RELATIONSHIPS BETWEEN ANGLES

In this chapter, we will explore relationships between angles that are key to solving geometry problems. You will learn about complementary angles (summing to 90°, forming a right angle), supplementary angles (summing to 180°, forming a straight angle), opposite angles at a vertex (formed by intersecting lines), and angles created by parallel lines and a transversal, including corresponding, alternate, and co-interior angles. These concepts build on your understanding of right angles (90°), straight angles (180°), and full angles (360°).

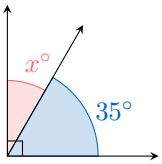
A COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Definition **Complementary**

Two angles are **complementary** if their sum is 90°. Complementary angles together form a right angle, like the corner of a square.

Ex: Calculate the measure of the unknown angle $x°$ if it is complementary to a 35° angle.

Answer:



The sum of complementary angles is 90°.

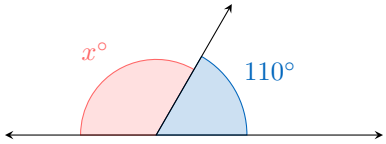
$$\begin{aligned} x° + 35° &= 90° \\ x° &= 90° - 35° \quad \text{(subtract } 35°\text{)} \\ &= 55° \end{aligned}$$

Definition **Supplementary**

Two angles are **supplementary** if their sum is 180°. Supplementary angles together form a straight line.

Ex: Calculate the measure of the unknown angle $x°$ if it is supplementary to a 110° angle.

Answer:



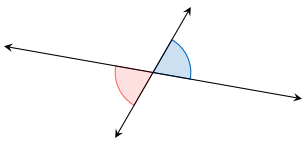
The sum of supplementary angles is 180°.

$$\begin{aligned} x° + 110° &= 180° \\ x° &= 180° - 110° \quad \text{(subtract } 110°\text{)} \\ &= 70° \end{aligned}$$

B OPPOSITE ANGLES AT A VERTEX

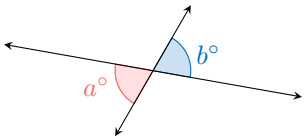
Definition **Opposite Angles at a Vertex**

Opposite angles at a vertex are angles that are opposite each other at the point where two lines intersect, sharing a common vertex.



Proposition **Equality of Opposite Angles at a Vertex**

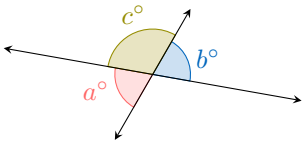
Opposite angles at a vertex are equal.



$$a° = b°$$

Proof

Consider two lines intersecting at a point, forming opposite angles $a°$ and $b°$, and an adjacent angle $c°$.



Since $a°$ and $c°$ form a straight angle:

$$a° + c° = 180° \quad \text{(straight angle)}$$

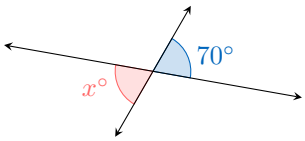
Similarly, $b°$ and $c°$ form a straight angle:

$$b° + c° = 180° \quad \text{(straight angle)}$$

Thus, $a° + c° = b° + c°$. Subtracting $c°$ from both sides gives:

$$a° = b°$$

Ex: Calculate the measure of the unknown angle $x°$.



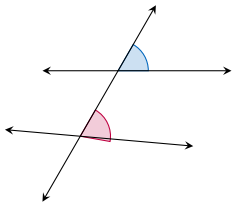
Answer: Opposite angles at a vertex are equal.

$$x° = 70° \quad \text{(opposite angles are equal)}$$

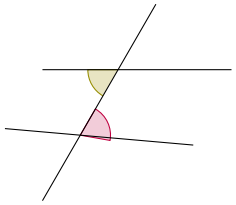
C CORRESPONDING, ALTERNATE, AND CO-INTERIOR ANGLES

Definition **Corresponding, Alternate, and Co-interior Angles**

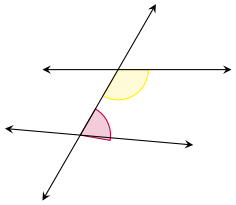
- **Corresponding angles** are on the same side of a transversal that intersects two lines and are in matching corners relative to the intersected lines, like the top-right corners of each intersection.



- **Alternate angles** are on opposite sides of a transversal and lie between the two intersected lines, forming a "Z" shape.

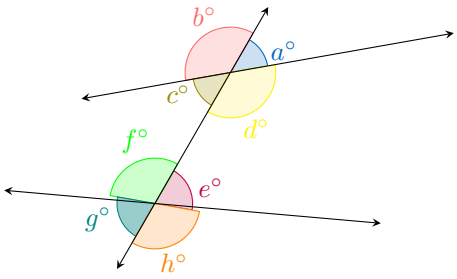


- **Co-interior angles** are on the same side of a transversal and between the two intersected lines, forming a "C" shape.



Ex: Identify the following for the given diagram:

1. The corresponding angles.
2. The alternate angles.
3. The co-interior angles.



Answer:

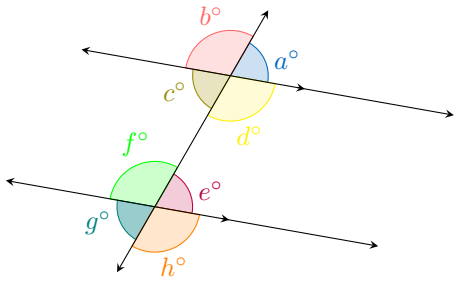
1. Corresponding angles: $a°$ and $e°$, $b°$ and $f°$, $c°$ and $g°$, $d°$ and $h°$.
2. Alternate angles: $c°$ and $e°$, $d°$ and $f°$.
3. Co-interior angles: $c°$ and $f°$, $d°$ and $e°$.

D PROPERTIES OF PARALLEL LINES

Proposition **Properties of Parallel Lines**

If two lines are parallel and intersected by a transversal, then:

- Corresponding angles are equal.
- Alternate angles are equal.
- Co-interior angles are supplementary (sum to 180°).

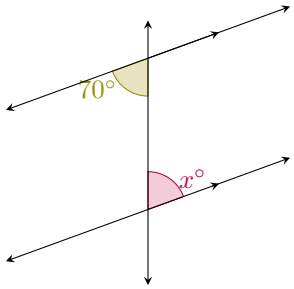


corresponding angles: $a^\circ = e^\circ$, $b^\circ = f^\circ$, $c^\circ = g^\circ$, $d^\circ = h^\circ$

alternate angles: $c^\circ = e^\circ$, $d^\circ = f^\circ$

co-interior angles: $c^\circ + f^\circ = 180°$, $d^\circ + e^\circ = 180°$

Ex: Calculate the measure of the unknown angle x°, given that the lines are parallel.

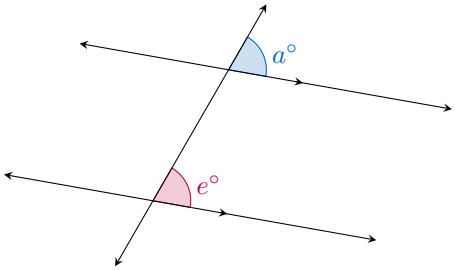


Answer: Since the angles are alternate and the lines are parallel, they are equal.

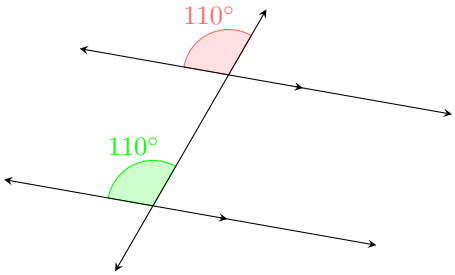
$$x^\circ = 70^\circ \quad \text{(alternate angles are equal)}$$

Proposition **Parallel Lines from Equal Angles**

If any pair of corresponding angles or alternate angles are equal, then the lines are parallel.



Ex: Show that the lines are parallel, given the angle measures.



Answer: Since the corresponding angles are equal ($110° = 110°$), the lines are parallel.