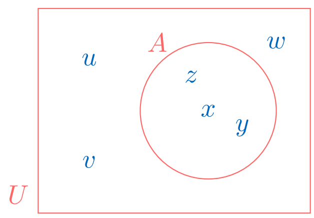
Answer: The set A', the complement of A, includes the elements outside the circle labeled A in the Venn diagram.

- The elements outside A are 2, 4, and 6.



- Therefore, $A' = \{2, 4, 6\}$.

MCQ 95: For this Venn diagram:



Find A'.

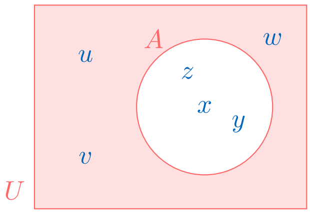
☒ $A' = \{u, v, w\}$

☐ $A' = \{x, y, z\}$

☐ $A' = \{u, v, w, x, y, z\}$

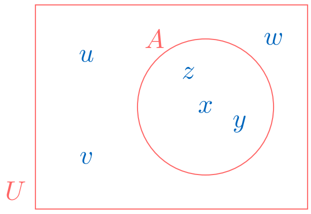
Answer: The set A', the complement of A, includes the elements that are outside the circle labeled A in the Venn diagram.

- The elements outside A are u, v, and w.



- Therefore, $A' = \{u, v, w\}$.

MCQ 96: For this Venn diagram:



Find the universal set U.

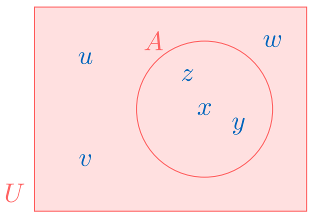
☐ $U = \{u, v, w\}$

☐ $U = \{x, y, z\}$

☒ $U = \{u, v, w, x, y, z\}$

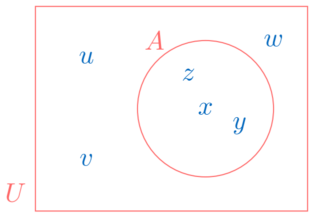
Answer:

- The universe U includes all the elements represented in the diagram.



- Therefore, $U = \{u, v, w, x, y, z\}$.

MCQ 97: For this Venn diagram:



Find A.

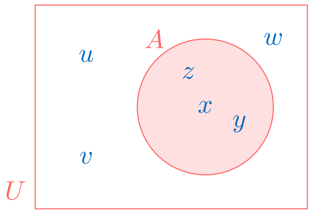
☐ $A = \{u, v, w\}$

☒ $A = \{x, y, z\}$

☐ $A = \{u, v, w, x, y, z\}$

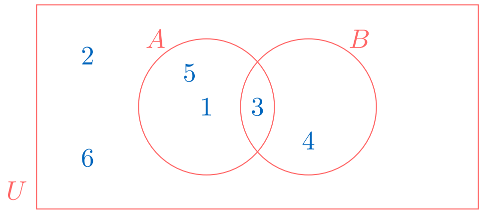
Answer: The set A includes the elements that are inside the circle labeled A in the Venn diagram.

- The elements inside A are x, y, and z.



- Therefore, $A = \{x, y, z\}$.

MCQ 98: For this Venn diagram:



Find A.

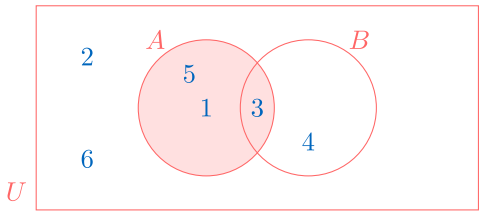
☐ $A = \{2, 4, 6\}$

☒ $A = \{1, 3, 5\}$

☐ $A = \{1, 2, 3, 4, 5, 6\}$

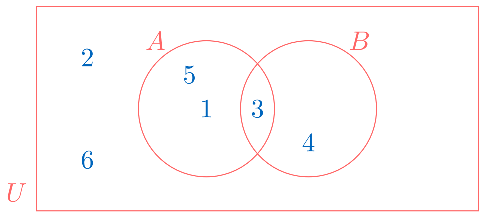
Answer: The set A includes the elements that are inside the circle labeled A in the Venn diagram.

- The elements inside A are 1, 3, and 5.



- Answer: $A = \{1, 3, 5\}$

MCQ 99: For this Venn diagram:



Find A'.

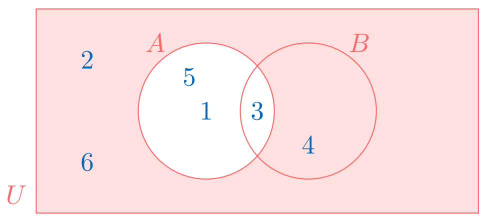
☒ $A' = \{2, 4, 6\}$

☐ $A' = \{1, 3, 5\}$

☐ $A' = \{1, 2, 3, 4, 5, 6\}$

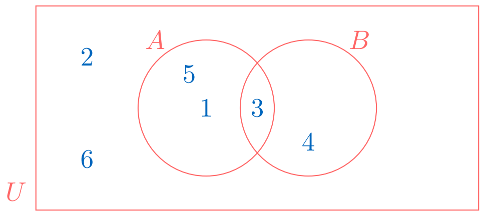
Answer: The set A', the complement of A, includes the elements that are outside the circle labeled A in the Venn diagram.

- The elements outside A are 2, 4, and 6.



- Answer: $A' = \{2, 4, 6\}$

MCQ 100: For this Venn diagram:

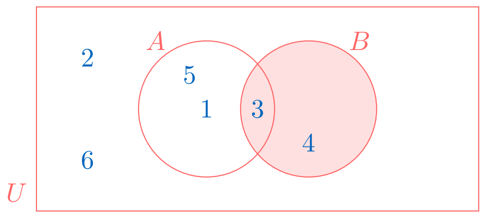


Find B.

- ☐ $B = \{4\}$
- ☒ $B = \{3, 4\}$
- ☐ $B = \{1, 3, 4, 5\}$
- ☐ $B = \{2, 6\}$

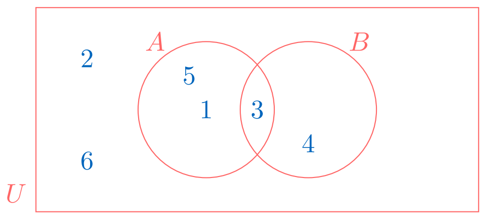
Answer: The set B includes the elements that are inside the circle labeled B in the Venn diagram.

- The elements inside B are 3 and 4.



- Therefore, $B = \{3, 4\}$.

MCQ 101: For this Venn diagram:

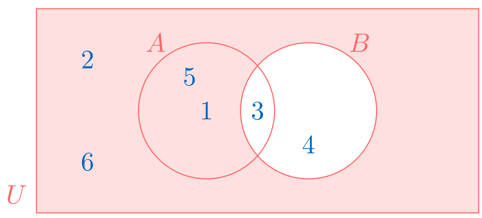


Find B'.

- ☐ $B' = \{4\}$
- ☐ $B' = \{3, 4\}$
- ☒ $B' = \{1, 2, 5, 6\}$
- ☐ $B' = \{2, 6\}$

Answer: The set B', the complement of B, includes the elements that are outside the circle labeled B in the Venn diagram.

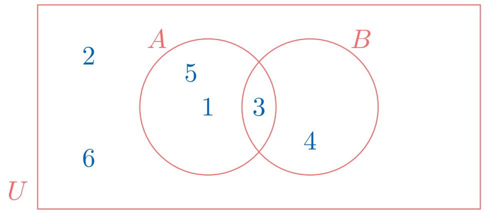
- The elements outside B are 1, 2, 5, and 6.



- Therefore, $B' = \{1, 2, 5, 6\}$.

B.3.2 IDENTIFYING ELEMENTS USING VENN DIAGRAMS

MCQ 102: For this Venn diagram:

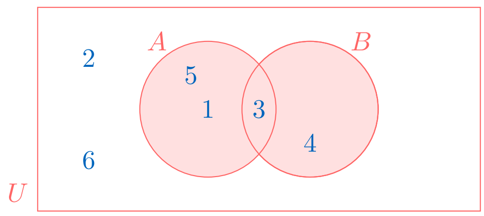


Find $A \cup B$.

- ☒ $A \cup B = \{1, 3, 4, 5\}$
- ☐ $A \cup B = \{1, 2, 3, 4, 5, 6\}$
- ☐ $A \cup B = \{2, 4, 6\}$
- ☐ $A \cup B = \{1, 3, 4\}$

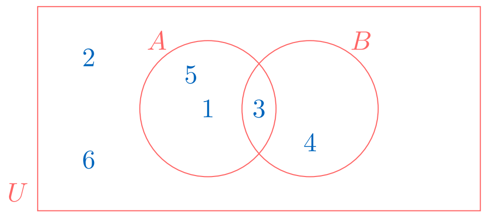
Answer: The set $A \cup B$ includes all elements that are inside either the circle labeled A, the circle labeled B, or both, in the Venn diagram.

- The elements inside A are 1, 3, 5.
- The elements inside B are 3, 4.
- The union (all unique elements from A or B) is 1, 3, 4, 5.



Therefore, $A \cup B = \{1, 3, 4, 5\}$.

MCQ 103: For this Venn diagram:

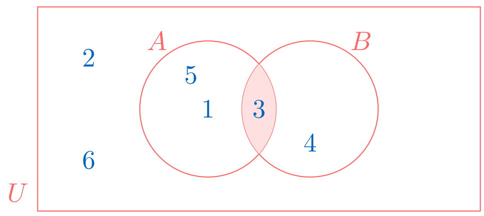


Find $A \cap B$.

- ☐ $A \cap B = \{1, 3, 5\}$
- ☒ $A \cap B = \{3\}$
- ☐ $A \cap B = \{3, 4\}$
- ☐ $A \cap B = \{2, 6\}$

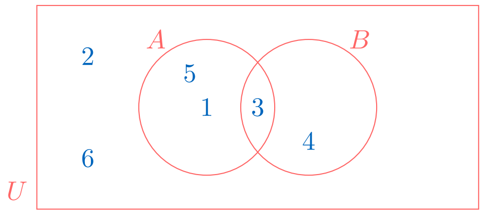
Answer: The set $A \cap B$ includes all elements that are inside both the circle labeled A and the circle labeled B in the Venn diagram.

- The only element inside both A and B is 3.



Therefore, $A \cap B = \{3\}$.

MCQ 104: For this Venn diagram:

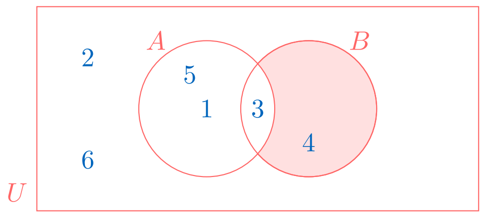


Find $A' \cap B$.

- ☐ $A' \cap B = \{2, 6\}$
- ☒ $A' \cap B = \{4\}$
- ☐ $A' \cap B = \{4, 3\}$
- ☐ $A' \cap B = \{1, 3, 4, 5\}$

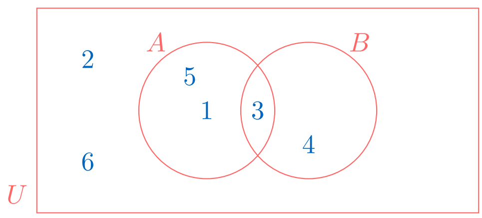
Answer: The set $A' \cap B$ includes all elements that are outside the circle A but inside the circle B.

- Elements outside A are 2, 4, 6.
- Elements inside B are 3, 4.
- The only element in B but not in A is 4.



Therefore, $A' \cap B = \{4\}$.

MCQ 105: For this Venn diagram:

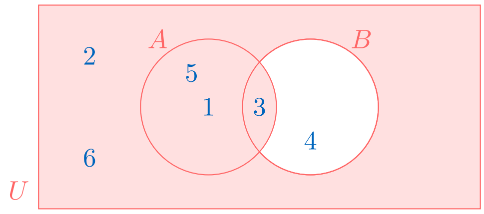


Find $A \cup B'$.

- ☐ $A \cup B' = \{1, 2, 5, 6\}$
- ☐ $A \cup B' = \{2, 4, 6\}$
- ☒ $A \cup B' = \{1, 2, 3, 5, 6\}$
- ☐ $A \cup B' = \{1, 3, 4, 5\}$

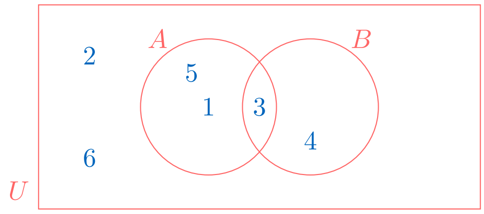
Answer: The set $A \cup B'$ includes all elements that are inside A or outside B in the Venn diagram.

- Elements in A are 1, 3, 5.
- Elements outside B are 1, 2, 5, 6.
- The union of these sets is 1, 2, 3, 5, 6.



Therefore, $A \cup B' = \{1, 2, 3, 5, 6\}$.

MCQ 106: For this Venn diagram:

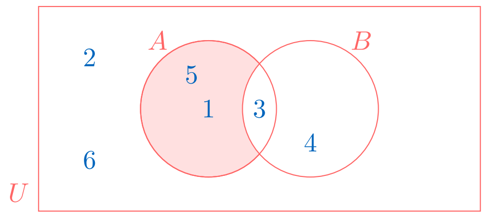


Find $A \cap B'$.

- ☒ $A \cap B' = \{1, 5\}$
- ☐ $A \cap B' = \{2, 6\}$
- ☐ $A \cap B' = \{3, 4\}$
- ☐ $A \cap B' = \{1, 3, 4, 5\}$

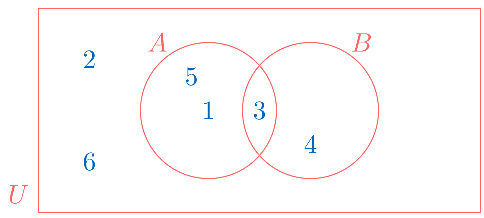
Answer: The set $A \cap B'$ includes all elements that are inside A but outside B in the Venn diagram.

- Elements inside A are 1, 3, 5.
- Elements outside B are 1, 2, 5, 6.
- The intersection is $\{1, 5\}$.



Therefore, $A \cap B' = \{1, 5\}$.

MCQ 107: For this Venn diagram:



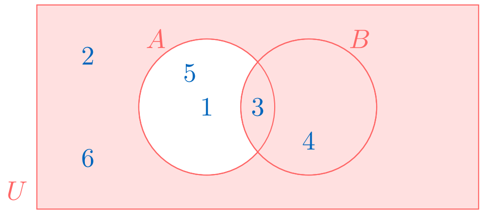
Find $A' \cup B$.

- ☐ $A' \cup B = \{1, 2, 3, 4, 5, 6\}$
- ☐ $A' \cup B = \{1, 3, 4, 5\}$
- ☐ $A' \cup B = \{2, 4, 6\}$
- ☒ $A' \cup B = \{2, 3, 4, 6\}$

Answer: The set $A' \cup B$ includes all elements that are either outside A or inside B in the Venn diagram.

- Elements outside A are 2, 4, 6.

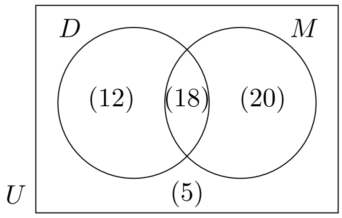
- Elements in B are 3, 4.
- The union is 2, 3, 4, 6.



Therefore, $A' \cup B = \{2, 3, 4, 6\}$.

B.3.3 SOLVING WORD PROBLEMS WITH VENN DIAGRAMS

Ex 108: The Venn diagram shows the number of students in a school who participate in the drama club (D) and the music club (M).



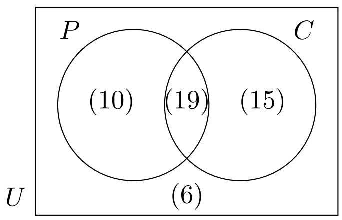
How many students:

- are in the school? $\boxed{55}$ students
- participate in the music club? $\boxed{38}$ students
- participate in both clubs? $\boxed{18}$ students
- do not participate in either club? $\boxed{5}$ students
- participate in at least one club? $\boxed{50}$ students

Answer:

- Total students in the school: $12 + 18 + 20 + 5 = \boxed{55}$.
- Music club participants: $18 + 20 = \boxed{38}$.
- Both clubs: $\boxed{18}$.
- Neither club: $\boxed{5}$.
- At least one club: $12 + 18 + 20 = \boxed{50}$.

Ex 109: The Venn diagram shows the number of participants in a community center attending painting (P) and cooking (C) classes.



How many participants:

- attend the community center? $\boxed{50}$ participants
- attend cooking classes? $\boxed{34}$ participants
- attend both classes? $\boxed{19}$ participants
- attend neither class? $\boxed{6}$ participants
- attend at least one class? $\boxed{44}$ participants

Answer:

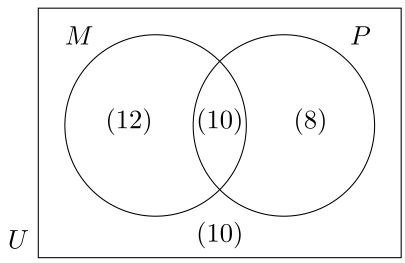
- Total participants: $10 + 19 + 15 + 6 = \boxed{50}$.
- Cooking class participants: $19 + 15 = \boxed{34}$.
- Both classes: $\boxed{19}$.
- Neither class: $\boxed{6}$.
- At least one class: $10 + 19 + 15 = \boxed{44}$.

Ex 110: In a class of 40 students, 22 like mathematics (M), 18 like physics (P), and 10 like both. How many students:

- like at least one subject? $\boxed{30}$
- like mathematics but not physics? $\boxed{12}$
- like exactly one subject? $\boxed{20}$
- like neither subject? $\boxed{10}$

Answer:

- Both subjects: 10
- Only mathematics: $22 - 10 = 12$
- Only physics: $18 - 10 = 8$
- Like at least one subject: $12 + 10 + 8 = \boxed{30}$
- Like mathematics but not physics: 12
- Like exactly one subject: $12 + 8 = \boxed{20}$
- Like neither subject: $40 - 30 = \boxed{10}$



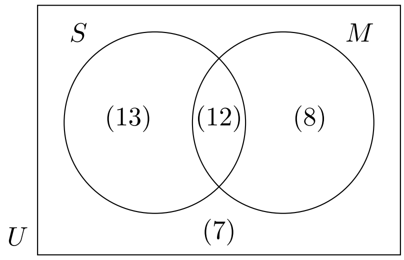
Ex 111: In a group of 40 employees, 25 work in sales (S), 20 in marketing (M), and 12 in both. How many employees:

- work in at least one department? $\boxed{33}$
- work in sales but not marketing? $\boxed{13}$
- work in exactly one department? $\boxed{21}$
- work in neither department? $\boxed{7}$

Answer:

- Both departments: 12

- Only sales: $25 - 12 = 13$
- Only marketing: $20 - 12 = 8$
- Work in at least one department: $13 + 12 + 8 = \boxed{33}$
- Work in sales but not marketing: 13
- Work in exactly one department: $13 + 8 = \boxed{21}$
- Work in neither department: $40 - 33 = \boxed{7}$



C NUMBER SETS

C.1 NUMBER SETS

C.1.1 CHECKING MEMBERSHIP

Ex 112: $6 \boxed{\in} \mathbb{Z}$

Answer:

- The set of **integers**, denoted $\mathbb{Z}$, contains all whole numbers (positive, negative, and zero):

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

- 6 is a whole number.
- So, $6 \in \mathbb{Z}$.

Ex 113: $-2 \boxed{\notin} \mathbb{N}$

Answer:

- The set of **natural numbers**, denoted $\mathbb{N}$, is usually defined as all non-negative whole numbers:

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

- -2 is negative and not in this set.
- So, $-2 \notin \mathbb{N}$.

Ex 114: $-\dfrac{2}{3} \boxed{\in} \mathbb{Q}$

Answer:

- The set of **rational numbers**, denoted $\mathbb{Q}$, is the set of all numbers that can be written as $\frac{p}{q}$ with $p \in \mathbb{Z}$ and $q \in \mathbb{Z}$, $q \neq 0$.
- $-\frac{2}{3} = \frac{-2}{3}$ fits this form ($p = -2$, $q = 3$).
- So, $-\frac{2}{3} \in \mathbb{Q}$.

Ex 115: $0.1 \boxed{\in} \mathbb{R}$

Answer:

- The set of **real numbers**, denoted $\mathbb{R}$, includes all points on the number line (rationals and irrationals).
- 0.1 is a decimal number, thus a real number.
- So, $0.1 \in \mathbb{R}$.

Ex 116: $3 \boxed{\in} \mathbb{Q}$

Answer:

- Every integer can be written as a fraction: $3 = \frac{3}{1}$.
- The set of rationals is $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z},\ q \neq 0\}$.
- So, $3 \in \mathbb{Q}$.

Ex 117: $\sqrt{2} \boxed{\notin} \mathbb{Q}$

Answer:

- $\sqrt{2}$ is not a rational number: it cannot be written as a fraction $\frac{p}{q}$ with $p, q \in \mathbb{Z}$, $q \neq 0$.
- **Proof by contradiction:** If $\sqrt{2} = \frac{p}{q}$ (with p, q coprime), then $2q^2 = p^2$, so p and q are both even, which contradicts that they are coprime.
- So, $\sqrt{2} \notin \mathbb{Q}$.

C.2 INTERVALS

C.2.1 CONVERTING SETS TO INTERVAL NOTATION

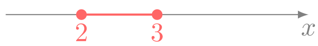
Ex 118: Express the set $\{x \in \mathbb{R} \mid -1 < x\}$ using interval notation.

Answer:

- $\{x \in \mathbb{R} \mid -1 < x\} = (-1, +\infty)$. The finite endpoint -1 is excluded; the interval extends to $+\infty$
- 

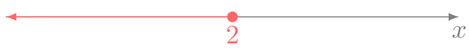
Ex 119: Express the set $\{x \in \mathbb{R} \mid 2 \leq x \leq 3\}$ using interval notation.

Answer:

- $\{x \in \mathbb{R} \mid 2 \leq x \leq 3\} = [2, 3]$. The endpoints 2 and 3 belong to the interval, so we use closed endpoints.
- 

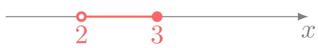
Ex 120: Express the set $\{x \in \mathbb{R} \mid x \leq 2\}$ using interval notation.

Answer:

- $\{x \in \mathbb{R} \mid x \leq 2\} = (-\infty, 2]$. The endpoint $-\infty$ does not belong to the interval (open), while 2 does (closed).
- 

Ex 121: Express the set $\{x \in \mathbb{R} \mid 2 < x \leq 3\}$ using interval notation.

Answer:

- $\{x \in \mathbb{R} \mid 2 < x \leq 3\} = (2, 3]$. The endpoint 2 does not belong to the interval (open), while 3 does (closed).
- 

C.2.2 CONVERTING NUMBER LINE GRAPHS TO INTERVAL NOTATION

Ex 122: Express the interval shown on the number line below using interval notation:

Answer: The number line shows a closed interval from 2 to 3, where both endpoints are included. Therefore, the interval is $[2, 3]$

Ex 123: Express the interval shown on the number line below using interval notation:

Answer: The number line shows an interval starting just after -0.5 and ending at 3, where -0.5 is excluded and 3 is included. Therefore, the interval is $(-0.5, 3]$.

Ex 124: Express the interval shown on the number line below using interval notation:

Answer: The number line shows an interval extending infinitely to the left (negative infinity) and ending just before 2, where 2 is not included. Therefore, the interval is $(-\infty, 2)$.

Ex 125: Express the interval shown on the number line below using interval notation:

Answer: The number line shows an interval starting at $\frac{1}{2}$ (included) and extending infinitely to the right. Therefore, the interval is $\left[\frac{1}{2}, +\infty\right)$.

C.2.3 CHECKING MEMBERSHIP

Ex 126: $2 \boxed{\notin} (2, 3)$

Answer: The number 2 does not belong to the interval $(2, 3)$ because the parentheses indicate that both endpoints are excluded.

Ex 127: $-0.5 \boxed{\in} (-1, 1)$

Answer: The number -0.5 belongs to the interval $(-1, 1)$ because $-1 < -0.5 < 1$.

Ex 128: $\frac{3}{2} \boxed{\in} [1, 2]$

Answer: The number $\frac{3}{2}$ belongs to the interval $[1, 2]$ because $1 \leq \frac{3}{2} \leq 2$.

Ex 129: $-3 \boxed{\in} (-\infty, 2)$

Answer: The number -3 belongs to the interval $(-\infty, 2)$ because $-3 < 2$.

C.2.4 SOLVING LINEAR INEQUALITIES

Ex 130: Find the solution set S of the inequality:

$$2x - 1 \geq 0$$

Express your answer in interval notation.

Answer: To solve the inequality:

$$\begin{aligned} 2x - 1 &\geq 0 && \text{(given)} \\ 2x &\geq 1 && \text{(add 1 to both sides)} \\ x &\geq \frac{1}{2} && \text{(divide by 2)} \end{aligned}$$

The solution set is all real numbers greater than or equal to $\frac{1}{2}$. In interval notation:

$$S = \left[\frac{1}{2}, +\infty\right)$$

Ex 131: Find the solution set S of the inequality:

$$-2x - 1 \geq 0$$

Express your answer in interval notation.

Answer: To solve the inequality:

$$\begin{aligned} -2x - 1 &\geq 0 && \text{(given)} \\ -2x &\geq 1 && \text{(add 1 to both sides)} \\ x &\leq -\frac{1}{2} && \text{(divide by } -2\text{, inequality reverses)} \end{aligned}$$

The solution set is all real numbers less than or equal to $-\frac{1}{2}$. In interval notation:

$$S = \left(-\infty, -\frac{1}{2}\right]$$

Ex 132: Find the solution set S of the inequality:

$$-2x + 4 < 2$$

Express your answer in interval notation.

Answer: To solve the inequality:

$$\begin{aligned} -2x + 4 &< 2 && \text{(given)} \\ -2x &< -2 && \text{(subtract 4 from both sides)} \\ x &> 1 && \text{(divide by } -2\text{, inequality reverses)} \end{aligned}$$

The solution set is all real numbers greater than 1. In interval notation:

$$S = (1, +\infty)$$

Ex 133: Find the solution set S of the inequality:

$$3x + 2 < -2x + 12$$

Express your answer in interval notation.

Answer: To solve the inequality:

$$\begin{aligned} 3x + 2 &< -2x + 12 \\ 5x + 2 &< 12 && \text{(add } 2x \text{ to both sides)} \\ 5x &< 10 && \text{(subtract 2 from both sides)} \\ x &< 2 && \text{(divide by 5)} \end{aligned}$$

The solution set is all real numbers less than 2. In interval notation:

$$S = (-\infty, 2)$$