REFERENCE FUNCTIONS

Introduction In mathematics, **reference functions** are fundamental building blocks that help us understand more complex relationships. This chapter explores the following functions:
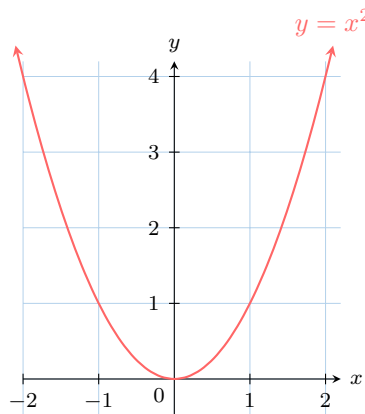
- **Square Function**: $f(x) = x^2$

- **Square Root Function**: $f(x) = \sqrt{x}$

- **Cube Function**: $f(x) = x^3$

- **Inverse Function**: $f(x) = \frac{1}{x}$

For each function, we will investigate its definition and equation, its graph and properties and Real-world examples.

A SQUARE FUNCTION

Definition square function

The **square function** is given by $f(x) = x^2$. This means that each input value x is multiplied by itself to give the output.

- **Domain**: All real numbers ($\mathbb{R}$)
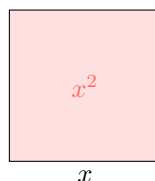
- **Shape**: A **parabola** opening upwards.

Proposition Properties

- For any real number x, $x^2 \geqslant 0$.

- The square function is strictly decreasing on $(-\infty, 0]$ and strictly increasing on $[0, +\infty)$.

x	$-\infty$	0	$+\infty$
x^2		0	

- The square function is even (its graph is symmetric with respect to the y-axis).

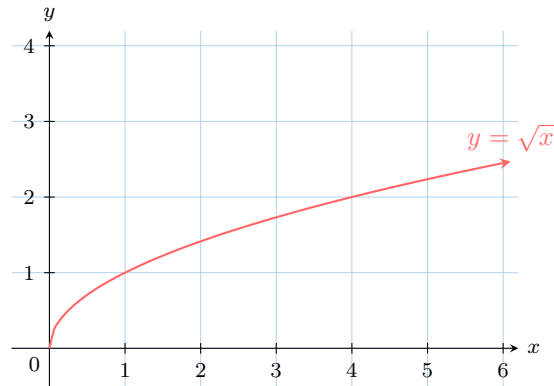**Ex:** A square has a side length x meters. Its area is given by $A(x) = x^2$.

For example, for $x = 4$ meters, $A(4) = 4^2 = 16$ square meters.

B SQUARE ROOT FUNCTION

Definition **Square Root Function**

The **square root function** is given by $f(x) = \sqrt{x}$. It is the inverse of the square function, where the output is the non-negative value that, when squared, gives the input.
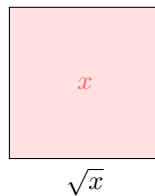

- **Domain**: $[0, +\infty)$ (non-negative real numbers)

- **Shape**: A curve that increases rapidly for small x and more slowly as x grows.

Proposition **Properties**

The square root function is strictly increasing on $[0, +\infty)$.

x	0	$+\infty$
$\sqrt{x}$	0	

Ex: Let a square have an area of x square meters. The length of the side of the square is $l(x) = \sqrt{x}$.
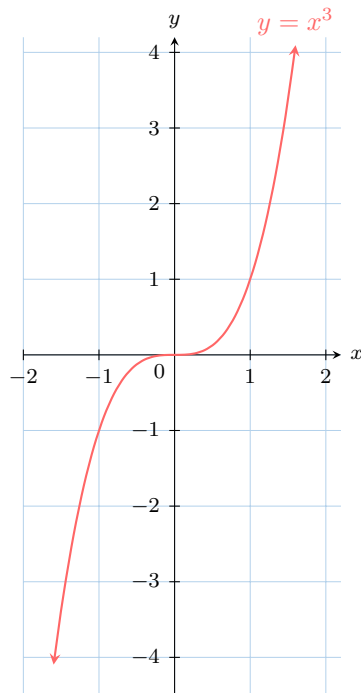

For example, for $x = 25$ square meters, $l(x) = \sqrt{25} = 5$ meters.

C CUBE FUNCTION

Definition Cube Function

The **cube function** is given by $f(x) = x^3$.

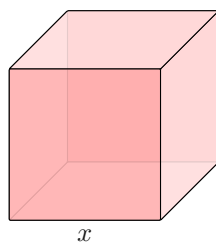**Domain**: All real numbers ($\mathbb{R}$)

Proposition Properties

- The cube function is strictly increasing on $(-\infty, +\infty)$.

x	$-\infty$	$+\infty$
x^3	$\nearrow$	

- The cube function is odd (its graph is symmetric symmetric about the origin).

Ex: The volume of a cube with a side length x meters is $V(x) = x^3$.
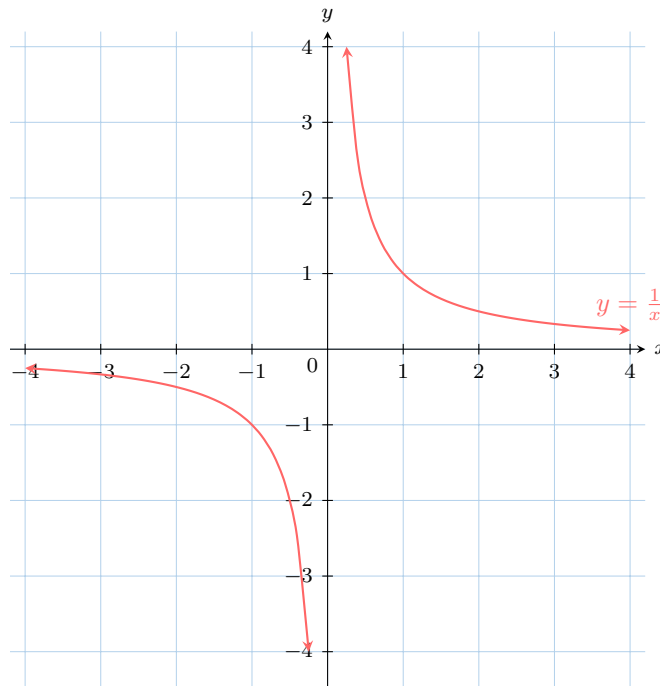

For example, for $x = 3$ meters, $V(3) = 3^3 = 27$ cubic meters.

D INVERSE FUNCTION

Definition Inverse Function

The **inverse Function** is given by $f(x) = \dfrac{1}{x}$. It represents a reciprocal relationship, where the output is the reciprocal of the input.

- **Domain**: $\mathbb{R}^*$ $(x \neq 0)$

- **Shape**: a **hyperbola**.

Proposition Properties

- The inverse function is strictly decreasing on $(-\infty, 0)$ and strictly decreasing on $(0, +\infty)$.

x	$-\infty$		0		$+\infty$
$\dfrac{1}{x}$		↘		↘	

- The inverse function is odd (its graph is symmetric symmetric about the origin).