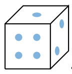
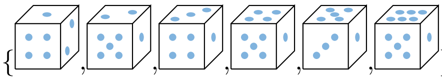
SET THEORY

A SET

Definition Set

A **set** is a collection of objects, called elements.
We list its elements between curly brackets.

Ex: List all possible results when rolling a standard die .

Answer: $E = \{1, 2, 3, 4, 5, 6\} = \{$$\}$.

Definition Element

- An **element** is an object contained in a set.
- $\in$ means "is an element of" or "belongs to".
- $\notin$ means "is not an element of" or "does not belong to".

Ex: $2 \in \{1, 2, 3, 4, 5, 6\}$ and $7 \notin \{1, 2, 3, 4, 5, 6\}$.

Definition Equal sets

Two sets are **equal** if they have exactly the same elements.

Ex: Determine if the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal.

Answer: Yes, the sets $\{2, 6, 4\}$ and $\{2, 4, 6\}$ are equal because they contain the same elements: 2, 4, and 6.

Ex: Determine if the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are equal.

Answer: No, the sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are not equal because element 3 belongs to $\{1, 2, 3\}$ but not to $\{1, 2, 4\}$.

Definition Empty Set

The **empty set** is a set with no elements. It is written as $\{\}$ or $\emptyset$.

B ORDERED PAIR

Definition Ordered Pair

An **ordered pair**, denoted (a, b) or ab, is a pair of objects in which their order is significant. The ordered pair (a, b) is different from the ordered pair (b, a) unless $a = b$.

Ex: In a sprint relay race, two runners are paired up. Let L be Louis and H be Hugo. The ordered pair (L, H) means Louis runs first, then passes the baton to Hugo. The ordered pair (H, L) means Hugo runs first, then passes to Louis. These are different races.

C SUBSETS

Definition Subset

A set A is a **subset** of a set B if every element in A is also in B. We write this as $A \subseteq B$.

Ex: Is $A \subseteq B$ when $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$?

Answer: Check each element: 2, 4, and 6 from A are all in $B = \{1, 2, 3, 4, 5, 6\}$. Since every element of A is in B, $A \subseteq B$.

D INTERSECTION AND UNION

Definition Intersection

The **intersection** of two sets A and B, written $A \cap B$, is the set of elements that are in both A and B.

Ex: What is the intersection $\{1, 2, 3\} \cap \{2, 3, 4\}$?

Answer: For the intersection $\cap$, include all common element: 2 3. Donc

$$\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

Definition Union

The **union** of two sets A and B, written $A \cup B$, is the set of all elements in A or B (or both).

Ex: What is the union $\{1, 2, 3\} \cup \{2, 3, 4\}$?

Answer: For the union $\cup$, include all elements from both sets without repeats: $1, 2, 3, 4$. So,

$$\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

E CARDINALITY

Definition Cardinality

$n(A)$ denotes the number of elements in the set A.

Ex: $n(\{1, 2, 3, 4, 5, 6\}) = 6$.

F COMPLEMENT

Definition Universal set

A **universal set** is the set of all elements considered.

Definition Complement

The **complement** of a set A, denoted A', consists of all elements in U that are not in A. Sets A and A' are said to be **complementary**.

Ex: Given the universe $U = \{1, 2, 3, 4, 5, 6\}$ and the set $A = \{1, 3, 5\}$, find the complement A'.

Answer: Start with the universe $U = \{1, 2, 3, 4, 5, 6\}$.
The set $A = \{1, 3, 5\}$ includes 1, 3, and 5.
The complement A' is all the elements in U that are not in A:

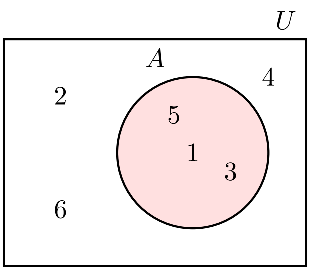
$$A' = \{2, 4, 6\}$$

G VENN DIAGRAMS

Definition Venn Diagram

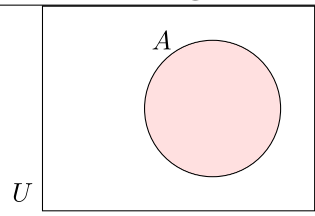
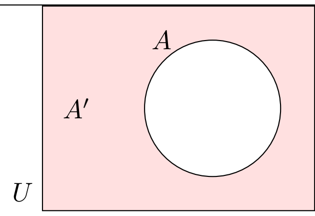
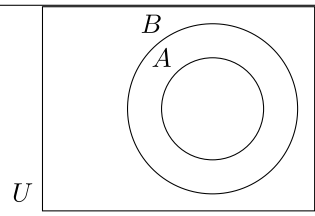
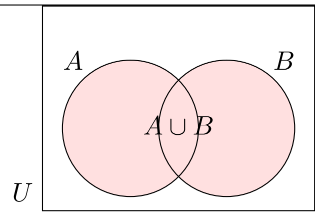
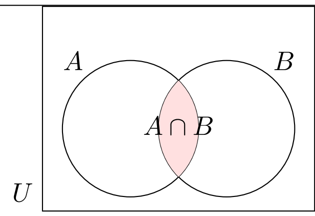
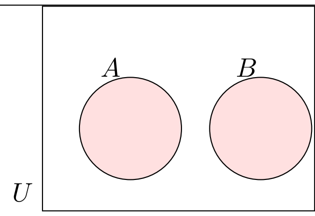
A **Venn diagram** uses a rectangle to show the universal set U and circles to represent other sets within it.

Ex: Here's a Venn diagram for $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$:



Definition **Key Venn Diagram Concepts**

This table shows common set operations and their Venn diagrams:

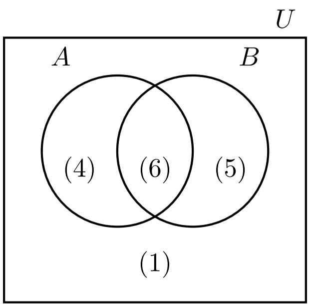
Notation	**Meaning**	**Venn Diagram**
A	Set A	
A'	Complement of A (everything in U not in A)	
$A \subseteq B$	A is a subset of B	
$A \cup B$	Union of A and B (all elements in A or B)	
$A \cap B$	Intersection of A and B (elements in both)	
$A \cap B = \{\}$	A and B are disjoint (no common elements)	

Venn diagrams help solve problems by showing the number of elements in each region.

Definition **Counting Elements**

In a Venn diagram, we use brackets around numbers to show how many elements are in each region.

Ex: Consider this Venn diagram:



Here, there are 6 elements in both A and B, 4 in A but not B, 5 in B but not A, and 1 outside both. Total elements: A has $4 + 6 = 10$, B has $6 + 5 = 11$.