RELATIONSHIPS BETWEEN ANGLES

In this chapter, we will explore relationships between angles that are key to solving geometry problems. You will learn about complementary angles (summing to 90°, forming a right angle), supplementary angles (summing to 180°, forming a straight angle), opposite angles at a vertex (formed by intersecting lines), and angles created by parallel lines and a transversal, including corresponding, alternate, and co-interior angles. These concepts build on your understanding of right angles (90°), straight angles (180°), and full angles (360°).
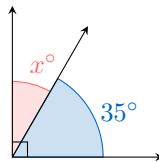
A COMPLEMENTARY AND SUPPLEMENTARY ANGLES

> **Definition Complementary**
>
> Two angles are **complementary** if their sum is 90°. Complementary angles together form a right angle, like the corner of a square.

Ex: Calculate the measure of the unknown angle $x°$ if it is complementary to a 35° angle.

Answer:

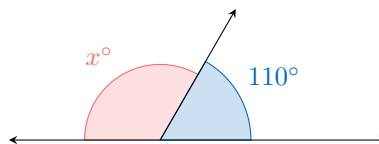The sum of complementary angles is 90°.

$$x° + 35° = 90°$$
$$x° = 90° - 35° \quad \text{(subtract 35°)}$$
$$= 55°$$

> **Definition Supplementary**
>
> Two angles are **supplementary** if their sum is 180°. Supplementary angles together form a straight line.

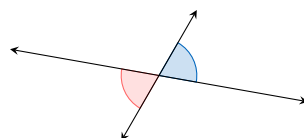**Ex:** Calculate the measure of the unknown angle $x°$ if it is supplementary to a 110° angle.

Answer:

The sum of supplementary angles is 180°.

$$x° + 110° = 180°$$
$$x° = 180° - 110° \quad \text{(subtract 110°)}$$
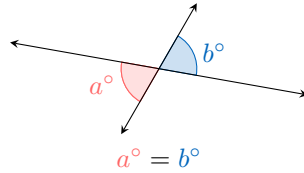$$= 70°$$

B OPPOSITE ANGLES AT A VERTEX

> **Definition Opposite Angles at a Vertex**
>
> **Opposite angles at a vertex** are angles that are opposite each other at the point where two lines intersect, sharing a common vertex.
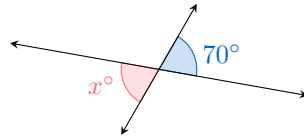
> **Proposition Equality of Opposite Angles at a Vertex**
>
> Opposite angles at a vertex are equal.
>
>
>
> $$a° = b°$$
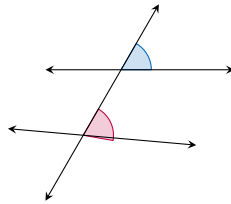
Ex: Calculate the measure of the unknown angle $x°$.

Answer: Opposite angles at a vertex are equal.

$$x° = 70° \quad \text{(opposite angles are equal)}$$
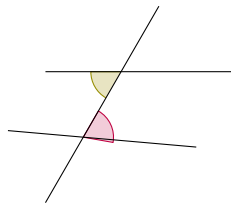
C CORRESPONDING, ALTERNATE, AND CO-INTERIOR ANGLES

> **Definition Corresponding, Alternate, and Co-interior Angles**
>
> - **Corresponding angles** are on the same side of a transversal that intersects two lines and are in matching corners relative to the intersected lines, like the top-right corners of each intersection.
>
>
>
> - **Alternate angles** are on opposite sides of a transversal and lie between the two intersected lines, forming a "Z" shape.
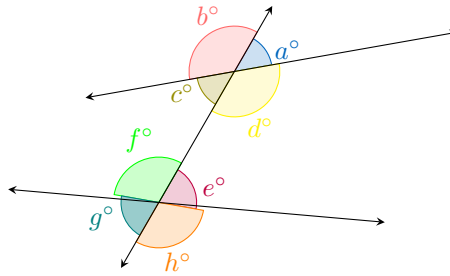>
>
>
> - **Co-interior angles** are on the same side of a transversal and between the two intersected lines, forming a "C" shape.
>
>
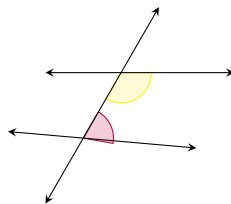
Ex: Identify the following for the given diagram:

1. The corresponding angles.

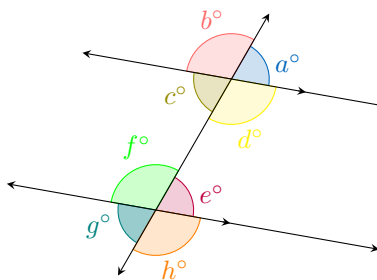2. The alternate angles.

3. The co-interior angles.

Answer:

1. Corresponding angles: $a°$ and $e°$, $b°$ and $f°$, $c°$ and $g°$, $d°$ and $h°$.

2. Alternate angles: $c°$ and $e°$, $d°$ and $f°$.

3. Co-interior angles: $c°$ and $f°$, $d°$ and $e°$.

D PROPERTIES OF PARALLEL LINES

> **Proposition Properties of Parallel Lines**
>
> If two lines are parallel and intersected by a transversal, then:
>
> - Corresponding angles are equal.
> - Alternate angles are equal.
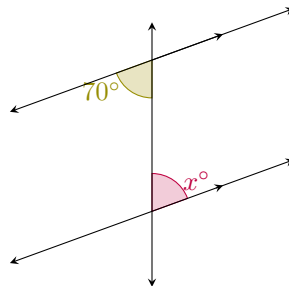> - Co-interior angles are supplementary (sum to 180°).
>
>
>
> corresponding angles: $a° = e°$, $b° = f°$, $c° = g°$, $d° = h°$
>
> alternate angles: $c° = e°$, $d° = f°$
>
> co-interior angles: $c° + f° = 180°$, $d° + e° = 180°$

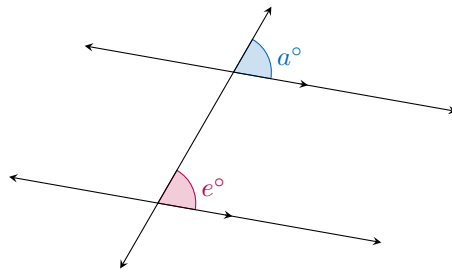**Ex:** Calculate the measure of the unknown angle $x°$, given that the lines are parallel.

Answer: Since the angles are alternate and the lines are parallel, they are equal.
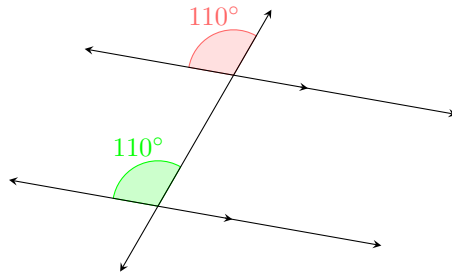
$$x° = 70° \quad \text{(alternate angles are equal)}$$

> **Proposition Parallel Lines from Equal Angles**
>
> If any pair of corresponding angles or alternate angles are equal, then the lines are parallel.

Ex: Show that the lines are parallel, given the angle measures.

Answer: Since the corresponding angles are equal $(110° = 110°)$, the lines are parallel.