

SEQUENCES

A NUMERICAL SEQUENCE

A.1 FINDING THE VALUE OF A SPECIFIC TERM (u_n)

Ex 1: Using the table below, find u_4 .

n	1	2	3	4	5	6
u_n	3	5	7	9	11	13

$$u_4 = \boxed{}$$

Ex 2: Using the table below, find u_5 .

n	1	2	3	4	5	6
u_n	2	6	12	20	30	42

$$u_5 = \boxed{}$$

Ex 3: Using the table below, find u_7 .

n	1	2	3	4	5	6	7	8
u_n	4	9	16	25	36	49	64	81

$$u_7 = \boxed{}$$

Ex 4: Using the table below, find u_8 .

n	1	2	3	4	5	6	7	8
u_n	1	3	7	15	31	63	127	255

$$u_8 = \boxed{}$$

B RECURSIVE DEFINITION

B.1 CALCULATING TERMS FROM A RECURSIVE RULE

Ex 5: A sequence is defined recursively by:

- $u_1 = 5$.
- $u_{n+1} = u_n + 3$.

Find the first four terms of this sequence.

- $u_1 = \boxed{}$
- $u_2 = \boxed{}$
- $u_3 = \boxed{}$
- $u_4 = \boxed{}$

Ex 6: A sequence is defined recursively by:

- $u_0 = 1$.
- $u_{n+1} = u_n + \frac{1}{2}$.

Find the first four terms of this sequence (from u_0 to u_3).

- $u_0 = \boxed{}$

- $u_1 = \boxed{}$
- $u_2 = \boxed{}$
- $u_3 = \boxed{}$

Ex 7: A sequence is defined recursively by:

- $u_0 = 0$.
- $u_{n+1} = 2u_n + 1$.

Find the first four terms of this sequence (from u_0 to u_3).

- $u_0 = \boxed{}$
- $u_1 = \boxed{}$
- $u_2 = \boxed{}$
- $u_3 = \boxed{}$

Ex 8: A sequence is defined recursively by:

- $u_0 = 3$.
- $u_{n+1} = -u_n + 1$.

Find the first four terms of this sequence (from u_0 to u_3).

- $u_0 = \boxed{}$
- $u_1 = \boxed{}$
- $u_2 = \boxed{}$
- $u_3 = \boxed{}$

B.2 MODELING REAL SITUATIONS WITH SEQUENCES


Ex 9: A scientist observes a culture of bacteria. Initially (at day 0), there are $u_0 = 5$ bacteria. Each day, the number of bacteria doubles. Let u_n be the number of bacteria at day n .

Part A: Define the Sequence Recursively

- The initial term is $u_0 = \boxed{}$
- The recursive rule is $u_{n+1} = \boxed{} \times u_n$.

Part B: Calculate the Terms for the First Five Days

- $u_1 = \boxed{}$ bacteria
- $u_2 = \boxed{}$ bacteria
- $u_3 = \boxed{}$ bacteria
- $u_4 = \boxed{}$ bacteria
- $u_5 = \boxed{}$ bacteria

Ex 10:  Let u_n be the number of steps I walk on day n . On day 0, I walk $u_0 = 1000$ steps. Each day, I walk 500 more steps than the previous day.

Part A: Define the Sequence Recursively

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = u_n + \boxed{}$.

Part B: Calculate the Number of Steps for the Next Five Days

- $u_1 = \boxed{}$ steps
- $u_2 = \boxed{}$ steps
- $u_3 = \boxed{}$ steps
- $u_4 = \boxed{}$ steps
- $u_5 = \boxed{}$ steps

Ex 11: Let u_n be the amount of money you have at the start of week n . At the start of week 0, you have $u_0 = 20$ dollars. Each week, you receive an allowance of \$10.

Part A: Define the Sequence Recursively

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = u_n + \boxed{}$.

Part B: Calculate the Amount of Money for the Next Five Weeks

- $u_1 = \boxed{}$ dollars
- $u_2 = \boxed{}$ dollars
- $u_3 = \boxed{}$ dollars
- $u_4 = \boxed{}$ dollars
- $u_5 = \boxed{}$ dollars

B.3 IDENTIFYING THE RECURSIVE RULE

Ex 12: Given the sequence $(3, 5, 7, 9, 11, 13, \dots)$, starting with index $n = 0$. Find its recursive definition.

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = \boxed{}$

Ex 13: Given the sequence $(100, 90, 80, 70, 60, \dots)$, starting with index $n = 0$. Find its recursive definition.

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = \boxed{}$

Ex 14: Given the sequence $(2, 6, 18, 54, 162, \dots)$, starting with index $n = 0$. Find its recursive definition.

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = \boxed{}$

Ex 15: Given the sequence $(8, 4, 2, 1, 0.5, \dots)$, starting with index $n = 0$. Find its recursive definition.

- The initial term is $u_0 = \boxed{}$.
- The recursive rule is $u_{n+1} = \boxed{}$

B.4 MODELING WITH ARITHMETICO-GEOMETRIC SEQUENCES



Ex 16: A company has 200 employees in 2025. Each year, 10% of the employees leave the company, and the company hires 30 new employees.

Let (u_n) be the sequence corresponding to the number of employees in the company in $2025 + n$.

1. How many employees will there be in 2026?

2. How many employees will there be in 2027?

3. For all $n \in \mathbb{N}$, express u_{n+1} in terms of u_n .

$$u_{n+1} = \boxed{}$$



Ex 17: A gym has 200 members in 2025. Each year, the number of members increases by 10% through referrals, and the gym adds 20 new members from advertising.

Let (u_n) be the sequence corresponding to the number of members in the gym in $2025 + n$.

1. How many members will there be in 2026?

2. How many members will there be in 2027?

3. For all $n \in \mathbb{N}$, express u_{n+1} in terms of u_n .

$$u_{n+1} = \boxed{}$$

C EXPLICIT DEFINITION

C.1 CALCULATING TERMS USING AN EXPLICIT FORMULA



Ex 18: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$.

Calculate u_{100} .

$$u_{100} = \boxed{}$$



Ex 19: Consider the sequence defined by the explicit formula: $u_n = -5n + 100$.

Calculate u_{50} .

$$u_{50} = \boxed{}$$



Ex 20: Consider the sequence defined by the explicit formula: $u_n = 3 \times 2^n$. Calculate u_{10} .

$$u_{10} = \boxed{}$$



Ex 21: Consider the sequence defined by the explicit formula: $u_n = n^2 + 5$. Calculate u_{20} .

$$u_{20} = \boxed{}$$



Ex 22: Consider the sequence defined by the explicit formula: $u_n = \frac{n}{4} + 1$. Calculate u_{40} .

$$u_{40} = \boxed{}$$

C.2 FINDING THE EXPLICIT FORMULA FROM A PATTERN

Ex 23: For the sequence given in the table below, find the explicit formula for u_n .

n	0	1	2	3	4
u_n	0	3	6	9	12

$$u_n = \boxed{}$$

Ex 24: For the sequence given in the table below, find the explicit formula for u_n .

n	0	1	2	3	4	5
u_n	1	2	4	8	16	32

$$u_n = \boxed{}$$

Ex 25: For the sequence given in the table below, find the explicit formula for u_n .

n	1	2	3	4	5
u_n	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$$u_n = \boxed{}$$

Ex 26: For the sequence given in the table below, find the explicit formula for u_n .

n	0	1	2	3	4
u_n	1	3	5	7	9

$$u_n = \boxed{}$$

Ex 27: For the sequence given in the table below, find the explicit formula for u_n .

n	1	2	3	4	5
u_n	1	4	9	16	25

$$u_n = \boxed{}$$

C.3 FINDING EXPRESSIONS FOR ADJACENT TERMS

Ex 28: Consider the sequence defined by the explicit formula: $u_n = 3n + 2$. Calculate and simplify u_{n+1} .

$$u_{n+1} = \boxed{}$$

Ex 29: Consider the sequence defined by the explicit formula: $u_n = 5n - 2$. Calculate and simplify u_{n-1} .

$$u_{n-1} = \boxed{}$$

Ex 30: Consider the sequence defined by the explicit formula: $u_n = n^2 + 3n$. Calculate and simplify u_{n+1} .

$$u_{n+1} = \boxed{}$$

Ex 31: Consider the sequence defined by the explicit formula: $u_n = 10 - 4n$. Calculate and simplify u_{n-1} .

$$u_{n-1} = \boxed{}$$

D ARITHMETIC SEQUENCE

D.1 STUDYING AN ARITHMETIC SEQUENCE

Ex 32: Consider the sequence ($u_0 = 5$, $u_1 = 8$, $u_2 = 11$, $u_3 = 14$, $u_4 = 17$, ...).

- $u_1 - u_0 = \boxed{}$
 - $u_2 - u_1 = \boxed{}$
 - $u_3 - u_2 = \boxed{}$

2. Show that the sequence is arithmetic.

- ☐ The difference between consecutive terms is constant.
- ☐ The ratio of consecutive terms is constant.
- ☐ The terms alternate in sign.

3. What is its recursive rule?

$$u_{n+1} = \boxed{}$$

4. What is its explicit rule?

$$u_n = \boxed{}$$

5. Find the 50th term of the sequence.

$$u_{50} = \boxed{}$$

Ex 33: Consider the sequence ($u_0 = 4$, $u_1 = 9$, $u_2 = 14$, $u_3 = 19$, ...).

- $u_1 - u_0 = \boxed{}$

- $u_2 - u_1 = \boxed{}$
- $u_3 - u_2 = \boxed{}$

- Show that the sequence is arithmetic.
 - ☐ The difference between consecutive terms is constant.
 - ☐ The ratio of consecutive terms is constant.
 - ☐ The terms alternate in sign.
- What is its recursive rule?


$$u_{n+1} = \boxed{}$$

- What is its explicit rule?

$$u_n = \boxed{}$$

- Find the 50th term of the sequence.

$$u_{50} = \boxed{}$$

Ex 34:  Consider the sequence ($u_0 = 125$, $u_1 = 115$, $u_2 = 105$, $u_3 = 95$, ...).

- $u_1 - u_0 = \boxed{}$
 - $u_2 - u_1 = \boxed{}$
 - $u_3 - u_2 = \boxed{}$
- Show that the sequence is arithmetic.
 - ☐ The difference between consecutive terms is constant.
 - ☐ The ratio of consecutive terms is constant.
 - ☐ The terms alternate in sign.
- What is its recursive rule?

$$u_{n+1} = \boxed{}$$


- What is its explicit rule?

$$u_n = \boxed{}$$

- Find the 1000th term of the sequence.

$$u_{1000} = \boxed{}$$

D.2 MODELING REAL SITUATIONS WITH EXPLICIT FORMULAS


Ex 35:  You have an initial savings of \$30. Each week, you add \$10 to your savings. Let u_n be the total amount of money you have after n weeks.

- **Part A: Write the Explicit Formula**
The formula for the amount of money after n weeks is:

$$u_n = \boxed{}$$

- **Part B: Calculate a Future Value**
How much money will you have after 20 weeks?

$$u_{20} = \boxed{}$$


Ex 36:  You deposit \$1,500 in a savings account that pays simple interest at a rate of 4% per year. Let u_n be the total amount in the account after n years.

- **Part A: Write the Explicit Formula**
The interest earned each year is 4% of \$1,500, which is $0.04 \times 1500 = 60$ dollars.
The formula for the amount after n years is:

$$u_n = \boxed{}$$

- **Part B: Calculate a Future Value**
What will your account balance be after 20 years?

$$u_{20} = \boxed{} \text{ dollars}$$

Ex 37:  You start a stamp collection with 12 stamps. Each month, you add 4 new stamps. Let u_n be the total number of stamps after n months.


- **Part A: Write the Explicit Formula**
The formula for the number of stamps after n months is:

$$u_n = \boxed{}$$

- **Part B: Calculate a Future Value**
How many stamps will you have after 15 months?

$$u_{15} = \boxed{} \text{ stamps}$$


D.3 FINDING THE TERM NUMBER IN AN ARITHMETIC SEQUENCE

Ex 38:  An arithmetic sequence is defined by its initial term $u_0 = 8$ and a common difference $d = 4$. Determine the index n for which the term u_n has a value of 56.

$$n = \boxed{}$$

Ex 39: An arithmetic sequence is defined by its first term $u_1 = 10$ and a common difference $d = 5$. Determine the index n for which the term u_n has a value of 105.

$$n = \boxed{}$$

Ex 40:  An arithmetic sequence is defined by its first term $u_1 = 50$ and a common difference $d = -4$. Determine the index n for which the term u_n has a value of 10.

$$n = \boxed{}$$

Ex 41: An arithmetic sequence is defined by its initial term $u_0 = 1$ and a common difference $d = 2$. Determine the index n for which the term u_n has a value of 21.

$$n = \boxed{}$$

D.4 FINDING THE EXPLICIT FORMULA FROM TWO TERMS

Ex 42: An arithmetic sequence is given by two of its terms:
 $u_2 = 11$ and $u_6 = 31$.
 Determine the explicit formula for u_n .

$$u_n = \boxed{}$$

Ex 43: An arithmetic sequence is given by two of its terms:
 $u_5 = 20$ and $u_{10} = 5$.
 Determine the explicit formula for u_n .


$$u_n = \boxed{}$$

Ex 44: An arithmetic sequence is given by two of its terms:
 $u_3 = 4$ and $u_7 = 6$.
 Determine the explicit formula for u_n .

$$u_n = \boxed{}$$

E GEOMETRIC SEQUENCE

E.1 STUDYING A GEOMETRIC SEQUENCE

Ex 45:  Consider the sequence ($u_0 = 3$, $u_1 = 6$, $u_2 = 12$, $u_3 = 24$, ...).

- $u_1 \div u_0 = \boxed{}$
 - $u_2 \div u_1 = \boxed{}$
 - $u_3 \div u_2 = \boxed{}$

- Show that the sequence is geometric.
 - ☐ The ratio between consecutive terms is constant.
 - ☐ The difference between consecutive terms is constant.
 - ☐ The terms alternate in sign.
 The terms are all even.

- What is its recursive rule?


$$u_{n+1} = \boxed{}$$

- What is its explicit rule?

$$u_n = \boxed{}$$

- Find the 10th term of the sequence.

$$u_{10} = \boxed{}$$

Ex 46:  Consider the sequence ($u_0 = 1$, $u_1 = -1$, $u_2 = 1$, $u_3 = -1$, $u_4 = 1$, ...).

- $u_1 \div u_0 = \boxed{}$
 - $u_2 \div u_1 = \boxed{}$
 - $u_3 \div u_2 = \boxed{}$

- Show that the sequence is geometric.
 - ☐ The ratio between consecutive terms is constant.
 - ☐ The difference between consecutive terms is constant.
 - ☐ The terms alternate in sign.
 Every term is positive.
- What is its recursive rule?


$$u_{n+1} = \boxed{}$$

- What is its explicit rule?

$$u_n = \boxed{}$$

- Find the 10th term of the sequence.

$$u_{10} = \boxed{}$$

Ex 47:  Consider the sequence ($u_0 = 4$, $u_1 = 2$, $u_2 = 1$, $u_3 = 0.5$, $u_4 = 0.25$, ...).

- $u_1 \div u_0 = \boxed{}$
 - $u_2 \div u_1 = \boxed{}$
 - $u_3 \div u_2 = \boxed{}$

- Show that the sequence is geometric.
 - ☐ The ratio between consecutive terms is constant.
 - ☐ The difference between consecutive terms is constant.
 - ☐ The terms alternate in sign.
 The terms are increasing.

- What is its recursive rule?

$$u_{n+1} = \boxed{}$$


- What is its explicit rule?

$$u_n = \boxed{}$$

- Find the 10th term of the sequence.

$$u_{10} = \boxed{}$$

E.2 MODELING REAL SITUATIONS WITH EXPLICIT FORMULAS

Ex 48:  A scientist observes a culture of bacteria. Initially (at hour 0), there are $u_0 = 50$ bacteria. Each hour, the number of bacteria doubles. Let u_n be the number of bacteria after n hours.

- Part A: Write the Explicit Formula**


The formula for the number of bacteria after n hours is:

$$u_n = \boxed{}$$

- Part B: Calculate a Future Value**

How many bacteria will there be after 6 hours?

$$u_6 = \boxed{} \text{ bacteria}$$

Ex 49:  You invest \$2,000 in an account with compound interest that grows by 5% each year. Let u_n be the total amount in the account after n years.

• **Part A: Write the Explicit Formula**


To increase by 5%, we multiply by 1.05. The formula for the amount after n years is:

$$u_n = \boxed{}$$

• **Part B: Calculate a Future Value**

What will the balance be after 10 years? (Round to two decimal places)

$$u_{10} = \boxed{}$$

Ex 50:  A radioactive substance has an initial mass of 1000 grams. Its half-life is one year, meaning it loses half of its mass every year through nuclear decay. Let u_n be the mass of the substance after n years.

• **Part A: Write the Explicit Formula**

The formula for the mass remaining after n years is:

$$u_n = \boxed{}$$

• **Part B: Calculate a Future Value**


How much of the substance will remain after 5 years? (Round to two decimal places)

$$u_5 = \boxed{} \text{ grams}$$


E.3 FINDING THE TERM NUMBER IN A GEOMETRIC SEQUENCE

Ex 51: A geometric sequence is defined by its initial term $u_0 = 10$ and a common ratio $r = 2$. Determine the index n for which the term u_n has a value of 160.

$$n = \boxed{}$$


Ex 52:  A geometric sequence is defined by its initial term $u_0 = 1000$ and a common ratio $r = 0.5$. Determine the index n for which the term u_n has a value of 125.

$$n = \boxed{}$$


Ex 53:  A geometric sequence is defined by its first term $u_1 = 5$ and a common ratio $r = 3$. Determine the index n for which the term u_n has a value of 3645.

$$n = \boxed{}$$


E.4 FINDING THE EXPLICIT FORMULA FROM TWO TERMS

Ex 54:  A geometric sequence is given by two of its terms: $u_2 = 4$ and $u_6 = 64$. Assuming the common ratio is positive, determine the explicit formula for u_n .

$$u_n = \boxed{}$$

Ex 55:  A geometric sequence is given by two of its terms: $u_3 = 20$ and $u_5 = 5$. Assuming the common ratio is positive, determine the explicit formula for u_n .

$$u_n = \boxed{}$$

Ex 56:  A geometric sequence is given by two of its terms: $u_2 = 12$ and $u_5 = 96$. Assuming the common ratio is positive, determine the explicit formula for u_n (starting from u_1).

$$u_n = \boxed{}$$

F SERIES

F.1 CALCULATING TERMS AND PARTIAL SUMS

Ex 57: Consider the sequence $(u_n) = (2, 5, 8, 11, \dots)$, where the first term is u_0 . Find:

1. $u_0 = \boxed{}$

2. $u_1 = \boxed{}$

3. $u_2 = \boxed{}$

4. $S_0 = \boxed{}$

5. $S_1 = \boxed{}$

6. $S_2 = \boxed{}$

Ex 58: Consider the sequence $(u_n) = (10, 20, 40, 80, \dots)$, where the first term is u_1 . Find:

1. $u_1 = \boxed{}$

2. $u_2 = \boxed{}$

3. $u_3 = \boxed{}$

4. $S_1 = \boxed{}$

5. $S_2 = \boxed{}$

6. $S_3 = \boxed{}$

Ex 59: Consider the sequence $(u_n) = (100, 95, 90, 85, \dots)$, where the first term is u_1 . Find:

1. $u_1 = \boxed{}$

$$2. u_2 = \square$$

$$3. u_3 = \square$$

$$4. S_1 = \square$$

$$5. S_2 = \square$$

$$6. S_3 = \square$$

Ex 60: Consider the sequence $(u_n) = (64, 16, 4, 1, \dots)$, where the first term is u_0 . Find:

$$1. u_0 = \square$$

$$2. u_1 = \square$$

$$3. u_2 = \square$$

$$4. S_0 = \square$$

$$5. S_1 = \square$$

$$6. S_2 = \square$$

F.2 CALCULATING PARTIAL SUMS FROM AN EXPLICIT FORMULA

Ex 61: Consider the sequence (u_n) defined by the explicit formula $u_n = 2n + 1$, starting from $n = 1$. Calculate the partial sum S_4 .

$$S_4 = \square$$

Ex 62: Consider the sequence (u_n) defined by the explicit formula $u_n = 2^n$, starting from $n = 0$. Calculate the partial sum S_4 .

$$S_4 = \square$$

Ex 63: Consider the sequence (u_n) defined by the explicit formula $u_n = 15 - 10n$, starting from $n = 0$. Calculate the partial sum S_3 .

$$S_3 = \square$$

Ex 64: Consider the sequence (u_n) defined by the explicit formula $u_n = n^2$, starting from $n = 0$. Calculate the partial sum S_3 .

$$S_3 = \square$$

F.3 EVALUATING SUMS IN SIGMA NOTATION

Ex 65: Calculate the sum:

$$\sum_{i=1}^7 i = \square$$

Ex 66: Calculate the sum:

$$\sum_{k=0}^3 k^2 = \square$$

Ex 67: Calculate the sum:

$$\sum_{k=1}^3 \frac{1}{k} = \square$$

Ex 68: Calculate the sum:

$$\sum_{i=0}^2 4 \left(\frac{3}{2}\right)^i = \square$$

Ex 69: Calculate the sum:

$$\sum_{i=0}^3 (2i - 1) = \square$$

G SUM OF AN ARITHMETIC SEQUENCE

G.1 CALCULATING THE SUM OF AN ARITHMETIC SERIES: LEVEL 1

Ex 70: Calculate the sum of the first 7 positive integers.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = \square$$

Ex 71: Calculate the sum of the first 7 positive even integers.

$$2 + 4 + 6 + 8 + 10 + 12 + 14 = \square$$


Ex 72: Calculate the sum of the following arithmetic sequence.

$$11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 = \square$$


Ex 73: Calculate the sum of the following arithmetic sequence.

$$60 + 55 + 50 + 45 + 40 + 35 + 30 + 25 + 20 = \square$$


G.2 CALCULATING THE SUM OF AN ARITHMETIC SERIES: LEVEL 2

Ex 74:  Calculate the sum of the first 100 positive integers.


$$1 + 2 + 3 + \dots + 100 = \square$$

Ex 75:  Calculate the sum of the arithmetic sequence:


$$3 + 6 + 9 + 12 + \dots + 252 = \square$$

Ex 76:  Calculate the sum of the arithmetic sequence:

$$100 + 90 + 80 + \dots + 10 = \square$$

Ex 77:  Calculate the sum of the arithmetic sequence:

$$5 + 7 + 9 + \dots + 43 = \square$$

Ex 78:  Calculate the sum of the arithmetic sequence:

$$(-8) + (-4) + 0 + 4 + \dots + 40 = \square$$

G.3 CALCULATING THE SUM OF AN ARITHMETIC SERIES IN SIGMA NOTATION: LEVEL 1

Ex 79: Calculate the sum:

$$\sum_{i=1}^7 i = \square$$

Ex 80: Calculate the sum:

$$\sum_{i=1}^6 (2i + 1) = \square$$

Ex 81: Calculate the sum:

$$\sum_{i=2}^4 3i = \square$$


Ex 82: Calculate the sum:

$$\sum_{i=1}^5 (12 - 2i) = \square$$


G.4 CALCULATING THE SUM OF AN ARITHMETIC SERIES IN SIGMA NOTATION: LEVEL 2

Ex 83: Calculate the sum:

$$\sum_{i=1}^{84} 3i = \square$$

Ex 84:  Calculate the sum:

$$\sum_{i=0}^{20} (100 - 5i) = \square$$

Ex 85:  Calculate the sum:

$$\sum_{i=3}^{24} (5i + 2) = \square$$

G.5 PROVING ARITHMETIC SERIES FORMULAS

Ex 86: Use the formula for the sum of an arithmetic sequence to prove that the sum of the first n positive integers is:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Ex 87: Use the formula for the sum of an arithmetic sequence to prove that the sum of the first n positive even integers is:

$$\sum_{i=1}^n 2i = n(n+1)$$

Ex 88: Let (u_n) be an arithmetic sequence with initial term u_0 and common difference d .

Use the formula for the sum of an arithmetic sequence to prove that:

$$\sum_{i=0}^n u_i = \frac{(n+1)(2u_0 + nd)}{2}$$

H SUM OF A GEOMETRIC SEQUENCE

H.1 CALCULATING THE SUM OF A GEOMETRIC SERIES: LEVEL 1

Ex 89: Calculate the sum of the following geometric sequence.

$$1 + 2 + 4 + 8 + 16 + 32 = \square$$


Ex 90: Calculate the sum of the following geometric sequence.

$$S = 6 + 12 + 24 + 48 + 96 + 192 = \square$$


Ex 91: Calculate the sum of the following geometric sequence.

$$S = 32 + 16 + 8 + 4 + 2 = \square$$


H.2 CALCULATING THE SUM OF A GEOMETRIC SERIES: LEVEL 2

Ex 92:  Calculate the sum of the geometric sequence:


$$1 + 2 + 4 + 8 + \dots + 2048 = \square$$

Ex 93:  Calculate the sum of the geometric sequence:

$$5 + 15 + 45 + \dots + 3645 = \square$$

Ex 94:  Calculate the sum of the geometric sequence:

$$100 + 50 + 25 + \dots + 3.125 = \square$$

Ex 95:  Calculate the sum of the geometric sequence:

$$10 + 20 + 40 + \dots + 1280 = \square$$

H.3 CALCULATING THE SUM OF A GEOMETRIC SERIES IN SIGMA NOTATION: LEVEL 1

Ex 96: Calculate the sum:

$$\sum_{i=0}^5 2^i = \square$$


Ex 97: Calculate the sum:

$$\sum_{i=1}^4 3 \times (-2)^{i-1} = \square$$


Ex 98: Calculate the sum:

$$\sum_{i=0}^3 32 \left(\frac{1}{2}\right)^i = \square$$

H.4 CALCULATING THE SUM OF A GEOMETRIC SERIES IN SIGMA NOTATION: LEVEL 2

Ex 99:  Calculate the sum:

$$\sum_{i=1}^{15} 3 \times 2^{i-1} = \square$$

Ex 100:  Calculate the sum (round to two decimal places):

$$\sum_{i=0}^{10} 100 \times (0.8)^i \approx \square$$

Ex 101:  Calculate the sum:

$$\sum_{i=1}^{12} 4 \times (-2)^{i-1} = \square$$

I SUM OF AN INFINITE GEOMETRIC SERIES

I.1 DETERMINING CONVERGENCE OF GEOMETRIC SERIES

MCQ 102: Does the following infinite geometric series converge or diverge?

$$S = 2 + 6 + 18 + 54 + \dots$$

- ☐ Converges
☐ Diverges

MCQ 103: Does the following infinite geometric series converge or diverge?

$$S = 100 - 50 + 25 - 12.5 + \dots$$

- ☐ Converges
☐ Diverges

MCQ 104: Does the following infinite geometric series converge or diverge?

$$S = 1 - 2 + 4 - 8 + \dots$$

- ☐ Converges
☐ Diverges

MCQ 105: Does the following infinite geometric series converge or diverge?

$$S = 7 + 7 + 7 + 7 + \dots$$

- ☐ Converges
☐ Diverges


I.2 CALCULATING THE SUM OF AN INFINITE GEOMETRIC SERIES

Ex 106: Find the sum:

$$16 + 8 + 4 + 2 + \dots = \square$$

Ex 107: Find the sum of the following infinite geometric series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \square$$

Ex 108:  Find the sum of the following infinite geometric series:

$$27 - 9 + 3 - 1 + \frac{1}{3} - \dots = \square$$