SQUARE ROOTS

A SQUARE ROOTS

A.1 CALCULATING SQUARE ROOTS OF PERFECT SQUARES

Ex 1: Calculate:

$$\sqrt{4} = \boxed{2}$$

Answer: Since $2 \times 2 = 4$, we have $\sqrt{4} = 2$.

Ex 2: Without using a calculator, calculate:

$$\sqrt{36} = \boxed{6}$$

Answer: Since $6 \times 6 = 36$, we have $\sqrt{36} = 6$.

Ex 3: Calculate:

$$\sqrt{64} = \boxed{8}$$

Answer: Since $8 \times 8 = 64$, we have $\sqrt{64} = 8$.

Ex 4: Calculate:

$$\sqrt{49} = \boxed{7}$$

Answer: Since $7 \times 7 = 49$, we have $\sqrt{49} = 7$.

Ex 5: Calculate:

$$\sqrt{100} = \boxed{10}$$

Answer: Since $10 \times 10 = 100$, we have $\sqrt{100} = 10$.

Ex 6: Calculate:

$$\sqrt{81} = \boxed{9}$$

Answer: Since $9 \times 9 = 81$, we have $\sqrt{81} = 9$.

Ex 7: Calculate:

$$\sqrt{0} = \boxed{0}$$

Answer: Since $0 \times 0 = 0$, we have $\sqrt{0} = 0$.

A.2 CALCULATING SQUARE ROOTS OF FRACTIONS

Ex 8: Write in fraction form:

$$\sqrt{\frac{1}{4}} = \boxed{\frac{1}{2}}$$

Answer: Since $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, we have $\sqrt{\frac{1}{4}} = \frac{1}{2}$.

Ex 9: Write in fraction form:

$$\sqrt{\frac{1}{25}} = \boxed{\frac{1}{5}}$$

Answer: Since $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$, we have $\sqrt{\frac{1}{25}} = \frac{1}{5}$.

Ex 10: Write in fraction form:

$$\sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$$

Answer: Since $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, we have $\sqrt{\frac{1}{9}} = \frac{1}{3}$.

Ex 11: Write in fraction form:

$$\sqrt{\frac{1}{16}} = \boxed{\frac{1}{4}}$$

Answer: Since $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$, we have $\sqrt{\frac{1}{16}} = \frac{1}{4}$.

Ex 12: Write in fraction form:

$$\sqrt{\frac{9}{16}} = \boxed{\frac{3}{4}}$$

Answer: Since $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, we have $\sqrt{\frac{9}{16}} = \frac{3}{4}$.

Ex 13: Write in fraction form:

$$\sqrt{\frac{4}{9}} = \boxed{\frac{2}{3}}$$

Answer: Since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, we have $\sqrt{\frac{4}{9}} = \frac{2}{3}$.

B CALCULATING SQUARE ROOTS

B.1 USING A CALCULATOR

Ex 14: Using a calculator, evaluate $\sqrt{2}$ (round to 2 decimal places).

$$\sqrt{2} \approx \boxed{1.41}$$

Answer: By entering $\sqrt{2}$ and pressing the equal button, the calculator displays: $1.41421356237\ldots$
So $\sqrt{2} \approx 1.41$ (rounded to two decimal places).

Ex 15: Using a calculator, evaluate $\sqrt{10}$ (round to 2 decimal places).

$$\sqrt{10} \approx \boxed{3.16}$$

Answer: By entering $\sqrt{10}$ and pressing the equal button, the calculator displays: $3.16227766017\ldots$
So $\sqrt{10} \approx 3.16$ (rounded to two decimal places).

Ex 16: Using a calculator, evaluate $\sqrt{50}$ (round to 2 decimal places).

$$\sqrt{50} \approx \boxed{7.07}$$

Answer: By entering $\sqrt{50}$ and pressing the equal button, the calculator displays: $7.07106781187\ldots$
So $\sqrt{50} \approx 7.07$ (rounded to two decimal places).

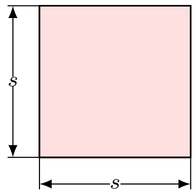
Ex 17: Using a calculator, evaluate $\sqrt{0.5}$ (round to 2 decimal places).

$$\sqrt{0.5} \approx \boxed{0.71}$$

Answer: By entering $\sqrt{0.5}$ and pressing the equal button, the calculator displays: $0.70710678118\ldots$
So $\sqrt{0.5} \approx 0.71$ (rounded to two decimal places).

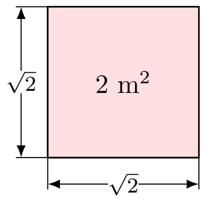
B.2 FINDING THE SIDE LENGTH OF A SQUARE

Ex 18: The area of a square is 2 m^2. What is the length of the side of the square, s?



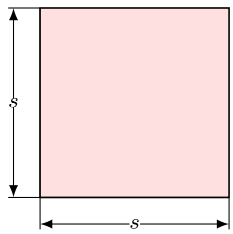
$s \approx \boxed{1.41}$ m (round your answer to 2 decimal places)

Answer: The area of a square is s^2, so $s^2 = 2$.
Therefore, $s = \sqrt{2} \approx 1.41$ m.



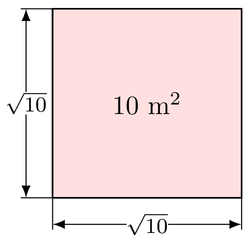
$$\sqrt{2} \times \sqrt{2} = 2$$

Ex 19: The area of a square is 10 m^2. What is the length of the side of the square, s?



$s \approx \boxed{3.16}$ m (round your answer to 2 decimal places)

Answer: The area of a square is s^2, so $s^2 = 10$.
Therefore, $s = \sqrt{10} \approx 3.16$ m (rounded to two decimal places).



$$\sqrt{10} \times \sqrt{10} = 10$$

C NTH ROOTS

C.1 CALCULATING CUBE ROOTS OF PERFECT CUBES

Ex 20: Calculate:

$$\sqrt[3]{8} = \boxed{2}$$

Answer: Since $2 \times 2 \times 2 = 8$, we have $\sqrt[3]{8} = 2$.

Ex 21: Without using a calculator, calculate:

$$\sqrt[3]{27} = \boxed{3}$$

Answer: Since $3 \times 3 \times 3 = 27$, we have $\sqrt[3]{27} = 3$.

Ex 22: Calculate:

$$\sqrt[3]{64} = \boxed{4}$$

Answer: Since $4 \times 4 \times 4 = 64$, we have $\sqrt[3]{64} = 4$.

Ex 23: Calculate:

$$\sqrt[3]{125} = \boxed{5}$$

Answer: Since $5 \times 5 \times 5 = 125$, we have $\sqrt[3]{125} = 5$.

Ex 24: Calculate:

$$\sqrt[3]{1\,000} = \boxed{10}$$

Answer: Since $10 \times 10 \times 10 = 1\,000$, we have $\sqrt[3]{1\,000} = 10$.

Ex 25: Calculate:

$$\sqrt[3]{0} = \boxed{0}$$

Answer: Since $0 \times 0 \times 0 = 0$, we have $\sqrt[3]{0} = 0$.

D LAWS OF RADICALS

D.1 WRITING AS A SINGLE ROOT: LEVEL 1

Ex 26: Write as a single square root:

$$\sqrt{3}\sqrt{4} = \boxed{\sqrt{12}}$$

Answer:

$$\begin{aligned}\sqrt{3}\sqrt{4} &= \sqrt{3 \times 4} \\ &= \sqrt{12}\end{aligned}$$

Ex 27: Write as a single square root:

$$\sqrt{5}\sqrt{20} = \boxed{\sqrt{100}}$$

Answer:

$$\begin{aligned}\sqrt{5}\sqrt{20} &= \sqrt{5 \times 20} \\ &= \sqrt{100}\end{aligned}$$

Ex 28: Write as a single square root:

$$\sqrt{6}\sqrt{6} = \boxed{\sqrt{36}}$$

Answer:

$$\begin{aligned}\sqrt{6}\sqrt{6} &= \sqrt{6 \times 6} \\ &= \sqrt{36}\end{aligned}$$

Ex 29: Write as a single square root:

$$\sqrt{9}\sqrt{4} = \boxed{\sqrt{36}}$$

Answer:

$$\begin{aligned}\sqrt{9}\sqrt{4} &= \sqrt{9\times 4}\\ &= \sqrt{36}\end{aligned}$$

Ex 30: Write as a single square root:

$$\sqrt{2}\sqrt{8} = \boxed{\sqrt{16}}$$

Answer:

$$\begin{aligned}\sqrt{2}\sqrt{8} &= \sqrt{2\times 8}\\ &= \sqrt{16}\end{aligned}$$

D.2 WRITING AS A SINGLE ROOT: LEVEL 2

Ex 31: Write as a single square root:

$$\sqrt{2}\sqrt{3}\sqrt{5} = \boxed{\sqrt{30}}$$

Answer:

$$\begin{aligned}\sqrt{2}\sqrt{3}\sqrt{5} &= \sqrt{2\times 3\times 5}\\ &= \sqrt{30}\end{aligned}$$

Ex 32: Write as a single square root:

$$\sqrt{5}\sqrt{2}\sqrt{10} = \boxed{\sqrt{100}}$$

Answer:

$$\begin{aligned}\sqrt{5}\sqrt{2}\sqrt{10} &= \sqrt{5\times 2\times 10}\\ &= \sqrt{100}\end{aligned}$$

Ex 33: Write as a single square root:

$$\left(\sqrt{3}\right)^3 = \boxed{\sqrt{27}}$$

Answer:

$$\begin{aligned}\left(\sqrt{3}\right)^3 &= \sqrt{3}\sqrt{3}\sqrt{3}\\ &= \sqrt{3\times 3\times 3}\\ &= \sqrt{27}\end{aligned}$$

Ex 34: Write as a single square root:

$$\left(\sqrt{2}\right)^3\sqrt{3} = \boxed{\sqrt{24}}$$

Answer:

$$\begin{aligned}\left(\sqrt{2}\right)^3\sqrt{3} &= \sqrt{2}\sqrt{2}\sqrt{2}\sqrt{3}\\ &= \sqrt{2\times 2\times 2\times 3}\\ &= \sqrt{24}\end{aligned}$$

D.3 UNDERSTANDING SQUARE ROOT OPERATIONS

MCQ 35: Is $\sqrt{2}+\sqrt{3}=\sqrt{2+3}$?

☐ True

☒ False

Answer: **The statement is incorrect.**
The product of square roots can be combined into a single root by multiplying inside:

$$\sqrt{2}\sqrt{3} = \sqrt{2\times 3} = \sqrt{6}$$

But the sum of square roots **cannot** be combined in this way:

$$\sqrt{2}+\sqrt{3} \neq \sqrt{2+3}$$

because $\sqrt{2}+\sqrt{3} \approx 1.41+1.73 = 3.14$ and $\sqrt{5}\approx 2.24$.

MCQ 36: Is $\sqrt{2}\sqrt{3}=\sqrt{6}$?

☒ True

☐ False

Answer: **The statement is correct.**
The product of square roots can be written as a single root:

$$\sqrt{2}\sqrt{3} = \sqrt{2\times 3} = \sqrt{6}$$

This is a property of square roots: $\sqrt{a}\sqrt{b}=\sqrt{ab}$.

MCQ 37: Is $\sqrt{3}+\sqrt{3}=\sqrt{3+3}$?

☐ True

☒ False

Answer: **The statement is incorrect.**
The sum of square roots **cannot** be simplified by adding inside:

$$\sqrt{3}+\sqrt{3} = 2\sqrt{3}$$

but

$$\sqrt{3+3} = \sqrt{6}$$

and $2\sqrt{3}\approx 3.46$ while $\sqrt{6}\approx 2.45$, so they are not equal.
The product of roots, however, can be written as a single root:

$$\sqrt{3}\sqrt{3} = \sqrt{3\times 3} = \sqrt{9} = 3$$

MCQ 38: Is $\sqrt{3}+\sqrt{3}=3$?

☐ True

☒ False

Answer: **The statement is incorrect.**
$\sqrt{3}+\sqrt{3}$ means you add two times the value of $\sqrt{3}$:

$$\sqrt{3}+\sqrt{3} = 2\sqrt{3} \approx 2\times 1.73 = 3.46$$

So $\sqrt{3}+\sqrt{3}\neq 3$.

D.4 SIMPLIFYING THE SQUARE ROOT OF A PERFECT SQUARE: LEVEL 1

Ex 39: Simplify:

$$\sqrt{4} = \boxed{2}$$

Answer:

$$\begin{aligned}\sqrt{4} &= \sqrt{2 \times 2} \\ &= 2\end{aligned}$$

Ex 40: Simplify:

$$\sqrt{36} = \boxed{6}$$

Answer:

$$\begin{aligned}\sqrt{36} &= \sqrt{6 \times 6} \\ &= 6\end{aligned}$$

Ex 41: Simplify:

$$\sqrt{10^2} = \boxed{10}$$

Answer:

$$\begin{aligned}\sqrt{10^2} &= \sqrt{10 \times 10} \\ &= 10\end{aligned}$$

Ex 42: Simplify:

$$\text{For } x \geq 0, \sqrt{x^2} = \boxed{x}$$

Answer: Let $x \geq 0$.

$$\begin{aligned}\sqrt{x^2} &= \sqrt{x \times x} \\ &= x\end{aligned}$$

Ex 43: Simplify:

$$\text{For } x \geq 0, \sqrt{(2x)^2} = \boxed{2x}$$

Answer: Let $x \geq 0$.

$$\begin{aligned}\sqrt{(2x)^2} &= \sqrt{(2x) \times (2x)} \\ &= 2x\end{aligned}$$

D.5 SIMPLIFYING THE SQUARE ROOT OF A PERFECT SQUARE: LEVEL 2

Ex 44: Simplify:

$$\text{For } x \geq 0, \sqrt{9x^2} = \boxed{3x}$$

Answer: Let $x \geq 0$.

$$\begin{aligned}\sqrt{9x^2} &= \sqrt{(3x) \times (3x)} \\ &= 3x\end{aligned}$$

Ex 45: Simplify:

$$\sqrt{x^4} = \boxed{x^2}$$

Answer:

$$\begin{aligned}\sqrt{x^4} &= \sqrt{x^2 \times x^2} \\ &= x^2\end{aligned}$$

Ex 46: Simplify:

$$\text{For } x \geq 0, \sqrt{4x^2} + x = \boxed{3x}$$

Answer: Let $x \geq 0$.

$$\begin{aligned}\sqrt{4x^2} + x &= \sqrt{(2x) \times (2x)} + x \\ &= 2x + x \\ &= 3x\end{aligned}$$

Ex 47: Simplify:

$$\sqrt{12}\sqrt{3} = \boxed{6}$$

Answer:

$$\begin{aligned}\sqrt{12}\sqrt{3} &= \sqrt{12 \times 3} \\ &= \sqrt{2 \times 2 \times 3 \times 3} \\ &= 2 \times 3 \\ &= 6\end{aligned}$$

D.6 SIMPLIFYING SQUARE ROOTS

Ex 48: Simplify:

$$\sqrt{18} = \boxed{3\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{18} &= \sqrt{3 \times 3 \times 2} \\ &= 3\sqrt{2}\end{aligned}$$

Ex 49: Simplify:

$$\sqrt{50} = \boxed{5\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{50} &= \sqrt{5 \times 5 \times 2} \\ &= 5\sqrt{2}\end{aligned}$$

Ex 50: Simplify:

$$\sqrt{32} = \boxed{4\sqrt{2}}$$

Answer:

$$\begin{aligned}\sqrt{32} &= \sqrt{4 \times 4 \times 2} \\ &= 4\sqrt{2}\end{aligned}$$

Ex 51: Simplify:

$$\sqrt{20} = \boxed{2\sqrt{5}}$$

Answer:

$$\begin{aligned}\sqrt{20} &= \sqrt{2 \times 2 \times 5} \\ &= 2\sqrt{5}\end{aligned}$$

D.7 SIMPLIFYING QUOTIENTS OF SQUARE ROOTS

Ex 52: Simplify:

$$\frac{\sqrt{10}}{\sqrt{5}} = \boxed{\sqrt{2}}$$

Answer:
$$\begin{aligned}\frac{\sqrt{10}}{\sqrt{5}} &= \sqrt{\frac{10}{5}}\\ &= \sqrt{2}\end{aligned}$$

Ex 53: Simplify:

$$\frac{\sqrt{75}}{\sqrt{25}} = \boxed{\sqrt{3}}$$

Answer:
$$\begin{aligned}\frac{\sqrt{75}}{\sqrt{25}} &= \sqrt{\frac{75}{25}}\\ &= \sqrt{3}\end{aligned}$$

Ex 54: Simplify:

$$\frac{\sqrt{18}}{\sqrt{3}} = \boxed{\sqrt{6}}$$

Answer:
$$\begin{aligned}\frac{\sqrt{18}}{\sqrt{3}} &= \sqrt{\frac{18}{3}}\\ &= \sqrt{6}\end{aligned}$$

Ex 55: Simplify:

$$\frac{\sqrt{20}}{\sqrt{2}} = \boxed{\sqrt{10}}$$

Answer:
$$\begin{aligned}\frac{\sqrt{20}}{\sqrt{2}} &= \sqrt{\frac{20}{2}}\\ &= \sqrt{10}\end{aligned}$$

E ALGEBRAIC OPERATIONS WITH RADICALS

E.1 ADDING AND SUBTRACTING LIKE RADICALS: LEVEL 1

Ex 56: Simplify:

$$2\sqrt{3} + 5\sqrt{3} = \boxed{7\sqrt{3}}$$

Answer:
$$\begin{aligned}2\sqrt{3} + 5\sqrt{3} &= (2+5)\sqrt{3} \quad \text{(factorisation)}\\ &= 7\sqrt{3}\end{aligned}$$

Ex 57: Simplify:

$$4\sqrt{5} + 7\sqrt{5} = \boxed{11\sqrt{5}}$$

Answer:
$$\begin{aligned}4\sqrt{5} + 7\sqrt{5} &= (4+7)\sqrt{5} \quad \text{(factorisation)}\\ &= 11\sqrt{5}\end{aligned}$$

Ex 58: Simplify:

$$3\sqrt{6} - \sqrt{6} = \boxed{2\sqrt{6}}$$

Answer:
$$\begin{aligned}3\sqrt{6} - \sqrt{6} &= (3-1)\sqrt{6} \quad \text{(factorisation)}\\ &= 2\sqrt{6}\end{aligned}$$

Ex 59: Simplify:

$$3\sqrt[3]{7} + 5\sqrt[3]{7} = \boxed{8\sqrt[3]{7}}$$

Answer:
$$\begin{aligned}3\sqrt[3]{7} + 5\sqrt[3]{7} &= (3+5)\sqrt[3]{7} \quad \text{(factorisation)}\\ &= 8\sqrt[3]{7}\end{aligned}$$

Ex 60: Simplify:

$$2\sqrt{2} - 4\sqrt{2} = \boxed{-2\sqrt{2}}$$

Answer:
$$\begin{aligned}2\sqrt{2} - 4\sqrt{2} &= (2-4)\sqrt{2} \quad \text{(factorisation)}\\ &= -2\sqrt{2}\end{aligned}$$

Ex 61: Simplify:

$$2\sqrt{7} - 5\sqrt{7} = \boxed{-3\sqrt{7}}$$

Answer:
$$\begin{aligned}2\sqrt{7} - 5\sqrt{7} &= (2-5)\sqrt{7} \quad \text{(factorisation)}\\ &= -3\sqrt{7}\end{aligned}$$

E.2 ADDING AND SUBTRACTING LIKE RADICALS: LEVEL 2

Ex 62: Simplify:

$$\sqrt{8} - \sqrt{2} = \boxed{\sqrt{2}}$$

Answer:
$$\begin{aligned}\sqrt{8} - \sqrt{2} &= \sqrt{2 \times 2 \times 2} - \sqrt{2}\\ &= 2\sqrt{2} - \sqrt{2}\\ &= (2-1)\sqrt{2}\\ &= \sqrt{2}\end{aligned}$$

Ex 63: Simplify:

$$\sqrt{12} + 3\sqrt{3} = \boxed{5\sqrt{3}}$$

Answer:
$$\begin{aligned}\sqrt{12} + 3\sqrt{3} &= \sqrt{2 \times 2 \times 3} + 3\sqrt{3}\\ &= 2\sqrt{3} + 3\sqrt{3}\\ &= (2+3)\sqrt{3}\\ &= 5\sqrt{3}\end{aligned}$$

Ex 64: Simplify:

$$5\sqrt{3} - \sqrt{12} = \boxed{3\sqrt{3}}$$

Answer:

$$\begin{aligned} 5\sqrt{3}-\sqrt{12} &= 5\sqrt{3}-\sqrt{2\times 2\times 3} \\ &= 5\sqrt{3}-2\sqrt{3} \\ &= (5-2)\sqrt{3} \\ &= 3\sqrt{3} \end{aligned}$$

Ex 65: Simplify:

$$2\sqrt{7}+3\sqrt{28}=\boxed{8\sqrt{7}}$$

Answer:

$$\begin{aligned} 2\sqrt{7}+3\sqrt{28} &= 2\sqrt{7}+3\times\sqrt{2\times 2\times 7} \\ &= 2\sqrt{7}+3\times 2\sqrt{7} \\ &= 2\sqrt{7}+6\sqrt{7} \\ &= (2+6)\sqrt{7} \\ &= 8\sqrt{7} \end{aligned}$$

F RATIONALIZING THE DENOMINATOR

F.1 WRITING WITH AN INTEGER DENOMINATOR: LEVEL 1

Ex 66: Simplify:

$$\frac{1}{\sqrt{2}}=\boxed{\frac{\sqrt{2}}{2}}$$

Answer:

$$\begin{aligned} \frac{1}{\sqrt{2}} &= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

Ex 67: Simplify:

$$\frac{2}{\sqrt{2}}=\boxed{\sqrt{2}}$$

Answer:

$$\begin{aligned} \frac{2}{\sqrt{2}} &= \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{2\sqrt{2}}{2} \\ &= \sqrt{2} \end{aligned}$$

Ex 68: Simplify:

$$\frac{2}{\sqrt{3}}=\boxed{\frac{2\sqrt{3}}{3}}$$

Answer:

$$\begin{aligned} \frac{2}{\sqrt{3}} &= \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{2\sqrt{3}}{3} \end{aligned}$$

Ex 69: Simplify:

$$\frac{3}{\sqrt{6}}=\boxed{\frac{\sqrt{6}}{2}}$$

Answer:

$$\begin{aligned} \frac{3}{\sqrt{6}} &= \frac{3}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{3\sqrt{6}}{6} \\ &= \frac{\sqrt{6}}{2} \end{aligned}$$

F.2 WRITING WITH AN INTEGER DENOMINATOR: LEVEL 2

Ex 70: Simplify:

$$\frac{3}{2\sqrt{3}}=\boxed{\frac{\sqrt{3}}{2}}$$

Answer:

$$\begin{aligned} \frac{3}{2\sqrt{3}} &= \frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{3\sqrt{3}}{2\times 3} \\ &= \frac{3\sqrt{3}}{6} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Ex 71: Simplify:

$$\frac{5}{\sqrt{8}}=\boxed{\frac{5\sqrt{2}}{4}}$$

Answer:

$$\begin{aligned} \frac{5}{\sqrt{8}} &= \frac{5}{\sqrt{4\times 2}} \\ &= \frac{5}{2\sqrt{2}} \\ &= \frac{5}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{5\sqrt{2}}{4} \end{aligned}$$

Ex 72: Simplify:

$$\frac{4}{3\sqrt{2}}=\boxed{\frac{2\sqrt{2}}{3}}$$

Answer:

$$\begin{aligned} \frac{4}{3\sqrt{2}} &= \frac{4}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{4\sqrt{2}}{3\times 2} \\ &= \frac{4\sqrt{2}}{6} \\ &= \frac{2\sqrt{2}}{3} \end{aligned}$$

Ex 73: Simplify:

$$\frac{6}{\sqrt{12}}=\boxed{\sqrt{3}}$$

Answer:

$$\begin{aligned}\frac{6}{\sqrt{12}} &= \frac{6}{\sqrt{4\times 3}}\\ &= \frac{6}{2\sqrt{3}}\\ &= \frac{3}{\sqrt{3}}\\ &= \frac{3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{3\sqrt{3}}{3}\\ &= \sqrt{3}\end{aligned}$$

F.3 WRITING IN THE FORM $a \pm \sqrt{b}$

Ex 74: Simplify:

$$\frac{1}{1+\sqrt{2}} = \boxed{\sqrt{2}-1}$$

Answer:

$$\begin{aligned}\frac{1}{1+\sqrt{2}} &= \frac{1}{(1+\sqrt{2})}\cdot\frac{(1-\sqrt{2})}{(1-\sqrt{2})}\\ &= \frac{1(1-\sqrt{2})}{\left(1^2-\left(\sqrt{2}\right)^2\right)} && \text{(difference of squares)}\\ &= \frac{1(1-\sqrt{2})}{1-2}\\ &= \frac{1-\sqrt{2}}{-1}\\ &= -(1-\sqrt{2})\\ &= \sqrt{2}-1\end{aligned}$$

Ex 75: Simplify:

$$\frac{1}{\sqrt{2}-1} = \boxed{\sqrt{2}+1}$$

Answer:

$$\begin{aligned}\frac{1}{\sqrt{2}-1} &= \frac{1}{(\sqrt{2}-1)}\cdot\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}\\ &= \frac{1(\sqrt{2}+1)}{((\sqrt{2})^2-(1)^2)} && \text{(difference of squares)}\\ &= \frac{\sqrt{2}+1}{2-1}\\ &= \frac{\sqrt{2}+1}{1}\\ &= \sqrt{2}+1\end{aligned}$$

Ex 76: Simplify:

$$\frac{4}{1-\sqrt{3}} = \boxed{-2-2\sqrt{3}}$$

Answer:

$$\begin{aligned}\frac{4}{1-\sqrt{3}} &= \frac{4}{(1-\sqrt{3})}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})}\\ &= \frac{4(1+\sqrt{3})}{\left(1^2-\left(\sqrt{3}\right)^2\right)} && \text{(difference of squares)}\\ &= \frac{4(1+\sqrt{3})}{1-3}\\ &= \frac{4(1+\sqrt{3})}{-2}\\ &= -2(1+\sqrt{3})\\ &= -2-2\sqrt{3}\end{aligned}$$

Ex 77: Simplify:

$$\frac{5}{\sqrt{3}-2} = \boxed{-5\sqrt{3}-10}$$

Answer:

$$\begin{aligned}\frac{5}{\sqrt{3}-2} &= \frac{5}{(\sqrt{3}-2)}\cdot\frac{(\sqrt{3}+2)}{(\sqrt{3}+2)}\\ &= \frac{5(\sqrt{3}+2)}{((\sqrt{3})^2-(2)^2)} && \text{(difference of squares)}\\ &= \frac{5(\sqrt{3}+2)}{3-4}\\ &= \frac{5(\sqrt{3}+2)}{-1}\\ &= -5(\sqrt{3}+2)\\ &= -5\sqrt{3}-10\end{aligned}$$

F.4 WRITING IN THE FORM $a + b\sqrt{2}$

Ex 78: Simplify:

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} = \boxed{2\sqrt{2}+3}$$

Answer:

$$\begin{aligned}\frac{\sqrt{2}+1}{\sqrt{2}-1} &= \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}\cdot\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}\\ &= \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{((\sqrt{2})^2-(1)^2)} && \text{(difference of squares)}\\ &= \frac{(\sqrt{2}+1)^2}{2-1}\\ &= (\sqrt{2}+1)^2\\ &= 2+2\sqrt{2}+1\\ &= 3+2\sqrt{2}\end{aligned}$$

Ex 79: Simplify:

$$\frac{2}{3-2\sqrt{2}} = \boxed{6+4\sqrt{2}}$$

Answer:

$$\begin{aligned}\frac{2}{3-2\sqrt{2}} &= \frac{2}{(3-2\sqrt{2})}\cdot\frac{(3+2\sqrt{2})}{(3+2\sqrt{2})}\\ &= \frac{2(3+2\sqrt{2})}{\left(3^2-(2\sqrt{2})^2\right)} && \text{(difference of squares)}\\ &= \frac{2(3+2\sqrt{2})}{9-8}\\ &= \frac{2(3+2\sqrt{2})}{1}\\ &= 6+4\sqrt{2}\end{aligned}$$

Ex 80: Simplify:

$$\tfrac{3}{\sqrt{2}-1} = \boxed{3+3\sqrt{2}}$$

Answer:

$$\begin{aligned}\frac{3}{\sqrt{2}-1} &= \frac{3}{(\sqrt{2}-1)}\cdot\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}\\ &= \frac{3(\sqrt{2}+1)}{\left((\sqrt{2})^2-(1)^2\right)} && \text{(difference of squares)}\\ &= \frac{3(\sqrt{2}+1)}{2-1}\\ &= 3(\sqrt{2}+1)\\ &= 3+3\sqrt{2}\end{aligned}$$

Ex 81: Simplify:

$$\tfrac{4+\sqrt{2}}{2-\sqrt{2}} = \boxed{5+3\sqrt{2}}$$

Answer:

$$\begin{aligned}\frac{4+\sqrt{2}}{2-\sqrt{2}} &= \frac{(4+\sqrt{2})}{(2-\sqrt{2})}\cdot\frac{(2+\sqrt{2})}{(2+\sqrt{2})}\\ &= \frac{(4+\sqrt{2})(2+\sqrt{2})}{\left(2^2-(\sqrt{2})^2\right)} && \text{(difference of squares)}\\ &= \frac{8+4\sqrt{2}+2\sqrt{2}+2}{4-2}\\ &= \frac{10+6\sqrt{2}}{2}\\ &= 5+3\sqrt{2}\end{aligned}$$

F.5 RATIONALIZING ALGEBRAIC DENOMINATORS

Ex 82: For $x>0$, rationalize the denominator of the expression:

$$\frac{1}{\sqrt{x}} = \boxed{\frac{\sqrt{x}}{x}}$$

Answer: We multiply the numerator and the denominator by $\sqrt{x}$.

$$\begin{aligned}\frac{1}{\sqrt{x}} &= \frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\\ &= \frac{\sqrt{x}}{x}\end{aligned}$$

Ex 83: For $x>0$, rationalize the denominator of the expression:

$$\frac{x}{\sqrt{x}} = \boxed{\sqrt{x}}$$

Answer: We multiply the numerator and the denominator by $\sqrt{x}$.

$$\begin{aligned}\frac{x}{\sqrt{x}} &= \frac{x}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\\ &= \frac{x\sqrt{x}}{x}\\ &= \sqrt{x}\end{aligned}$$

Alternatively, since $x=\sqrt{x}\times\sqrt{x}$, we can simplify directly:

$$\frac{x}{\sqrt{x}} = \frac{\sqrt{x}\times\sqrt{x}}{\sqrt{x}} = \sqrt{x}$$

Ex 84: For $x>0$, rationalize the denominator of the expression:

$$\frac{5}{\sqrt{2x}} = \boxed{\frac{5\sqrt{2x}}{2x}}$$

Answer: We multiply the numerator and the denominator by $\sqrt{2x}$.

$$\begin{aligned}\frac{5}{\sqrt{2x}} &= \frac{5}{\sqrt{2x}}\cdot\frac{\sqrt{2x}}{\sqrt{2x}}\\ &= \frac{5\sqrt{2x}}{2x}\end{aligned}$$

Ex 85: For $x>0$, simplify the expression by rationalizing the denominator:

$$\frac{x+1}{\sqrt{x}} = \boxed{\frac{(x+1)\sqrt{x}}{x}}$$

Answer: We multiply the numerator and the denominator by $\sqrt{x}$.

$$\begin{aligned}\frac{x+1}{\sqrt{x}} &= \frac{x+1}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\\ &= \frac{(x+1)\sqrt{x}}{x}\end{aligned}$$

This can also be written as $\frac{x\sqrt{x}+\sqrt{x}}{x}$.

F.6 RATIONALIZING BINOMIAL ALGEBRAIC DENOMINATORS

Ex 86: For $x\geq 0$ and $x\neq 1$, rationalize the denominator of the expression:

$$\frac{1}{1+\sqrt{x}} = \boxed{\frac{1-\sqrt{x}}{1-x}}$$

Answer: The denominator is $1+\sqrt{x}$. Its conjugate is $1-\sqrt{x}$. We multiply the numerator and denominator by this conjugate.

$$\begin{aligned}\frac{1}{1+\sqrt{x}} &= \frac{1}{(1+\sqrt{x})}\times\frac{(1-\sqrt{x})}{(1-\sqrt{x})}\\ &= \frac{1(1-\sqrt{x})}{1^2-(\sqrt{x})^2} && \text{Using } (a+b)(a-b)=a^2-b^2\\ &= \frac{1-\sqrt{x}}{1-x}\end{aligned}$$

Ex 87: For $x\geq 0$ and $x\neq 9$, rationalize the denominator of the expression:

$$\frac{x-9}{\sqrt{x}-3} = \boxed{\sqrt{x}+3}$$

Answer: The denominator is $\sqrt{x}-3$. Its conjugate is $\sqrt{x}+3$. We multiply the numerator and denominator by this conjugate.

$$\begin{aligned}\frac{x-9}{\sqrt{x}-3} &= \frac{x-9}{(\sqrt{x}-3)} \times \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)}\\ &= \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x})^2-3^2}\\ &= \frac{(x-9)(\sqrt{x}+3)}{x-9}\\ &= \boldsymbol{\sqrt{x}+3}\end{aligned}$$

Ex 88: For $x \geq 0$ and $x \neq 1$, rationalize the denominator of the expression:

$$\frac{\sqrt{x}}{\sqrt{x}+1} = \boxed{\frac{x-\sqrt{x}}{x-1}}$$

Answer: The denominator is $\sqrt{x}+1$. Its conjugate is $\sqrt{x}-1$.

$$\begin{aligned}\frac{\sqrt{x}}{\sqrt{x}+1} &= \frac{\sqrt{x}}{(\sqrt{x}+1)} \times \frac{(\sqrt{x}-1)}{(\sqrt{x}-1)}\\ &= \frac{\sqrt{x}(\sqrt{x}-1)}{(\sqrt{x})^2-1^2}\\ &= \frac{x-\sqrt{x}}{x-1}\end{aligned}$$

F.7 APPLYING MULTIPLE RADICAL SKILLS

Ex 89: Consider the expression $\sqrt{75}-\sqrt{12}$.

1. Show that $\sqrt{75}-\sqrt{12} = 3\sqrt{3}$.
2. Hence, simplify the expression $\dfrac{18}{\sqrt{75}-\sqrt{12}}$ by rationalizing the denominator.

Answer:

1. First, simplify each radical by finding the largest perfect square factor.

$$\begin{aligned}\sqrt{75}-\sqrt{12} &= \sqrt{25\times 3}-\sqrt{4\times 3}\\ &= \sqrt{25}\sqrt{3}-\sqrt{4}\sqrt{3}\\ &= 5\sqrt{3}-2\sqrt{3}\\ &= (5-2)\sqrt{3} = \boldsymbol{3\sqrt{3}}\end{aligned}$$

2.

$$\begin{aligned}\frac{18}{\sqrt{75}-\sqrt{12}} &= \frac{18}{3\sqrt{3}} && \text{(using question a)}\\ &= \frac{6}{\sqrt{3}}\\ &= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{6\sqrt{3}}{3}\\ &= \boldsymbol{2\sqrt{3}}\end{aligned}$$

Ex 90: Consider the expression $\dfrac{10}{3-\sqrt{7}}$.

1. Write down the conjugate of $3-\sqrt{7}$.
2. Express $\dfrac{10}{3-\sqrt{7}}$ in the form $a+b\sqrt{7}$, where $a, b \in \mathbb{Z}$.

Answer:

1. The conjugate of $3-\sqrt{7}$ is $\boldsymbol{3+\sqrt{7}}$.
2. We rationalize the denominator by multiplying the numerator and denominator by the conjugate.

$$\begin{aligned}\frac{10}{3-\sqrt{7}} &= \frac{10}{(3-\sqrt{7})} \times \frac{(3+\sqrt{7})}{(3+\sqrt{7})}\\ &= \frac{10(3+\sqrt{7})}{3^2-(\sqrt{7})^2}\\ &= \frac{10(3+\sqrt{7})}{9-7}\\ &= \frac{10(3+\sqrt{7})}{2}\\ &= 5(3+\sqrt{7}) = \boldsymbol{15+5\sqrt{7}}\end{aligned}$$

Ex 91: Consider the expression $(2+\sqrt{3})^2$.

1. Expand and simplify $(2+\sqrt{3})^2$ into the form $a+b\sqrt{3}$, where $a, b \in \mathbb{Z}$.
2. Hence, find the value of $(2+\sqrt{3})^4$.

Answer:

1. We expand the expression using the perfect square identity $(x+y)^2 = x^2+2xy+y^2$.

$$\begin{aligned}(2+\sqrt{3})^2 &= (2)^2+2(2)(\sqrt{3})+(\sqrt{3})^2\\ &= 4+4\sqrt{3}+3\\ &= \boldsymbol{7+4\sqrt{3}}\end{aligned}$$

2. We can write $(2+\sqrt{3})^4$ as $\left((2+\sqrt{3})^2\right)^2$. Using the result from part (a):

$$\begin{aligned}(2+\sqrt{3})^4 &= (7+4\sqrt{3})^2\\ &= (7)^2+2(7)(4\sqrt{3})+(4\sqrt{3})^2\\ &= 49+56\sqrt{3}+(16\times 3)\\ &= 49+56\sqrt{3}+48\\ &= \boldsymbol{97+56\sqrt{3}}\end{aligned}$$