SEQUENCES

A NUMERICAL SEQUENCE

A.1 FINDING TERM

Ex 1:

n	1	2	3	4	5	6
n^{th} **term**	**3**	**5**	**7**	**9**	**11**	**13**

What is the 4^{th} term of this sequence?

$$\boxed{9}$$

Answer: For $n = 4$, the 4^{th} **term** is **9**.

Ex 2:

n	1	2	3	4	5	6
n^{th} **term**	**2**	**6**	**12**	**20**	**30**	**42**

What is the 5^{th} term of this sequence?

$$\boxed{30}$$

Answer: For $n = 5$, the 5^{th} **term** is **30**.

Ex 3:

n	1	2	3	4	5	6	7	8
n^{th} **term**	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**

What is the 7^{th} term of this sequence?

$$\boxed{64}$$

Answer: For $n = 7$, the 7^{th} **term** is **64**.

Ex 4:

n	1	2	3	4	5	6	7	8
n^{th} **term**	**1**	**3**	**7**	**15**	**31**	**63**	**127**	**255**

What is the 8^{th} term of this sequence?

$$\boxed{255}$$

Answer: For $n = 8$, the 8^{th} **term** is **255**.

B RECURSIVE DEFINITION

B.1 CALCULATING THE FIRST TERMS

Ex 5: Find the first five terms in the sequence: start at 7 and **add** 4 each time.

n	1	2	3	4	5
n^{th} **term**	7	11	15	19	23

Answer:

$$7 \xrightarrow{+4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4} 23$$

The first five terms are: 7, 11, 15, 19, 23.

Ex 6: Find the first five terms in the sequence: start at 100 and **subtract** 15 each time.

n	1	2	3	4	5
n^{th} **term**	100	85	70	55	40

Answer:

$$100 \xrightarrow{-15} 85 \xrightarrow{-15} 70 \xrightarrow{-15} 55 \xrightarrow{-15} 40$$

The first five terms are: 100, 85, 70, 55, 40.

Ex 7: Find the first five terms in the sequence: start at 2 and **multiply** by 2 each time.

n	1	2	3	4	5
n^{th} **term**	2	4	8	16	32

Answer:

$$2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} 32$$

The first five terms are: 2, 4, 8, 16, 32.

Ex 8: Find the first five terms in the sequence: start at 81 and **divide** by 3 each time.

n	1	2	3	4	5
n^{th} **term**	81	27	9	3	1

Answer:

$$81 \xrightarrow{\div 3} 27 \xrightarrow{\div 3} 9 \xrightarrow{\div 3} 3 \xrightarrow{\div 3} 1$$

The first five terms are: 81, 27, 9, 3, 1.

B.2 IDENTIFYING THE RECURSIVE RULE

Ex 9:

n	1	2	3	4	5	6	...
n^{th} **term**	**3**	**5**	**7**	**9**	**11**	**13**	...

- The sequence starts at $\boxed{3}$.
- The rule is $\boxed{\textbf{Add}}$ $\boxed{2}$

Answer:

- The sequence starts at 3 (the first term).
- The rule is add 2:

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11 \xrightarrow{+2} 13$$

Ex 10:

n	1	2	3	4	5	6	...
n^{th} **term**	**60**	**55**	**50**	**45**	**40**	**35**	...

- The sequence starts at $\boxed{60}$.
- The rule is $\boxed{\textbf{Subtract}}$ $\boxed{5}$

Answer:

- The sequence starts at 60 (the first term).
- The rule is subtract 5:

$$60 \xrightarrow{-5} 55 \xrightarrow{-5} 50 \xrightarrow{-5} 45 \xrightarrow{-5} 40 \xrightarrow{-5} 35$$

Ex 11:

n	1	2	3	4	5	6	...
n^{th} **term**	**64**	**32**	**16**	**8**	**4**	**2**	...

- The sequence starts at $\boxed{64}$.
- The rule is $\boxed{\textbf{Divide}}$ $\boxed{2}$

Answer:

- The sequence starts at 64 (the first term).
- The rule is divide by 2:

$$64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2$$

Ex 12:

n	1	2	3	4	5	...
n^{th} **term**	**1**	**10**	**100**	1 000	10 000	...

- The sequence starts at $\boxed{1}$.
- The rule is $\boxed{\textbf{Multiply}}$ $\boxed{10}$

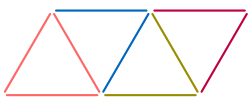
Answer:

- The sequence starts at 1 (the first term).
- The rule is multiply by 10:

$$1 \xrightarrow{\times 10} 10 \xrightarrow{\times 10} 100 \xrightarrow{\times 10} 1\,000 \xrightarrow{\times 10} 10\,000$$

B.3 IDENTIFYING THE RECURSIVE RULE IN GEOMETRIC PATTERNS

Ex 13: Observe the following pattern made with sticks:



Fill in the table below:

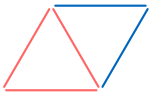
Number of triangles	1	2	3	4
Number of sticks	$\boxed{3}$	$\boxed{5}$	$\boxed{7}$	$\boxed{9}$

What rule can you find for the number of sticks?
Start with $\boxed{3}$ sticks. Add $\boxed{2}$ sticks for each new triangle.

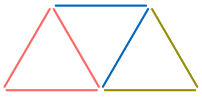
Answer:

- For 1 triangle, the number of sticks is 3.

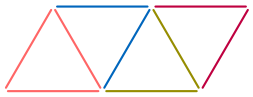
- For 2 triangles, the number of sticks is 5.



- For 3 triangles, the number of sticks is 7.



- For 4 triangles, the number of sticks is 9.



- **Rule:** Start with 3 sticks, and add 2 sticks for each additional triangle.

Ex 14: Observe the following pattern made with sticks:

Fill in the table below:

Number of squares	1	2	3	4
Number of sticks	$\boxed{4}$	$\boxed{7}$	$\boxed{10}$	$\boxed{13}$

What rule can you find for the number of sticks?
Start with $\boxed{4}$ sticks. Add $\boxed{3}$ sticks for each new square.

Answer:

- For 1 square, the number of sticks is 4.

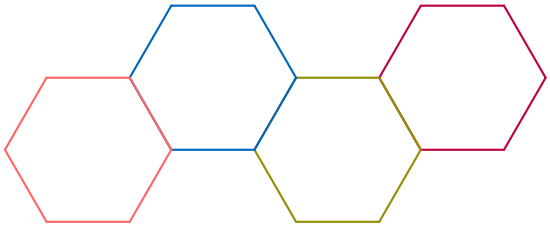
- For 2 squares, the number of sticks is 7.

- For 3 squares, the number of sticks is 10.

- For 4 squares, the number of sticks is 13.

- **Rule:** Start with 4 sticks, and add 3 sticks for each additional square.

Ex 15: Observe the following pattern made with sticks:



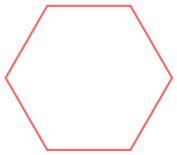
Fill in the table below:

Number of hexagons	1	2	3	4
Number of sticks	6	11	16	21

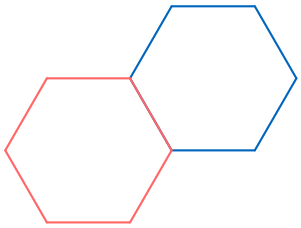
What rule can you find for the number of sticks?
Start with 6 sticks. Add 5 sticks for each new hexagon.

Answer:

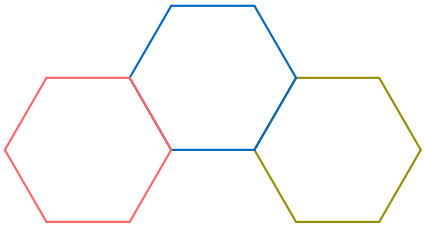
- For 1 hexagon, the number of sticks is 6.



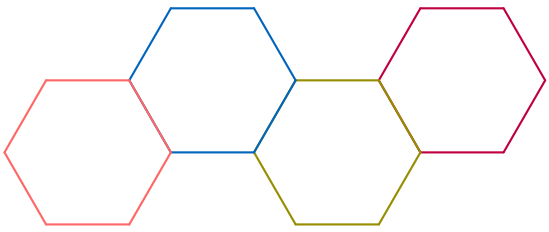
- For 2 hexagons, the number of sticks is 11.



- For 3 hexagons, the number of sticks is 16.



- For 4 hexagons, the number of sticks is 21.



- **Rule:** Start with 6 sticks, and add 5 sticks for each additional hexagon.

Ex 16: Observe the following pattern made with sticks:

Fill in the table below:

Diagram number	1	2	3	4
Number of sticks	4	6	8	10

What rule can you find for the number of sticks?
Start with 4 sticks. Add 2 sticks for the next diagram.

Answer:

- For diagram number 1, the number of sticks is 4.

- For diagram number 2, the number of sticks is 6.

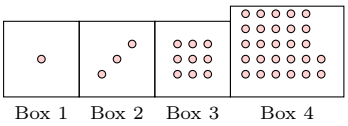
- For diagram number 3, the number of sticks is 8.

- For diagram number 4, the number of sticks is 10.

- **Rule:** Start with 4 sticks, and add 2 sticks for each new diagram.

B.4 IDENTIFYING THE RECURSIVE RULE IN DOT PATTERNS

Ex 17: Observe the following pattern made with dots:

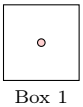


What rule can you find for the number of dots?
Start with 1 dot. Multiply by 3 the number of dots for each new box.
Fill in the table below:

Box	1	2	3	4
Number of dots	1	3	9	27

Answer:

- For 1 box, the number of dots is 1.

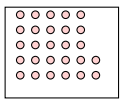


- For 2 boxes, the number of dots is 3.

- For 3 boxes, the number of dots is 9.

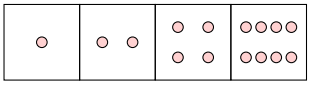

Box 3

- For 4 boxes, the number of dots is 27.


Box 4

- **Rule:** Start with 1 dot, and multiply by 3 the number of dots for each additional box.
  General formula: If n is the number of boxes, then the number of dots is 3^{n-1}.

Ex 18: Observe the following pattern made with dots:


Box 1 Box 2 Box 3 Box 4

What rule can you find for the number of dots?
Start with $\boxed{1}$ dot. Multiply by $\boxed{2}$ the number of dots for each new box.
Fill in the table below:

Box	1	2	3	4
Number of dots	1	2	4	8

Answer:

- For 1 box, the number of dots is 1.


Box 1

- For 2 boxes, the number of dots is 2.


Box 2

- For 3 boxes, the number of dots is 4.

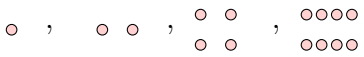

Box 3

- For 4 boxes, the number of dots is 8.


Box 4

- **Rule:** Start with 1 dot, and multiply by 2 the number of dots for each additional box.
  General formula: If n is the number of boxes, then the number of dots is 2^{n-1}.

Ex 19: Observe the following dot pattern:



Fill in the table below:

Diagram number	1	2	3	4
Number of dots	1	2	4	8

What rule can you find for the number of dots?
Start with $\boxed{1}$ dot. Multiply by $\boxed{2}$ the number of dots for each new diagram.

Answer:

- For diagram number 1, the number of dots is 1.

- For diagram number 2, the number of dots is 2.

- For diagram number 3, the number of dots is 4.

- For diagram number 4, the number of dots is 8.

- **Rule:** Start with 1 dot, and multiply by 2 the number of dots for the next diagram.

C ARITHMETIC SEQUENCE

C.1 FINDING NEXT TERM IN ARITHMETIC SEQUENCE

Ex 20: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	3	5	7	9	11	13

Answer: The 6^{th} term is 13, because each term increases by 2.

1	2	3	4	5	6
3	5	7	9	11	13
	+2	+2	+2	+2	+2

Ex 21: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	3	8	13	18	23	28

Answer: The 6^{th} term is 28, because each term increases by 5.

1	2	3	4	5	6
3	8	13	18	23	28
	+5	+5	+5	+5	+5

Ex 22: What is the 5^{th} term of this sequence?

n	1	2	3	4	5
n^{th} term	20	18	16	14	12

Answer: The 5^{th} term is 12, because each term decreases by 2.

1	2	3	4	5
20	18	16	14	12
	−2	−2	−2	−2

Ex 23: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	80	70	60	50	40	30

Answer: The 6^{th} term is 30, because each term decreases by 10.

1	2	3	4	5	6
80	70	60	50	40	30
	−10	−10	−10	−10	−10

D GEOMETRIC SEQUENCE

D.1 FINDING NEXT TERM IN GEOMETRIC SEQUENCE

Ex 24: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	2	4	8	16	32	64

Answer: The 6^{th} term is 64, because each term is multiplied by 2.

1	2	3	4	5	6
2	4	8	16	32	64
	×2	×2	×2	×2	×2

Ex 25: What is the 5^{th} term of this sequence?

n	1	2	3	4	5
n^{th} term	1	3	9	27	81

Answer: The 5^{th} term is 81, because each term is multiplied by 3.

1	2	3	4	5
1	3	9	27	81
	×3	×3	×3	×3

Ex 26: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	64	32	16	8	4	2

Answer: The 6^{th} term is 2, because each term is divided by 2.

1	2	3	4	5	6
64	32	16	8	4	2
	÷2	÷2	÷2	÷2	÷2

Ex 27: What is the 5^{th} term of this sequence?

n	1	2	3	4	5
n^{th} term	243	81	27	9	3

Answer: The 5^{th} term is 3, because each term is divided by 3.

1	2	3	4	5
243	81	27	9	3
	÷3	÷3	÷3	÷3

Ex 28: What is the 6^{th} term of this sequence?

n	1	2	3	4	5	6
n^{th} term	3	6	12	24	48	96

Answer: The 6^{th} term is 96, because each term is multiplied by 2.

1	2	3	4	5	6
3	6	12	24	48	96
	×2	×2	×2	×2	×2