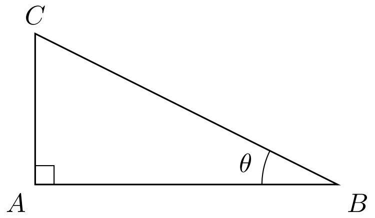
RIGHT-TRIANGLE TRIGONOMETRY

A SIDES OF A RIGHT-ANGLED TRIANGLE

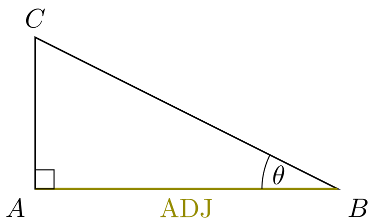
A.1 IDENTIFYING TRIANGLE SIDES

MCQ 1: In the triangle below, identify the adjacent side to the angle θ:



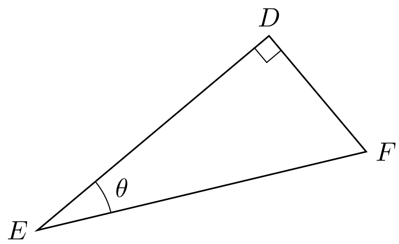
- ☒ $\overline{AB}$
- ☐ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



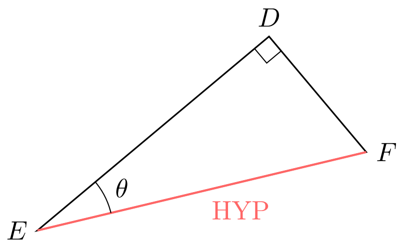
The adjacent side to the angle θ is $\overline{AB}$.

MCQ 2: In the triangle below, identify the hypotenuse relative to the angle θ:



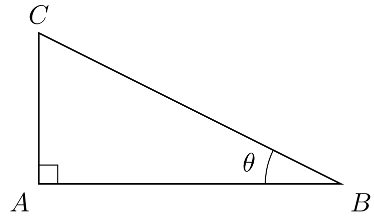
- ☐ $\overline{DE}$
- ☐ $\overline{DF}$
- ☒ $\overline{EF}$

Answer:



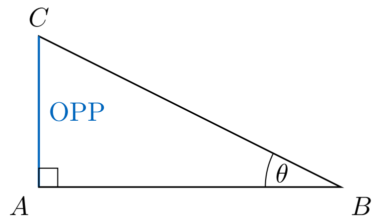
The hypotenuse relative to the angle θ is $\overline{EF}$.

MCQ 3: In the triangle below, identify the opposite side to the angle θ:



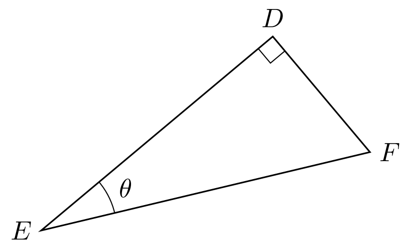
- ☐ $\overline{AB}$
- ☒ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



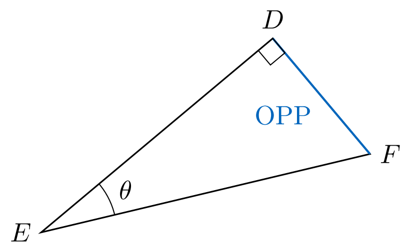
The opposite side to the angle θ is $\overline{AC}$.

MCQ 4: In the triangle below, identify the opposite side to the angle θ:



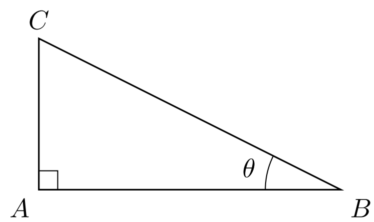
- ☐ $\overline{DE}$
- ☒ $\overline{DF}$
- ☐ $\overline{EF}$

Answer:



The opposite side to the angle θ is $\overline{DF}$.

MCQ 5: In the triangle below, identify the hypotenuse relative to the angle θ:

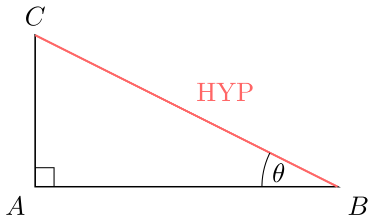


☐ $\overline{AB}$

☐ $\overline{AC}$

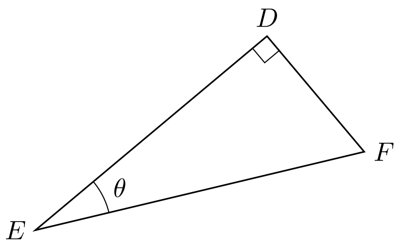
☒ $\overline{BC}$

Answer:



The hypotenuse relative to the angle θ is $\overline{BC}$.

MCQ 6: In the triangle below, identify the adjacent side to the angle θ:

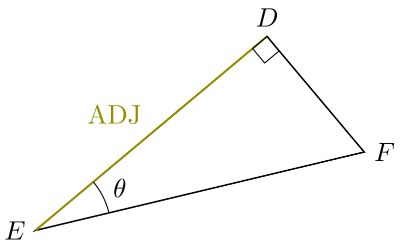


☒ $\overline{DE}$

☐ $\overline{DF}$

☐ $\overline{EF}$

Answer:

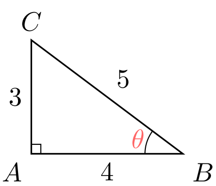


The adjacent side to the angle θ is $\overline{DE}$.

B TRIGONOMETRIC RATIOS

B.1 CALCULATING TRIGONOMETRIC RATIOS

Ex 7:



Calculate $\cos(\theta)$.

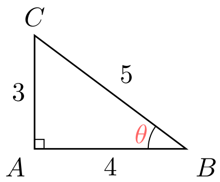
$$\cos(\theta) = \boxed{\frac{4}{5}}$$

Answer: Relative to θ:

- Adjacent side: $AB = 4$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}} \\ &= \frac{4}{5}\end{aligned}$$

Ex 8:



Calculate $\sin(\theta)$.

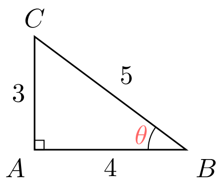
$$\sin(\theta) = \boxed{\frac{3}{5}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{3}{5}\end{aligned}$$

Ex 9:



Calculate $\tan(\theta)$.

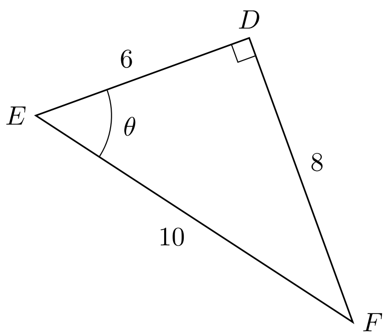
$$\tan(\theta) = \boxed{\frac{3}{4}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Adjacent side: $AB = 4$

$$\begin{aligned}\tan(\theta) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{3}{4}\end{aligned}$$

Ex 10:



Calculate $\sin(\theta)$.

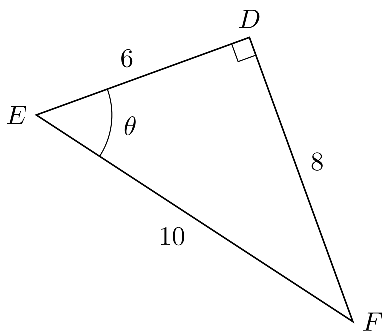
$$\sin(\theta) = \boxed{\frac{4}{5}}$$

Answer: Relative to θ:

- Opposite side: $DF = 8$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}}\\ &= \frac{8}{10}\\ &= \frac{4}{5}\end{aligned}$$

Ex 11:



Calculate $\tan(\theta)$.

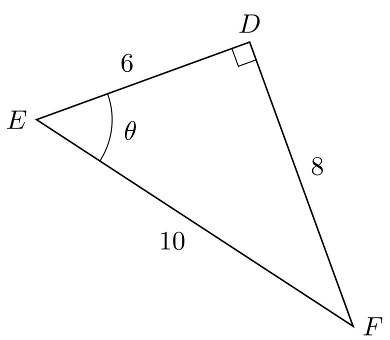
$$\tan(\theta) = \boxed{\frac{4}{3}}$$

Answer: Relative to θ:

- Opposite side: $DF = 8$
- Adjacent side: $DE = 6$

$$\begin{aligned}\tan(\theta) &= \frac{\text{OPP}}{\text{ADJ}}\\ &= \frac{8}{6}\\ &= \frac{4}{3}\end{aligned}$$

Ex 12:



Calculate $\cos(\theta)$.

$$\cos(\theta) = \boxed{\frac{3}{5}}$$

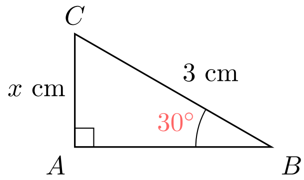
Answer: Relative to θ:

- Adjacent side: $DE = 6$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}}\\ &= \frac{6}{10}\\ &= \frac{3}{5}\end{aligned}$$

B.2 CALCULATING SIDE LENGTHS

Ex 13:



Calculate x.

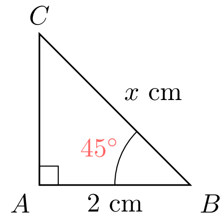
$x \approx$ $\boxed{1.50}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = x$
- Hypotenuse: $BC = 3$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}}\\ \sin(30°) &= \frac{x}{3}\\ x &= 3 \times \sin(30°)\\ x &= 1.50\,\text{cm}\end{aligned}$$

Ex 14:



Calculate x.

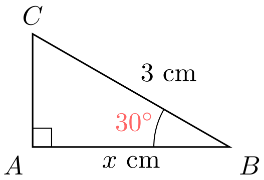
$x \approx$ $\boxed{2.83}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 45°$:

- Adjacent side: $AB = 2$
- Hypotenuse: $BC = x$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}}\\ \cos(45°) &= \frac{2}{x}\\ x &= \frac{2}{\cos(45°)}\\ x &\approx 2.83\,\text{cm} \quad \text{(round to 2 decimal places)}\end{aligned}$$

Ex 15:



Calculate x.

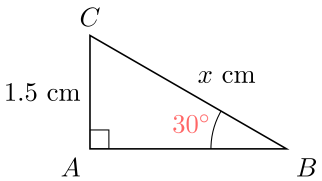
$x \approx \boxed{2.60}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Adjacent side: $AB = x$
- Hypotenuse: $BC = 3$

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}}$$
$$\cos(30°) = \frac{x}{3}$$
$$x = 3 \times \cos(30°)$$
$$x \approx 2.60\,\text{cm} \quad \text{(round to 2 decimal places)}$$

Ex 16:



Calculate x.

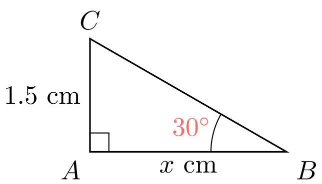
$x \approx \boxed{3.00}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Hypotenuse: $BC = x$

$$\sin(\theta) = \frac{\text{OPP}}{\text{HYP}}$$
$$\sin(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\sin(30°)}$$
$$x = 3.00\,\text{cm}$$

Ex 17:



Calculate x.

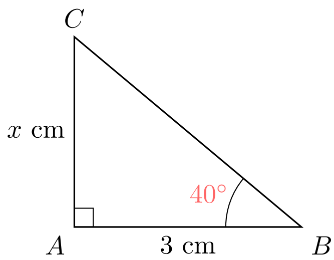
$x \approx \boxed{2.60}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Adjacent side: $AB = x$

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\tan(30°)}$$
$$x \approx 2.60\,\text{cm} \quad \text{(round to 2 decimal places)}$$

Ex 18:



Calculate x.

$x \approx \boxed{2.52}$ cm (round to 2 decimal places)

Answer: Relative to $\theta = 40°$:

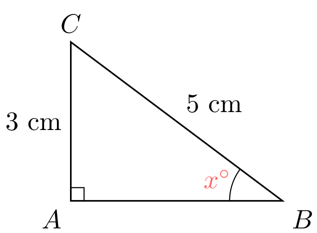
- Opposite side: $AC = x$
- Adjacent side: $AB = 3$

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(40°) = \frac{x}{3}$$
$$x = 3 \times \tan(40°)$$
$$x \approx 2.52\,\text{cm} \quad \text{(round to 2 decimal places)}$$

C INVERSE TRIGONOMETRIC FUNCTIONS

C.1 CALCULATING ANGLES

Ex 19:



Calculate the angle $x°$.

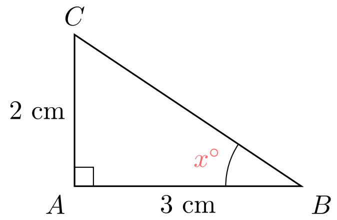
$x° \approx \boxed{36.9}°$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 5$ cm

$$x^\circ = \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right)$$
$$= \sin^{-1}\left(\frac{3}{5}\right)$$
$$\approx 36.9^\circ \quad \text{(round to 1 decimal place)}$$

Ex 20:



Calculate the angle x°.

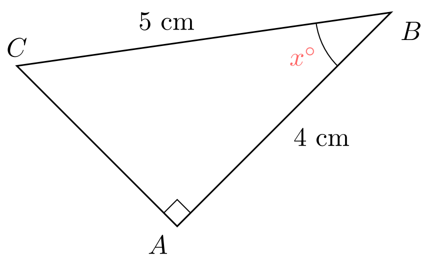
$x^\circ \approx \boxed{33.7}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 2$ cm
- Adjacent side: $AB = 3$ cm

$$x^\circ = \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right)$$
$$= \tan^{-1}\left(\frac{2}{3}\right)$$
$$\approx 33.7^\circ \quad \text{(round to 1 decimal place)}$$

Ex 21:



Calculate the angle x°.

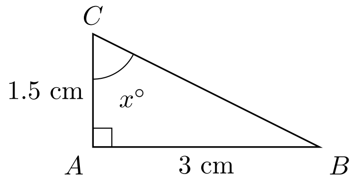
$x^\circ \approx \boxed{36.9}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Adjacent side: $AB = 4$ cm
- Hypotenuse: $BC = 5$ cm

$$x^\circ = \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right)$$
$$= \cos^{-1}\left(\frac{4}{5}\right)$$
$$\approx 36.9^\circ \quad \text{(round to 1 decimal place)}$$

Ex 22:



Calculate the angle x°.

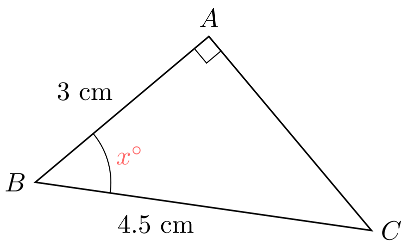
$x^\circ \approx \boxed{63.4}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AB = 3$ cm
- Adjacent side: $AC = 1.5$ cm

$$x^\circ = \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right)$$
$$= \tan^{-1}\left(\frac{3}{1.5}\right)$$
$$\approx 63.4^\circ \quad \text{(round to 1 decimal place)}$$

Ex 23:



Calculate the angle x°.

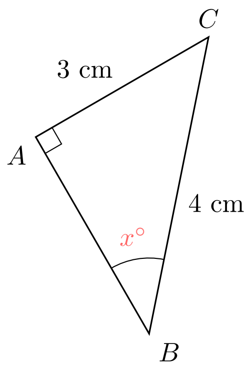
$x^\circ \approx \boxed{48.2}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Adjacent side: $AB = 3$ cm
- Hypotenuse: $BC = 4.5$ cm

$$x^\circ = \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right)$$
$$= \cos^{-1}\left(\frac{3}{4.5}\right)$$
$$\approx 48.2^\circ \quad \text{(round to 1 decimal place)}$$

Ex 24:



Calculate the angle x°.

$x^\circ \approx \boxed{48.6}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 4$ cm

$$\begin{aligned} x^\circ &= \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right) \\ &= \sin^{-1}\left(\frac{3}{4}\right) \\ &\approx 48.6^\circ \quad \text{(round to 1 decimal place)} \end{aligned}$$

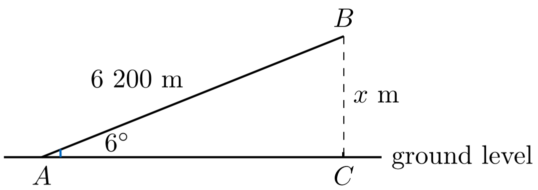
D SOLVING REAL-WORLD TRIGONOMETRY PROBLEMS

D.1 SOLVING REAL-WORLD TRIGONOMETRY PROBLEMS

Ex 25: A cyclist in France rides up a long incline with an average rise of 6°. If he rides for 6 200 m, how far has he climbed vertically?

$\boxed{648}$ m (round to the nearest integer)

Answer:



The cyclist rides 6.2 km (6200 m) up an incline with an angle of 6°. This forms a right triangle ABC, with the right angle at C, hypotenuse $AB = 6200$ m, and the vertical height $BC = x$.
Applying the sine definition:

- Hypotenuse: $AB = 6200$ m
- Opposite side: $BC = x$
- Angle: 6°

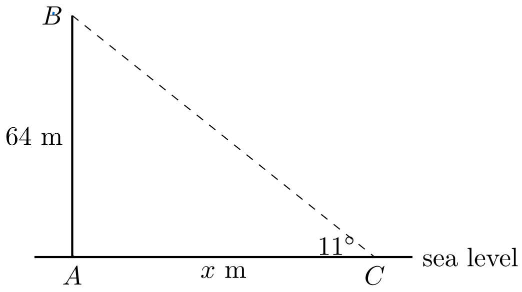
$$\begin{aligned} \sin(6^\circ) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{x}{6200} \\ x &= 6200 \times \sin(6^\circ) \\ &\approx 648\,\text{m} \quad \text{(round to the nearest integer)} \end{aligned}$$

Thus, the cyclist has climbed a vertical height of approximately 648 m.

Ex 26: The lamp in a lighthouse is 64 m above sea level. The angle of depression from the lamp to a fishing boat is 11°. How far horizontally is the boat from the lighthouse?

$\boxed{329}$ m (round to the nearest integer)

Answer:



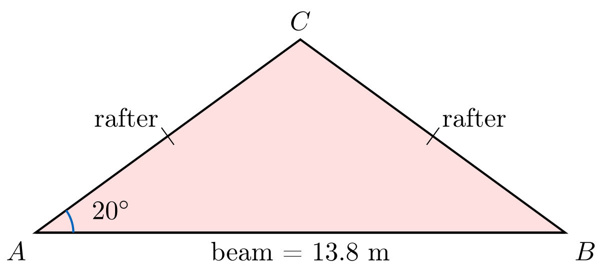
The lighthouse lamp (B) is 64 m above sea level (A). The angle of depression from B to the fishing boat (C) is 11°, which matches the angle of elevation from C to B.
This forms a right triangle ABC with the right angle at A, vertical side $AB = 64$ m, and horizontal side $AC = x$.

- Opposite side (to 11°): $AB = 64$ m
- Adjacent side: $AC = x$
- Angle: 11°

$$\begin{aligned} \tan(11^\circ) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{64}{x} \\ x &= \frac{64}{\tan(11^\circ)} \\ &\approx 329\,\text{m} \quad \text{(round to the nearest integer)} \end{aligned}$$

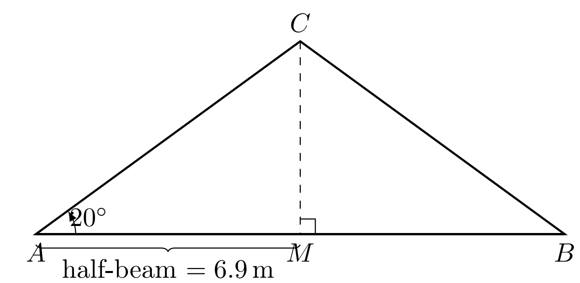
Thus, the horizontal distance from the boat to the lighthouse is approximately 329 m.

Ex 27: For the triangular roof truss illustrated, find the length of a rafter if the beam is 13.8 m and the pitch is 20°.



$\boxed{7.34}$ m (round to 2 decimal places)

Answer: Because the roof truss is isosceles, dropping a perpendicular from the ridge to the midpoint of the beam forms a right triangle whose hypotenuse is the rafter. Applying the cosine definition:

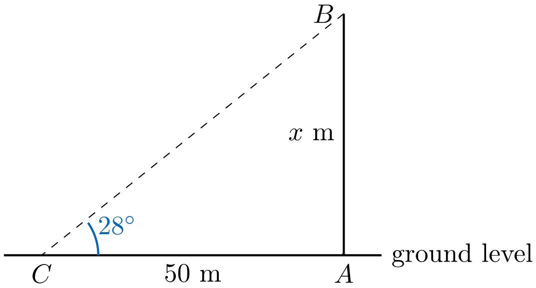


$$\begin{aligned}\cos(20^\circ) &= \frac{\text{adjacent (half-beam)}}{\text{hypotenuse (rafter)}}\\ \text{rafter} &= \frac{\text{half-beam}}{\cos(20^\circ)}\\ &= \frac{13.8/2}{\cos(20^\circ)}\\ &\approx 7.34\,\text{m} \quad \text{(round to 2 decimal places)}\end{aligned}$$

Ex 28: A person standing 50 m from the base of a tower looks up at the top with an angle of elevation of 28°. Find the height of the tower.

$\boxed{27}$ m (round to the nearest integer)

Answer:



The tower is vertical from base A to top B. The person at C is 50 m from A, with an angle of elevation of 28° from C to B. This forms a right triangle CAB with the right angle at A, opposite side $AB = x$ (height), adjacent side $CA = 50$ m.

- Opposite side (to 28°): $AB = x$
- Adjacent side: $CA = 50$ m
- Angle: 28°

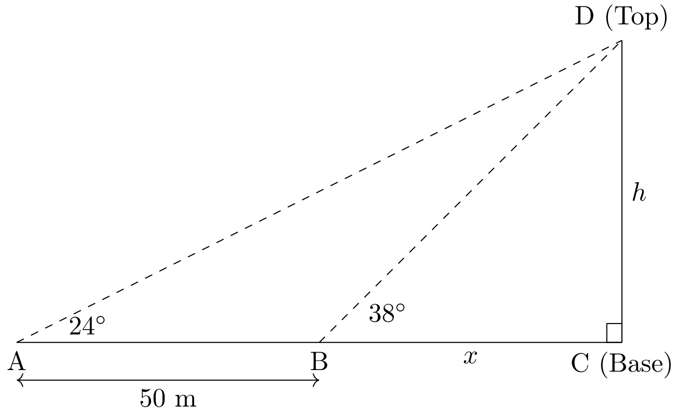
$$\begin{aligned}\tan(28^\circ) &= \frac{\text{OPP}}{\text{ADJ}}\\ &= \frac{x}{50}\\ x &= 50 \times \tan(28^\circ)\\ &\approx 27\,\text{m} \quad \text{(round to the nearest integer)}\end{aligned}$$

Thus, the height of the tower is approximately 27 m.

D.2 SOLVING MULTI-STEP TRIGONOMETRIC PROBLEMS

Ex 29: From a point A on the ground, the angle of elevation to the top of a building is 24°. From a point B, which is 50 m closer to the building, the angle of elevation is 38°. Find the height of the building, correct to one decimal place.

Answer: First, we draw a diagram to represent the situation. Let h be the height of the building (side CD), and let x be the distance from point B to the base of the building, C. The distance from A to C is therefore $x + 50$.



This setup creates two right-angled triangles, $\triangle ADC$ and $\triangle BDC$. We can establish a system of two equations using the tangent ratio:

1. In $\triangle ADC$: $\tan(24^\circ) = \dfrac{h}{x + 50}$
2. In $\triangle BDC$: $\tan(38^\circ) = \dfrac{h}{x}$

From equation (2), we can express x in terms of h:

$$x = \frac{h}{\tan(38^\circ)}$$

Now, substitute this expression for x into equation (1):

$$\tan(24^\circ) = \frac{h}{\frac{h}{\tan(38^\circ)} + 50}$$

To solve for h, we rearrange the equation:

$$\tan(24^\circ)\left(\frac{h}{\tan(38^\circ)} + 50\right) = h$$

$$\frac{h \cdot \tan(24^\circ)}{\tan(38^\circ)} + 50 \cdot \tan(24^\circ) = h$$

Now, group the terms containing h:

$$50 \cdot \tan(24^\circ) = h - \frac{h \cdot \tan(24^\circ)}{\tan(38^\circ)}$$

Factor out h:

$$50 \cdot \tan(24^\circ) = h\left(1 - \frac{\tan(24^\circ)}{\tan(38^\circ)}\right)$$

Finally, isolate h:

$$h = \frac{50 \cdot \tan(24^\circ)}{1 - \frac{\tan(24^\circ)}{\tan(38^\circ)}}$$

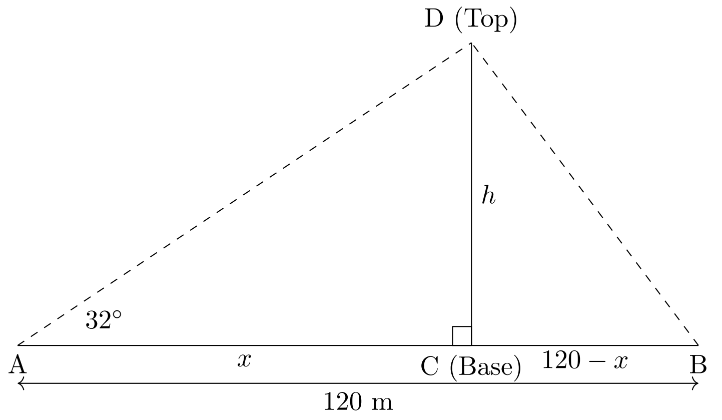
Using a calculator:

$$h \approx 51.752...$$

The height of the building is approximately 51.8 m.

Ex 30: Two observers, on opposite sides of a radio tower, are standing on a straight line that passes through the base of the tower. The observers are 120 m apart. The angle of elevation from the first observer to the top of the tower is 32°, and from the second observer, it is 48°. Find the height of the tower, correct to one decimal place.

Answer: First, we draw a diagram. Let h be the height of the tower (CD). Let A and B be the positions of the observers. Let the distance from observer A to the base C be x. The distance from observer B to the base C is then $120 - x$.



This setup creates two right-angled triangles, yielding a system of two equations:

1. In $\triangle BDC$: $\tan(48°) = \dfrac{h}{120-x}$
2. In $\triangle ADC$: $\tan(32°) = \dfrac{h}{x}$

From equation (2), we can express x in terms of h:

$$x = \frac{h}{\tan(32°)}$$

Now, substitute this expression for x into equation (1):

$$\tan(48°) = \frac{h}{120 - \frac{h}{\tan(32°)}}$$

To solve for h, we rearrange the equation:

$$\tan(48°)\left(120 - \frac{h}{\tan(32°)}\right) = h$$

$$120 \cdot \tan(48°) - \frac{h \cdot \tan(48°)}{\tan(32°)} = h$$

Now, group the terms containing h:

$$120 \cdot \tan(48°) = h + \frac{h \cdot \tan(48°)}{\tan(32°)}$$

Factor out h:

$$120 \cdot \tan(48°) = h\left(1 + \frac{\tan(48°)}{\tan(32°)}\right)$$

Finally, isolate h:

$$h = \frac{120 \cdot \tan(48°)}{1 + \frac{\tan(48°)}{\tan(32°)}}$$

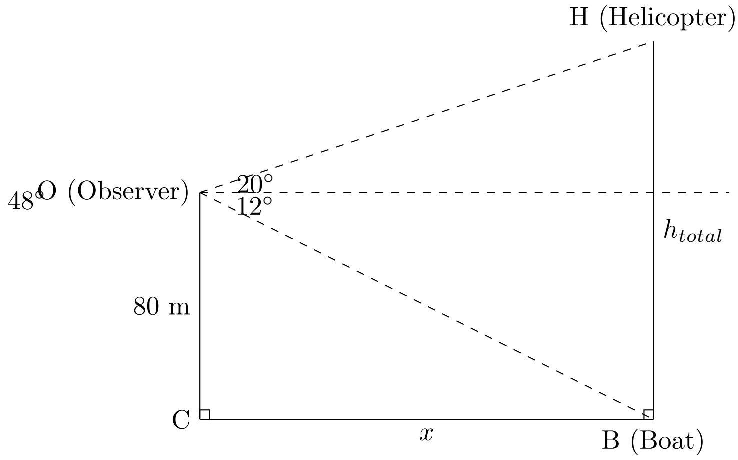
Using a calculator:

$$h \approx \frac{133.253}{1 + 1.7774} \approx \frac{133.253}{2.7774} \approx 47.985...$$

The height of the tower is approximately 48.0 m.

Ex 31: An observer stands on top of a vertical cliff of height 80 m. She measures the angle of depression to a boat at sea as 12°. At the same instant, she measures the angle of elevation to a helicopter flying directly above the boat as 20°. Find the height of the helicopter above the sea, correct to one decimal place.

Answer: First, we draw a diagram. Let O be the observer at the top of the cliff, and C be the base. The cliff height is $OC = 80$ m. Let B be the boat and H be the helicopter. Let x be the horizontal distance from the cliff to the boat, CB. Let h_{extra} be the height of the helicopter above the observer's level. The total height is $h_{total} = 80 + h_{extra}$.



This setup gives two right-angled triangles sharing the horizontal distance x. The angle of depression to the boat is 12°, so by alternate interior angles, the angle of elevation from the boat, $\angle OBC$, is also 12°. We have a system of two equations:

1. $\tan(20°) = \dfrac{h_{extra}}{x}$
2. $\tan(12°) = \dfrac{80}{x}$

From equation (2), we can express x:

$$x = \frac{80}{\tan(12°)}$$

Now, substitute this expression for x into equation (1):

$$\tan(20°) = \frac{h_{extra}}{\frac{80}{\tan(12°)}}$$

To solve for h_{extra}, we rearrange the equation:

$$h_{extra} = \tan(20°) \cdot \left(\frac{80}{\tan(12°)}\right) = \frac{80 \cdot \tan(20°)}{\tan(12°)}$$

Using a calculator:

$$h_{extra} \approx \frac{80 \cdot (0.36397...)}{0.21255...} \approx 136.98...$$

The total height of the helicopter above the sea is the sum of the cliff height and this extra height:

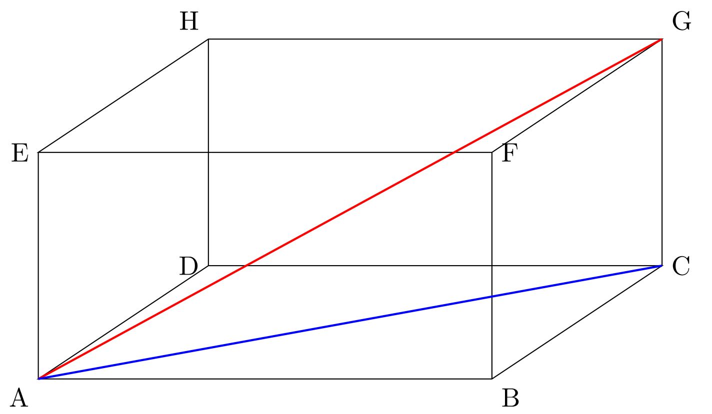
$$h_{total} = 80 + h_{extra} \approx 80 + 136.98... \approx 216.98...$$

The height of the helicopter above the sea is approximately 217.0 m.

E ANGLE BETWEEN A LINE AND A PLANE

E.1 APPLYING TRIGONOMETRY IN 3D SPACE

Ex 32: The diagram shows a cuboid with dimensions $AB = 8$ cm, $BC = 6$ cm, and $CG = 5$ cm.



Find:

1. The exact length of the space diagonal AG.
2. The angle that the diagonal AG makes with the base plane $ABCD$.

Answer:

1. First, find the length of the diagonal of the base, AC, using the right-angled triangle ABC.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AC^2 &= 8^2 + 6^2 \\ AC^2 &= 100 \\ AC &= \sqrt{100} \\ AC &= 10 \text{ cm} \end{aligned}$$

Now, consider the right-angled triangle ACG. The space diagonal AG is the hypotenuse.

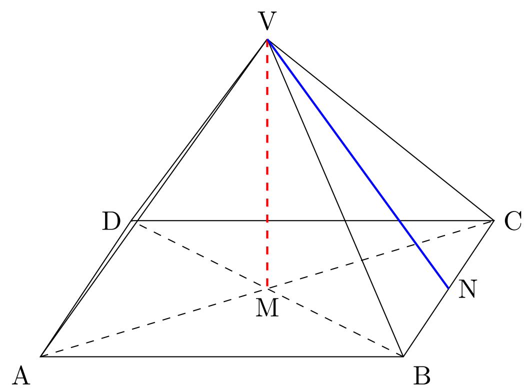
$$\begin{aligned} AG^2 &= AC^2 + CG^2 \\ AG^2 &= 10^2 + 5^2 \\ AG^2 &= 125 \\ AG &= \sqrt{125} \\ AG &= 5\sqrt{5} \text{ cm} \end{aligned}$$

2. The angle that AG makes with the base plane $ABCD$ is the angle $\angle GAC$. We use the triangle ACG. Let $\theta = \angle GAC$.

- OPP $= CG = 5$ cm
- ADJ $= AC = 10$ cm

$$\begin{aligned} \tan(\theta) &= \frac{\text{OPP}}{\text{ADJ}} \\ \tan(\theta) &= \frac{5}{10} \\ \tan(\theta) &= 0.5 \\ \theta &= \tan^{-1}(0.5) \\ \theta &\approx 26.6^\circ \end{aligned}$$

Ex 33: The diagram shows a right pyramid with a square base $ABCD$ of side length 10 cm. The vertex V is vertically above the center of the base, M, and the height of the pyramid VM is 12 cm.



Find:

1. The exact length of the slant height VN of the triangular face VBC.
2. The angle between the face VBC and the base plane $ABCD$.

Answer:

1. First, identify the right-angled triangle VMN inside the pyramid. The length MN is half the side length of the square base, so $MN = 10/2 = 5$ cm. The height is given as $VM = 12$ cm. The slant height VN is the hypotenuse.

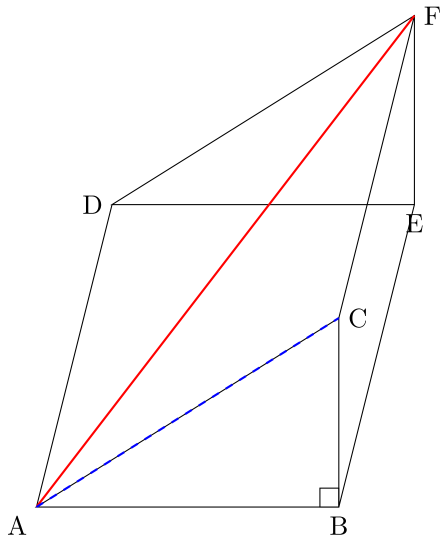
$$\begin{aligned} VN^2 &= VM^2 + MN^2 \\ VN^2 &= 12^2 + 5^2 \\ VN^2 &= 144 + 25 \\ VN^2 &= 169 \\ VN &= \sqrt{169} \\ VN &= 13 \text{ cm} \end{aligned}$$

2. The angle between the face VBC and the base $ABCD$ is the angle $\angle VNM$ in the same right-angled triangle VMN. Let $\phi = \angle VNM$.

- OPP $= VM = 12$ cm (height of pyramid)
- ADJ $= MN = 5$ cm (half the base side)

$$\begin{aligned} \tan(\phi) &= \frac{\text{OPP}}{\text{ADJ}} \\ \tan(\phi) &= \frac{12}{5} \\ \phi &= \tan^{-1}(2.4) \\ \phi &\approx 67.4^\circ \end{aligned}$$

Ex 34: The diagram shows a right wedge-shaped prism whose cross-section ABC is a right-angled triangle. The dimensions are $AB = 15$ cm, $BC = 8$ cm, and the length of the prism is 30 cm (so $CF = 30$ cm).



Find:

1. The exact length of the space diagonal AF.

2. The angle that the diagonal AF makes with the vertical face $BCFE$.

Answer:

1. First, find the length of the diagonal on the base, AC, using the right-angled triangle ABC.

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 15^2 + 8^2$$
$$AC^2 = 225 + 64 = 289$$
$$AC = \sqrt{289} = 17 \text{ cm}$$

Now, consider the right-angled triangle ACF. In a right prism, CF is perpendicular to the base, so $AC \perp CF$ and ACF is right-angled at C. The space diagonal AF is the hypotenuse.

$$AF^2 = AC^2 + CF^2$$
$$AF^2 = 17^2 + 30^2$$
$$AF^2 = 289 + 900 = 1189$$
$$AF = \sqrt{1189} \text{ cm}$$

(Note: 1189 has no square factors, so this is the exact length.)

2. The angle that AF makes with the vertical face $BCFE$ is the angle between the line AF and its projection onto that plane. Since the prism is right, the edge AB is perpendicular to the plane $BCFE$. Thus, the projection of point A onto the plane $BCFE$ is point B, and the projection of the line AF is the line BF. The required angle is $\angle AFB$ in the right-angled triangle ABF. First, find the length of BF. In right triangle BCF:

$$BF^2 = BC^2 + CF^2 = 8^2 + 30^2 = 64 + 900 = 964,$$

so $BF = \sqrt{964}$. Now in the right-angled triangle ABF, let $\psi = \angle AFB$.

- OPP $= AB = 15$ cm
- ADJ $= BF = \sqrt{964}$ cm (the diagonal of rectangle $BCFE$)

$$\tan(\psi) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(\psi) = \frac{15}{\sqrt{964}}$$
$$\psi = \tan^{-1}\left(\frac{15}{\sqrt{964}}\right)$$
$$\psi \approx 25.8^\circ$$