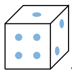
# SET THEORY

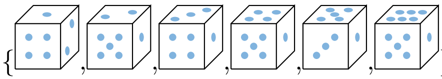
## A DEFINITIONS

### A.1 SET

**Definition Set**

A **set** is a collection of objects, called elements.
We list its elements between curly brackets.

**Ex:** List all possible results when rolling a standard die .

*Answer:* $E = \{1, 2, 3, 4, 5, 6\} = \{$  $\}$.

**Definition Element**

- An **element** is an object contained in a set.
- $\in$ means "is an element of" or "belongs to".
- $\notin$ means "is not an element of" or "does not belong to".

**Ex:** $2 \in \{1, 2, 3, 4, 5, 6\}$ and $7 \notin \{1, 2, 3, 4, 5, 6\}$.

**Definition Equal sets**

Two sets are **equal** if they have exactly the same elements.

**Ex:** Determine if the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal.

*Answer:* Yes, the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal because they contain the same elements: 2, 4, and 6.

**Ex:** Determine if the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are equal.

*Answer:* No, the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are not equal because element 3 belongs to $\{1, 2, 3\}$ but not to $\{1, 2, 4\}$.

**Definition Empty Set**

The **empty set** is a set with no elements. It is written as $\{\}$ or $\emptyset$.

### A.2 NATURAL NUMBERS

**Definition Natural Numbers**

The set of **natural numbers**, denoted $\mathbb{N}$, is the set of counting numbers starting from zero:

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \dots\}$$

### A.3 SUBSETS

**Definition Subset**

A set $A$ is a **subset** of a set $B$ if every element in $A$ is also in $B$. We write this as $A \subseteq B$.

**Ex:** Is $A \subseteq B$ when $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$?

*Answer:* Check each element: 2, 4, and 6 from $A$ are all in $B = \{1, 2, 3, 4, 5, 6\}$. Since every element of $A$ is in $B$, $A \subseteq B$.

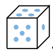
### A.4 SET-BUILDER NOTATION

**Definition Set-Builder Notation**

The **set-builder notation** is a way to describe a set by giving a rule that its elements must follow. It is written like this:

$$\{x \in E \mid \text{condition on } x\}$$

In this notation, $x$ represents an element from a set $E$, and the symbol $\mid$ (or sometimes :) separates $x$ from the condition it must meet. It reads: "the set of all $x$ in $E$ such that $x$ follows the given condition."

**Ex:** Let $E$ be the set of results from rolling a standard six-sided die .
What are the elements of the set $\{x \in E \mid x \text{ is even}\}$?

*Answer:* The set $\{x \in E \mid x \text{ is even}\}$ contains all the even numbers in $E$. Given $E = \{1, 2, 3, 4, 5, 6\}$, the even numbers are 2, 4, and 6, so:

$$\{x \in E \mid x \text{ is even}\} = \{2, 4, 6\}$$

### A.5 ORDERED PAIR AND N-TUPLE

**Definition Ordered Pair and n-Tuple**

An **ordered pair**, denoted $(a, b)$ or sometimes $ab$, is a pair of elements where the order matters.
More generally, an **ordered n-tuple**, denoted $(a_1, a_2, \dots, a_n)$, is a sequence of $n$ elements where the order of the elements is significant. An ordered pair is a special case when $n = 2$.

**Ex:** In a sprint relay race, two runners are paired up. Let $L$ be Louis and $H$ be Hugo. The ordered pair $(L, H)$ means Louis runs first, then passes the baton to Hugo. The ordered pair $(H, L)$ means Hugo runs first, then passes to Louis. These are different races.

### A.6 CARDINALITY

**Definition Cardinality**

$n(A)$ denotes the number of elements in the set $A$.

**Ex:** $n(\{1, 2, 3, 4, 5, 6\}) = 6 =$ 

**Definition Finite and Infinite Sets**

- A **finite set** has a finite number of elements. Informally, a finite set is a set which one could in principle count and finish counting.
- An **infinite set** is not a finite set.

**Ex:**

- $\{1, 2, 3\}$ is finite because it has exactly 3 elements .
- $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ is infinite because the counting goes on forever without stopping.

## B OPERATIONS

### B.1 COMPLEMENT

**Definition Universal set**

A **universal set** is the set of all elements considered.

**Definition Complement**

The **complement** of a set $A$, denoted $A'$, consists of all elements in $U$ that are not in $A$. Sets $A$ and $A'$ are said to be **complementary**.

**Ex:** Given the universe $U = \{1, 2, 3, 4, 5, 6\}$ and the set $A = \{1, 3, 5\}$, find the complement $A'$.

*Answer:* Start with the universe $U = \{1, 2, 3, 4, 5, 6\}$.
The set $A = \{1, 3, 5\}$ includes 1, 3, and 5.
The complement $A'$ is all the elements in $U$ that are not in $A$:

$$A' = \{2, 4, 6\}$$

## B.2 INTERSECTION AND UNION

**Definition Intersection**
The **intersection** of two sets $A$ and $B$, written $A \cap B$, is the set of elements that are in both $A$ and $B$.

**Ex:** What is the intersection $\{1, 2, 3\} \cap \{2, 3, 4\}$?

*Answer:* For the intersection $\cap$, include all common element: 2 3. Donc

$$\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

**Definition Union**
The **union** of two sets $A$ and $B$, written $A \cup B$, is the set of all elements in $A$ or $B$ (or both).

**Ex:** What is the union $\{1, 2, 3\} \cup \{2, 3, 4\}$?

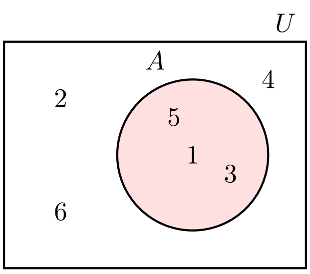
*Answer:* For the union $\cup$, include all elements from both sets without repeats: 1, 2, 3, 4. So,

$$\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

## B.3 VENN DIAGRAMS

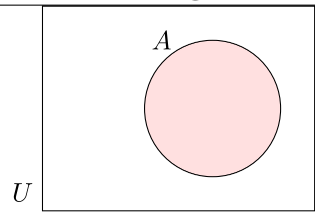
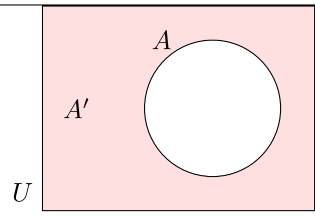
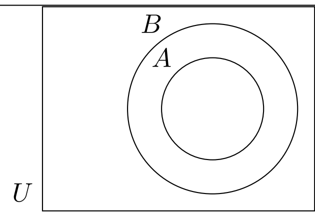
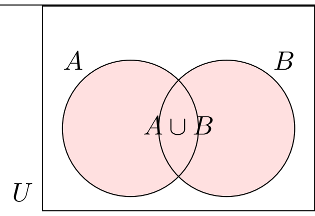
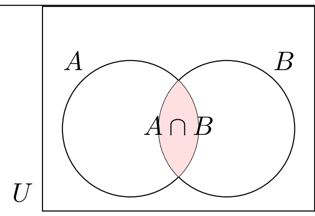
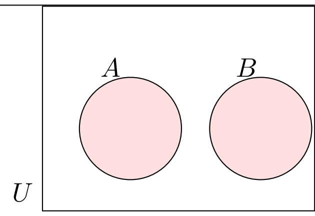
**Definition Venn Diagram**
A **Venn diagram** uses a rectangle to show the universal set $U$ and circles to represent other sets within it.

**Ex:** Here's a Venn diagram for $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$:



**Definition Key Venn Diagram Concepts**

This table shows common set operations and their Venn diagrams:

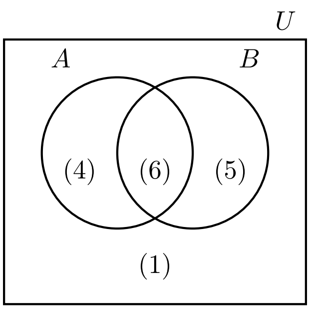
| Notation | Meaning | Venn Diagram |
|---|---|---|
| $A$ | Set $A$ |  |
| $A'$ | Complement of $A$ (everything in $U$ not in $A$) |  |
| $A \subseteq B$ | $A$ is a subset of $B$ |  |
| $A \cup B$ | Union of $A$ and $B$ (all elements in $A$ or $B$) |  |
| $A \cap B$ | Intersection of $A$ and $B$ (elements in both) |  |
| $A \cap B = \{\}$ | $A$ and $B$ are disjoint (no common elements) |  |

Venn diagrams help solve problems by showing the number of elements in each region.

**Definition Counting Elements**

In a Venn diagram, we use brackets around numbers to show how many elements are in each region.

**Ex:** Consider this Venn diagram:




Here, there are 6 elements in both $A$ and $B$, 4 in $A$ but not $B$, 5 in $B$ but not $A$, and 1 outside both. Total elements: $A$ has $4 + 6 = 10$, $B$ has $6 + 5 = 11$.

# C NUMBER SETS

## C.1 COMMON NUMBER SETS

Number sets are groups of numbers defined by specific properties, and they form the foundation of mathematics. We start with the simplest set, the natural numbers ($\mathbb{N}$), which we use for basic counting. From there, we build the integers ($\mathbb{Z}$) by adding negative numbers to include opposites. Next, we expand to the rational numbers ($\mathbb{Q}$) by allowing fractions, which cover division between integers. Finally, we reach the real numbers ($\mathbb{R}$) by filling in all the gaps with irrational numbers, creating a complete number line. Below are the most common number sets you'll encounter, each one growing from the previous set in this progression.

**Definition Common Number Sets**

- The **natural numbers**, denoted $\mathbb{N}$, are the counting numbers starting from zero:

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \dots\}$$

- The **integers**, denoted $\mathbb{Z}$, include all whole numbers—positive, negative, and zero:

$$\mathbb{Z} = \{\dots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots\}$$

- The **rational numbers**, denoted $\mathbb{Q}$, are numbers that can be written as a fraction $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$. Examples include $\frac{1}{2}$, $-3$, and $\frac{5}{4}$:

$$\mathbb{Q} = \left\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\right\}$$

- The **real numbers**, denoted $\mathbb{R}$, include all numbers on a continuous number line, such as rational numbers (like $\frac{1}{3}$) and irrational numbers (like $\sqrt{2}$ and $\pi$):

$$\mathbb{R} = \{\text{all numbers, rational and irrational}\}$$

**Proposition Relationships Between Number Sets**

The number sets are nested as follows:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

This means every natural number is an integer, every integer is a rational number, and every rational number is a real number.

## C.2 INTERVALS

**Definition Interval**

An **interval** is a set of all real numbers between two endpoints, which may or may not be included in the set.

**Ex:** The set of all real numbers between 0 and 1, including 1 but not 0, is an interval. It is written as $\{x \in \mathbb{R} \mid 0 < x \leq 1\}$.

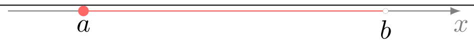
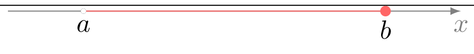
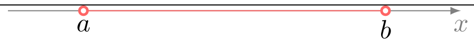
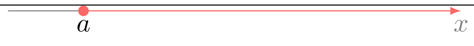
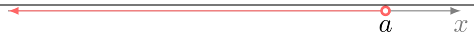
**Method Representing Intervals on a Number Line**

Intervals are often shown on a number line using these conventions:

1. An *open circle* (or parenthesis) means the endpoint is not included.
2. A *filled circle* (or bracket) means the endpoint is included.
3. An *arrow* shows that the interval extends to positive or negative infinity.

**Ex:** The number line representation of $\{x \in \mathbb{R} \mid 0 < x \leq 1\}$ is:

**Definition Interval notation**

| Interval Notation | Set-builder notation | Number line representation |
|---|---|---|
| $[a,b]$ | $\{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$ |  |
| $[a,b)$ | $\{x \in \mathbb{R} \mid a \leqslant x < b\}$ |  |
| $(a,b]$ | $\{x \in \mathbb{R} \mid a < x \leqslant b\}$ |  |
| $(a,b)$ | $\{x \in \mathbb{R} \mid a < x < b\}$ |  |
| $[a,+\infty)$ | $\{x \in \mathbb{R} \mid a \leqslant x\}$ |  |
| $(a,+\infty)$ | $\{x \in \mathbb{R} \mid a < x\}$ |  |
| $(-\infty,a]$ | $\{x \in \mathbb{R} \mid x \leqslant a\}$ |  |
| $(-\infty,a)$ | $\{x \in \mathbb{R} \mid x < a\}$ |  |