REFERENCE FUNCTIONS

A SQUARE FUNCTION

A.1 FINDING IMAGES AND ANTECEDENTS

Ex 1: For $f(x) = x^2$, fill the table of values:

x	-2	-1	0	1	2
$f(x)$	$\boxed{4}$	$\boxed{1}$	$\boxed{0}$	$\boxed{1}$	$\boxed{4}$

Answer explanation:

- $f(-2) = (-2)^2 = 4$ substituting x by (-2)
- $f(-1) = (-1)^2 = 1$ substituting x by (-1)
- $f(0) = (0)^2 = 0$ substituting x by (0)
- $f(1) = (1)^2 = 1$ substituting x by (1)
- $f(2) = (2)^2 = 4$ substituting x by (2)

So the table of values is:

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

Ex 2: For $f(x) = x^2$,

$$f(\sqrt{2}) = \boxed{2}$$

Answer explanation:

$$\begin{aligned} f(\sqrt{2}) &= \left(\sqrt{2}\right)^2 \\ &= 2 \end{aligned}$$

Ex 3: For $f(x) = x^2$,

$$f(3\sqrt{2}) = \boxed{18}$$

Answer explanation:

$$\begin{aligned} f(3\sqrt{2}) &= \left(3\sqrt{2}\right)^2 \\ &= 3^2 \times \left(\sqrt{2}\right)^2 \\ &= 9 \times 2 \\ &= 18 \end{aligned}$$

MCQ 4: If $x = -3$, then $x^2 = -9$.

☐ True

☒ False

Answer explanation: The statement "$x = -3$, then $x^2 = -9$" is incorrect. Squaring any real number, whether positive or negative, always results in a non-negative value. Specifically:

$$(-3)^2 = 9.$$

Thus, the correct value of x^2 is 9, not -9.
Correct Answer: False.

A.2 FINDING x SUCH THAT $f(x) = k$

Ex 5: For $f(x) = x^2$, find x such that $f(x) = 4$.

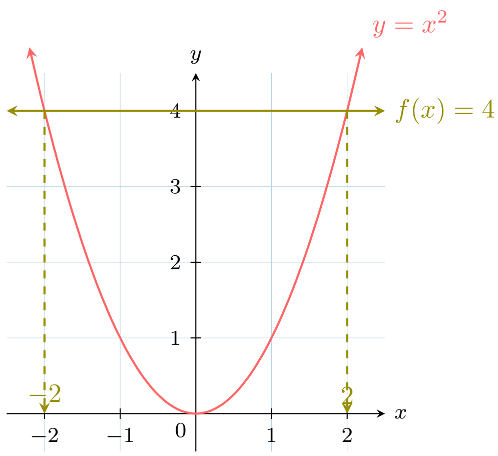
$$x = \boxed{-2} \text{ or } x = \boxed{2}.$$

Expected written answer:

$$\begin{aligned} & f(x) = 4 \\ \Leftrightarrow & x^2 = 4 \\ \Leftrightarrow & x = -\sqrt{4} \text{ or } x = \sqrt{4} \\ \Leftrightarrow & x = -2 \text{ or } x = 2 \end{aligned}$$

Answer explanation:

- Remember, when solving $x^2 = 4$, **do not forget the two solutions**: $x = -2$ and $x = 2$. Both satisfy the equation because:
$$(-2)^2 = 4 \quad \text{and} \quad 2^2 = 4.$$
- Graphically, the **values of x that satisfy** $f(x) = 4$ are determined as follows:



Ex 6: For $f(x) = x^2$, find x such that $f(x) = 1$.

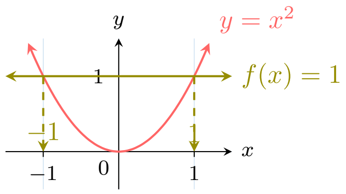
$$x = \boxed{-1} \text{ or } x = \boxed{1}.$$

Expected written answer:

$$\begin{aligned} & f(x) = 1 \\ \Leftrightarrow & x^2 = 1 \\ \Leftrightarrow & x = -\sqrt{1} \text{ or } x = \sqrt{1} \\ \Leftrightarrow & x = -1 \text{ or } x = 1 \end{aligned}$$

Answer explanation:

- Remember, when solving $x^2 = 1$, **do not forget the two solutions**: $x = -1$ and $x = 1$. Both satisfy the equation because:
$$(-1)^2 = 1 \quad \text{and} \quad 1^2 = 1.$$
- Graphically, the **values of x that satisfy** $f(x) = 1$ are determined as follows:



Ex 7: For $f(x) = x^2$, find x such that $f(x) = 0$.

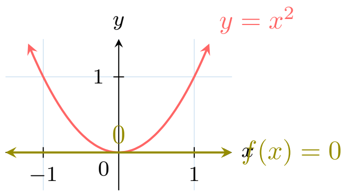
$$x = \boxed{0}.$$

Expected written answer:

$$\begin{aligned} &f(x) = 0 \\ \Leftrightarrow &x^2 = 0 \\ \Leftrightarrow &x = 0 \end{aligned}$$

Answer explanation:

- Remember, when solving $x^2 = 0$, there is only one solution: $x = 0$. This is because the square of x can only equal 0 if x itself is 0.
- Graphically, the **value of x that satisfies** $f(x) = 0$ is determined as follows:



Ex 8: For $f(x) = x^2$, find x such that $f(x) = -1$.

$$\boxed{\textbf{No solution}}$$

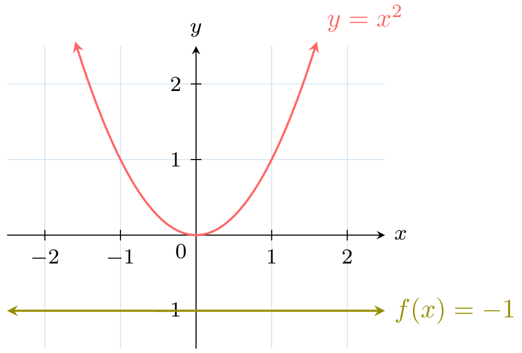
Expected written answer:

$$\begin{aligned} &f(x) = -1 \\ \Leftrightarrow &x^2 = -1 \end{aligned}$$

This equation has no solution because $x^2 \geq 0$ for all x.

Answer explanation:

- Recall that $x^2 \geq 0$ for all real values of x. This means $x^2 = -1$ is impossible because the square of any real number is non-negative.
- Graphically, there is no x for which $f(x) = -1$ because the graph of $y = x^2$ lies entirely above the x-axis.



MCQ 9: If $x^2 = 9$, then $x = 3$.

- ☐ True
- ☒ False

Answer explanation: If $x^2 = 9$, then x can be either 3 or -3 because both satisfy the equation:

$$3^2 = 9 \quad \text{and} \quad (-3)^2 = 9.$$

Thus, the correct statement should be:"If $x^2 = 9$, then $x = 3$ **or** $x = -3$."

A.3 COMPARING

Ex 10: Compare 2.2^2 and 2.4^2.

$$2.2^2 \boxed{<} 2.4^2$$

Expected written answer:

$$\begin{aligned} &2.2 < 2.4 \\ \Rightarrow &2.2^2 < 2.4^2 \quad (x \mapsto x^2 \text{ is strictly increasing on } [0, +\infty)) \end{aligned}$$

Answer explanation:

- $2.2 < 2.4$.
- Since the function $x \mapsto x^2$ is strictly increasing on the interval $[0, +\infty)$, the order between 2.2 and 2.4 is preserved when squaring the values.
- Thus, we conclude:
$$2.2^2 < 2.4^2.$$

Ex 11: Compare $(-1000)^2$ and $(-999)^2$.

$$(-1000)^2 \boxed{>} (-999)^2$$

Expected written answer:

$$\begin{aligned} &-1000 < -999 \\ \Rightarrow &(-1000)^2 > (-999)^2 \quad (x \mapsto x^2 \text{ is strictly decreasing on } (-\infty, 0]) \end{aligned}$$

Answer explanation:

- $-1000 < -999$.
- Since the function $x \mapsto x^2$ is strictly decreasing on the interval $(-\infty, 0]$, the order between -1000 and -999 is reversed when squaring the values.
- Thus, we conclude:
$$(-1000)^2 > (-999)^2.$$

Ex 12: Compare $(-10)^2$ and 10^2.

$$(-10)^2 \boxed{=} 10^2$$

Expected written answer:

- Solution 1:
$$(-10)^2 = (-10) \times (-10) = 100 \quad \text{and} \quad 10^2 = 10 \times 10 = 100,$$
$$\Rightarrow (-10)^2 = 10^2.$$

- Solution 2: The function $f(x) = x^2$ is an **even function**, meaning $f(x) = f(-x)$. Therefore:

$$(-10)^2 = 10^2.$$

Answer explanation:

- The square of a number, whether positive or negative, results in the same positive value:

$$(-10)^2 = (-10) \times (-10) = 100$$

$$10^2 = 10 \times 10 = 100.$$

- Therefore:

$$(-10)^2 = 10^2.$$

Ex 13: Compare $(200)^2$ and $(100)^2$.

$$(200)^2 \boxed{>} (100)^2$$

Expected written answer:

$$\begin{aligned} &200 > 100 \\ \Rightarrow &(200)^2 > (100)^2 \quad (x \mapsto x^2 \text{ is strictly increasing on } [0, +\infty)) \end{aligned}$$

Answer explanation:

- $200 > 100$.
- Since the function $x \mapsto x^2$ is strictly increasing on the interval $[0, +\infty)$, the order between 200 and 100 is preserved when squaring the values.
- Thus, we conclude:

$$(200)^2 > (100)^2.$$

Ex 14: Compare $(-14.5)^2$ and $(-14)^2$.

$$(-14.5)^2 \boxed{>} (-14)^2$$

Expected written answer:

$$\begin{aligned} &-14.5 < -14 \\ \Rightarrow &(-14.5)^2 > (-14)^2 \quad (x \mapsto x^2 \text{ is strictly decreasing on } (-\infty, 0]) \end{aligned}$$

Answer explanation:

- $-14.5 < -14$.
- Since the function $x \mapsto x^2$ is strictly decreasing on the interval $(-\infty, 0]$, the order between -14.5 and -14 is reversed when squaring the values.
- Thus, we conclude:

$$(-14.5)^2 > (-14)^2.$$

MCQ 15: Without using a calculator, order the following numbers in ascending order: $\left(\frac{1}{7}\right)^2, (-5)^2, \pi^2, (-1)^2$.

- ☐ $\left(\frac{1}{7}\right)^2 < (-5)^2 < \pi^2 < (-1)^2$
- ☒ $\left(\frac{1}{7}\right)^2 < (-1)^2 < \pi^2 < (-5)^2$
- ☐ $(-1)^2 < \left(\frac{1}{7}\right)^2 < \pi^2 < (-5)^2$
- ☐ $(-5)^2 < (-1)^2 < \left(\frac{1}{7}\right)^2 < \pi^2$

Answer explanation: Let's analyze the values of the given expressions without using a calculator:

- $\left(\frac{1}{7}\right)^2$: This number is very small because $\frac{1}{7} < 1$, and squaring a number less than 1 gives an even smaller result.
- $(-5)^2$: Squaring -5 gives 25, as $(-5)^2 = (-5) \times (-5) = 25$.
- π^2: Since $\pi \approx 3.14$, π^2 is slightly greater than 9, approximately 9.86.
- $(-1)^2$: Squaring -1 gives 1, as $(-1)^2 = (-1) \times (-1) = 1$.

Thus, the numbers in ascending order are:

$$\left(\frac{1}{7}\right)^2 < (-1)^2 < \pi^2 < (-5)^2.$$

A.4 DETERMINING AN INTERVAL

Ex 16: Let $3 < x$. Determine an interval for x^2

$$\boxed{9 < x^2}$$

Expected written answer:

$$\begin{aligned} &3 < x \\ \Rightarrow &3^2 < x^2 \quad (x \mapsto x^2 \text{ is strictly increasing on } [0, +\infty)) \\ \Rightarrow &9 < x^2 \end{aligned}$$

Answer explanation: When solving inequalities involving squares, it's essential to consider how the function behaves depending on the domain of the variable.

- **Starting with the given inequality:** We know $3 < x$.
- **Applying the squaring function:** The function $x \mapsto x^2$ is strictly increasing on the interval $[0, +\infty)$. Since $x > 3$, x lies in $[0, +\infty)$. Thus, squaring both sides preserves the inequality:

$$3^2 < x^2$$

- **Simplifying the inequality:**

$$9 < x^2$$

- **Conclusion:** The interval for x^2 is $x^2 \in (9, +\infty)$.

Ex 17: Let $x < -2$. Determine an interval for x^2

$$\boxed{9 < x^2}$$

Expected written answer:

$$\begin{aligned} &x < -2 \\ \Rightarrow &x^2 > (-2)^2 \quad (x \mapsto x^2 \text{ is strictly decreasing on } (-\infty, 0]) \\ \Rightarrow &x^2 > 4 \end{aligned}$$

Answer explanation: When solving inequalities involving squares, it's essential to consider how the function behaves depending on the domain of the variable.

- **Starting with the given inequality:** We know $x < -2$.
- **Applying the squaring function:** The function $x \mapsto x^2$ is strictly decreasing on the interval $(-\infty, 0]$. Since $x < -2$, squaring both sides flips the inequality direction:
$$x^2 > (-2)^2$$
- **Simplifying the inequality:**
$$x^2 > 4$$
- **Conclusion:** The interval for x^2 is $x^2 \in (4, +\infty)$.

Ex 18: Let $2 < x \leqslant 5$. Determine an interval for x^2

$$\boxed{4 < x^2 \leqslant 25}$$

Expected written answer:

$$\begin{aligned} & 2 < x \leqslant 5 \\ \Rightarrow\ & 2^2 < x^2 \leqslant 5^2 \quad (x \mapsto x^2 \text{ is strictly increasing on } [0, +\infty)) \\ \Rightarrow\ & 4 < x^2 \leqslant 25 \end{aligned}$$

Answer explanation: When solving inequalities involving squares, it's essential to consider how the function behaves depending on the domain of the variable.

- **Starting with the given inequality:** We know $2 < x \leqslant 5$.
- **Applying the squaring function:** The function $x \mapsto x^2$ is strictly increasing on the interval $[0, +\infty)$. Since $2 < x \leqslant 5$, squaring all terms preserves the inequality:
$$2^2 < x^2 \leqslant 5^2$$
- **Simplifying the inequality:**
$$4 < x^2 \leqslant 25$$
- **Conclusion:** The interval for x^2 is $x^2 \in (4, 25]$.

Ex 19: Let $-\sqrt{2} \leqslant x \leqslant 0$. Determine an interval for x^2

$$\boxed{0 \leqslant x^2 \leqslant 2}$$

Expected written answer:

$$\begin{aligned} & -\sqrt{2} \leqslant x \leqslant 0 \\ \Rightarrow\ & x^2 \leqslant (-\sqrt{2})^2 \quad (x \mapsto x^2 \text{ is decreasing on } (-\infty, 0]) \\ \Rightarrow\ & 0 \leqslant x^2 \leqslant 2 \end{aligned}$$

Answer explanation: When solving inequalities involving squares, it's essential to consider how the function behaves depending on the domain of the variable.

- **Starting with the given inequality:** We know $-\sqrt{2} \leqslant x \leqslant 0$.
- **Applying the squaring function:** The function $x \mapsto x^2$ is strictly decreasing on the interval $(-\infty, 0]$. Since x lies between $-\sqrt{2}$ and 0, squaring both sides gives:
$$0 \leqslant x^2 \leqslant (\sqrt{2})^2$$
- **Simplifying the inequality:**
$$0 \leqslant x^2 \leqslant 2$$
- **Conclusion:** The interval for x^2 is $x^2 \in [0, 2]$.

B SQUARE ROOT FUNCTION

B.1 FINDING IMAGES AND ANTECEDENTS

Ex 20: For $f(x) = \sqrt{x}$, fill the table of values:

x	0	1	4
$f(x)$	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$

Answer explanation:

- $f(0) = \sqrt{0}$ substituting x by (0)
  $= 0$
- $f(1) = \sqrt{1}$ substituting x by (1)
  $= 1$
- $f(4) = \sqrt{4}$ substituting x by (4)
  $= 2$

So the table of values is:

x	0	1	4
$f(x)$	0	1	2

Ex 21: For $f(x) = \sqrt{x}$,

$$f(25) = \boxed{5}$$

Answer explanation:

$$\begin{aligned} f(5) &= \sqrt{25} \\ &= \sqrt{5^2} \\ &= 5 \end{aligned}$$

Ex 22: For $f(x) = \sqrt{x}$,

$$f(100) = \boxed{10}$$

Answer explanation:

$$\begin{aligned} f(100) &= \sqrt{100} \\ &= \sqrt{10^2} \\ &= 10 \end{aligned}$$

Ex 23: Let $f(x) = \sqrt{x}$. Find $f(-1)$.

$$\boxed{\textbf{undefined}}$$

Answer explanation: The function $f(x) = \sqrt{x}$ is only defined for $x \geq 0$ because the square root of a negative number is not a real number. Therefore, $f(-1)$ is undefined.

B.2 FINDING x SUCH THAT $f(x) = k$

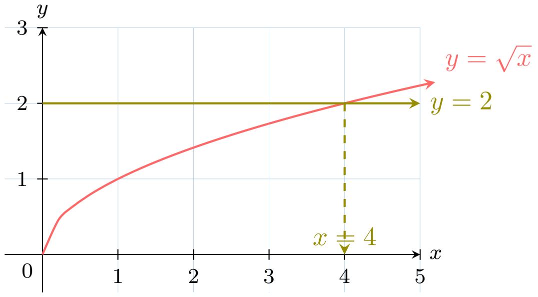
Ex 24: For $f(x) = \sqrt{x}$, find x such that $f(x) = 2$.

$$x = \boxed{4}.$$

Expected written answer:

$$\begin{aligned} & f(x) = 2 \\ \Leftrightarrow & \sqrt{x} = 2 \\ \Leftrightarrow & \left(\sqrt{x}\right)^2 = 2^2 \quad \text{(squaring both sides)} \\ \Leftrightarrow & x = 2 \end{aligned}$$

Answer explanation: Graphically, the **values of** x **that satisfy** $f(x) = 2$ are determined as follows:



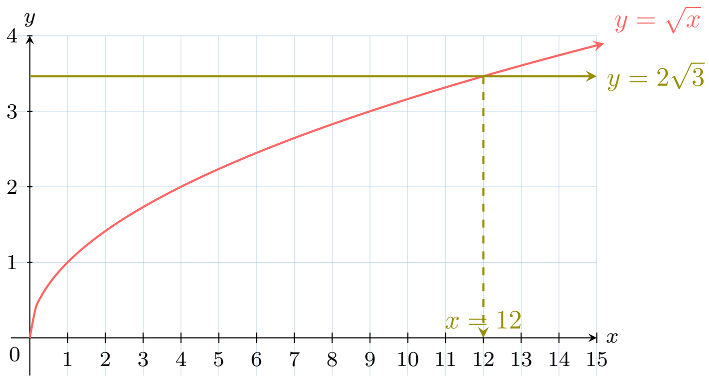
Ex 25: For $f(x) = \sqrt{x}$, find x such that $f(x) = 2\sqrt{3}$.

$$x = \boxed{12}.$$

Expected written answer:

$$\begin{aligned} & f(x) = 2\sqrt{3} \\ \Leftrightarrow & \sqrt{x} = 2\sqrt{3} \\ \Leftrightarrow & \left(\sqrt{x}\right)^2 = (2\sqrt{3})^2 \quad \text{(squaring both sides)} \\ \Leftrightarrow & x = 2^2 \times (\sqrt{3})^2 \\ \Leftrightarrow & x = 4 \times 3 \\ \Leftrightarrow & x = 12 \end{aligned}$$

Answer explanation: The **value of** x that satisfies $f(x) = 2\sqrt{3}$ is found at the intersection of the line $y = 2\sqrt{3}$ with the curve $y = \sqrt{x}$:



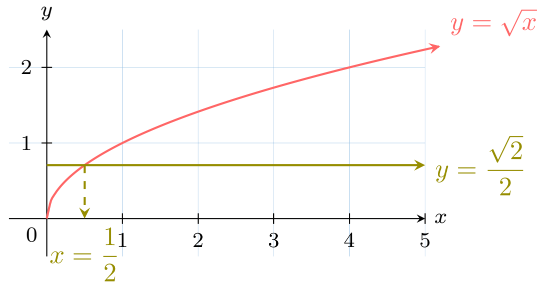
Ex 26: For $f(x) = \sqrt{x}$, find x such that $f(x) = \frac{\sqrt{2}}{2}$.

$$x = \boxed{0.5}.$$

Expected written answer:

$$\begin{aligned} & f(x) = \frac{\sqrt{2}}{2} \\ \Leftrightarrow & \sqrt{x} = \frac{\sqrt{2}}{2} \\ \Leftrightarrow & \left(\sqrt{x}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{(squaring both sides)} \\ \Leftrightarrow & x = \frac{(\sqrt{2})^2}{2^2} \\ \Leftrightarrow & x = \frac{2}{4} \\ \Leftrightarrow & x = \frac{1}{2} \\ \Leftrightarrow & x = 0.5 \end{aligned}$$

Answer explanation: The **value of** x that satisfies $f(x) = \frac{\sqrt{2}}{2}$ is found at the intersection of the line $y = \frac{\sqrt{2}}{2}$ with the curve $y = \sqrt{x}$:



Ex 27: For $f(x) = \sqrt{x}$, find x such that $f(x) = -1$.

$$\boxed{\text{No solution}}$$

Expected written answer:

$$\begin{aligned} & f(x) = -1 \\ \Leftrightarrow & \sqrt{x} = -1 \end{aligned}$$

This equation has no solution, because $\sqrt{x} \geq 0$.

B.3 COMPARING

Ex 28: Compare $\sqrt{2}$ and $\sqrt{3}$.

$$\sqrt{2} \boxed{<} \sqrt{3}$$

Expected written answer:

$$\begin{aligned} & 2 < 3 \\ \Rightarrow & \sqrt{2} < \sqrt{3} \quad (x \mapsto \sqrt{x} \text{ is strictly increasing on } [0, +\infty)) \end{aligned}$$

Answer explanation:

- $2 < 3$.
- Since the function $x \mapsto \sqrt{x}$ is strictly increasing on the interval $[0, +\infty)$, the order between 2 and 3 is preserved under the square root function.
- Thus, we conclude:
$$\sqrt{2} < \sqrt{3}.$$

Ex 29: Compare $\sqrt{100}$ and $\sqrt{99}$.

$$\sqrt{100} \boxed{>} \sqrt{99}$$

Expected written answer:

$$\begin{aligned} &100 > 99 \\ \Rightarrow &\sqrt{100} > \sqrt{99} \quad (x \mapsto \sqrt{x} \text{ is strictly increasing on } [0,+\infty)) \end{aligned}$$

Answer explanation:

- $100 > 99$.
- Since the function $x \mapsto \sqrt{x}$ is strictly increasing on the interval $[0,+\infty)$, the order between 100 and 99 is preserved under the square root function.
- Thus, we conclude:
$$\sqrt{100} > \sqrt{99}.$$

Ex 30: Compare $\sqrt{\pi}$ and $\sqrt{3}$.

$$\sqrt{\pi} \boxed{>} \sqrt{3}$$

Expected written answer:

$$\begin{aligned} &\pi > 3 \\ \Rightarrow &\sqrt{\pi} > \sqrt{3} \quad (x \mapsto \sqrt{x} \text{ is strictly increasing on } [0,+\infty)) \end{aligned}$$

Answer explanation:

- $\pi > 3$.
- Since the function $x \mapsto \sqrt{x}$ is strictly increasing on the interval $[0,+\infty)$, the order between π and 3 is preserved under the square root function.
- Thus, we conclude:
$$\sqrt{\pi} > \sqrt{3}.$$

C CUBE FUNCTION

C.1 FINDING IMAGES AND ANTECEDENTS

Ex 31: For $f(x) = x^3$, fill the table of values:

x	-2	-1	0	1	2
$f(x)$	$\boxed{-8}$	$\boxed{-1}$	$\boxed{0}$	$\boxed{1}$	$\boxed{8}$

Answer explanation:

- $f(-2) = (-2)^3$ substituting x by (-2)
  $= -8$
- $f(-1) = (-1)^3$ substituting x by (-1)
  $= -1$
- $f(0) = (0)^3$ substituting x by (0)
  $= 0$
- $f(1) = (1)^3$ substituting x by (1)
  $= 1$
- $f(2) = (2)^3$ substituting x by (2)
  $= 8$

So the table of values is:

x	-2	-1	0	1	2
$f(x)$	-8	-1	0	1	8

Ex 32: For $f(x) = x^3$,

$$f(10) = \boxed{1000}$$

Answer explanation:

$$\begin{aligned} f(10) &= (10)^3 \\ &= 1\,000 \end{aligned}$$

Ex 33: For $f(x) = x^3$,

$$f(-5) = \boxed{-125}$$

Answer explanation:

$$\begin{aligned} f(-5) &= (-5)^3 \\ &= -125 \end{aligned}$$

Ex 34: For $f(x) = x^3$,

$$f(-3) = \boxed{-27}$$

Answer explanation:

$$\begin{aligned} f(-3) &= (-3)^3 \\ &= -27 \end{aligned}$$

D INVERSE FUNCTION

D.1 FINDING IMAGES AND ANTECEDENTS

Ex 35: For $f(x) = \frac{1}{x}$, fill the table of values:

x	-2	-1	-0.5	0.5	1
$f(x)$	$\boxed{-0.5}$	$\boxed{-1}$	$\boxed{-2}$	$\boxed{2}$	$\boxed{1}$

Answer explanation:

- $f(-2) = \dfrac{1}{-2}$ substituting x by (-2)
  $= -0.5$
- $f(-1) = \dfrac{1}{-1}$ substituting x by (-1)
  $= -1$
- $f(-0.5) = \dfrac{1}{-0.5}$ substituting x by (-0.5)
  $= -2$
- $f(0.5) = \dfrac{1}{0.5}$ substituting x by (0.5)
  $= 2$
- $f(1) = \dfrac{1}{1}$ substituting x by (1)
  $= 1$
- $f(2) = \dfrac{1}{2}$ substituting x by (2)
  $= 0.5$

So the table of values is:

x	-2	-1	-0.5	0.5	1	2
$f(x)$	-0.5	-1	-2	2	1	0.5

Ex 36: For $f(x) = \frac{1}{x}$,

$$f\left(\frac{1}{2}\right) = \boxed{2}$$

Answer explanation:

$$\begin{aligned} f\left(\frac{1}{2}\right) &= \frac{1}{\frac{1}{2}} \\ &= 1 \times \frac{2}{1} \\ &= 2 \end{aligned}$$

D.2 FINDING x SUCH THAT $f(x) = k$

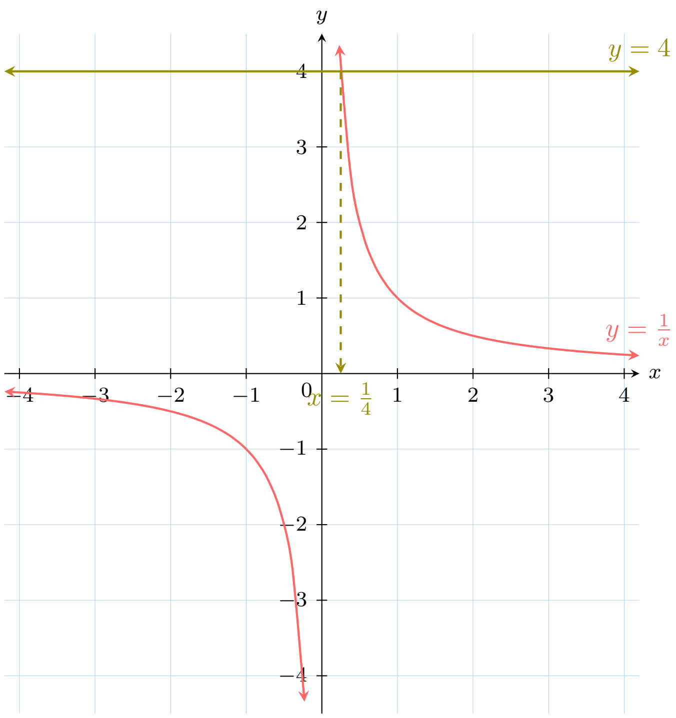
Ex 37: For $f(x) = \dfrac{1}{x}$, find x such that $f(x) = 4$.

$$x = \boxed{\frac{1}{4}}.$$

Expected written answer:

$$\begin{aligned} & f(x) = 4 \\ \Leftrightarrow & \frac{1}{x} = 4 \\ \Leftrightarrow & x \times 4 = 1 && \text{(cross-multiplication)} \\ \Leftrightarrow & x = \frac{1}{4} && \text{(divide both sides by 4)} \end{aligned}$$

Answer explanation: Graphically, the **value of x that satisfies** $f(x) = 4$ is determined as follows:



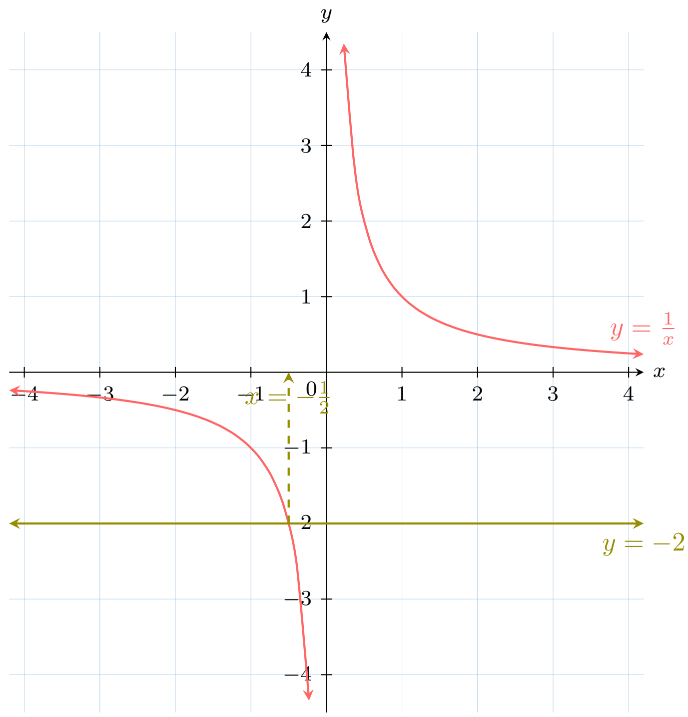
Ex 38: For $f(x) = \dfrac{1}{x}$, find x such that $f(x) = -2$.

$$x = \boxed{-\frac{1}{2}}.$$

Expected written answer:

$$\begin{aligned} & f(x) = -2 \\ \Leftrightarrow & \frac{1}{x} = -2 \\ \Leftrightarrow & x \times (-2) = 1 && \text{(cross-multiplication)} \\ \Leftrightarrow & x = -\frac{1}{2} && \text{(divide both sides by -2)} \end{aligned}$$

Answer explanation: Graphically, the **value of x that satisfies** $f(x) = -2$ is determined as follows:



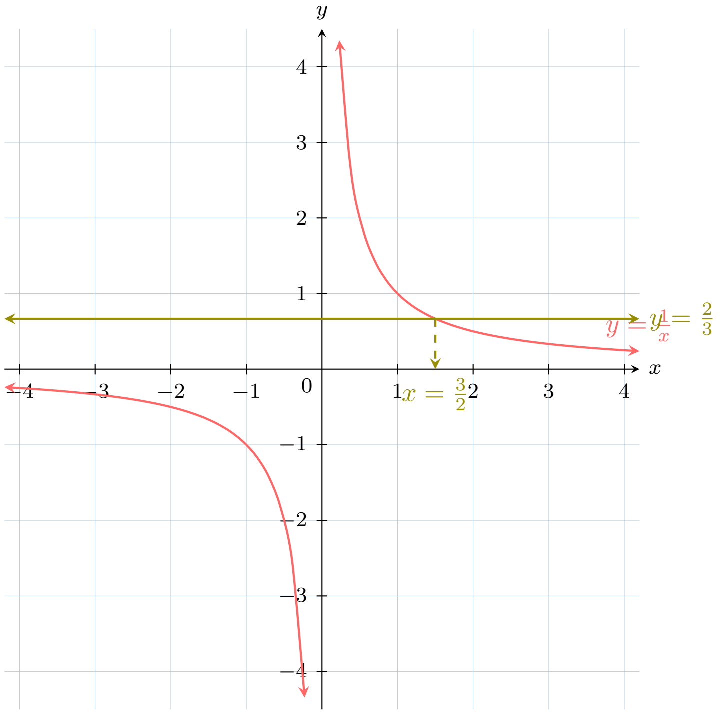
Ex 39: For $f(x) = \dfrac{1}{x}$, find x such that $f(x) = \dfrac{2}{3}$.

$$x = \boxed{\frac{3}{2}}.$$

Expected written answer:

$$\begin{aligned} & f(x) = \frac{2}{3} \\ \Leftrightarrow & \frac{1}{x} = \frac{2}{3} \\ \Leftrightarrow & x \times 2 = 1 \times 3 && \text{(cross-multiplication)} \\ \Leftrightarrow & x = \frac{3}{2} && \text{(divide both sides by 3)} \end{aligned}$$

Answer explanation: Graphically, the **value of x that satisfies** $f(x) = \frac{2}{3}$ is determined as follows:



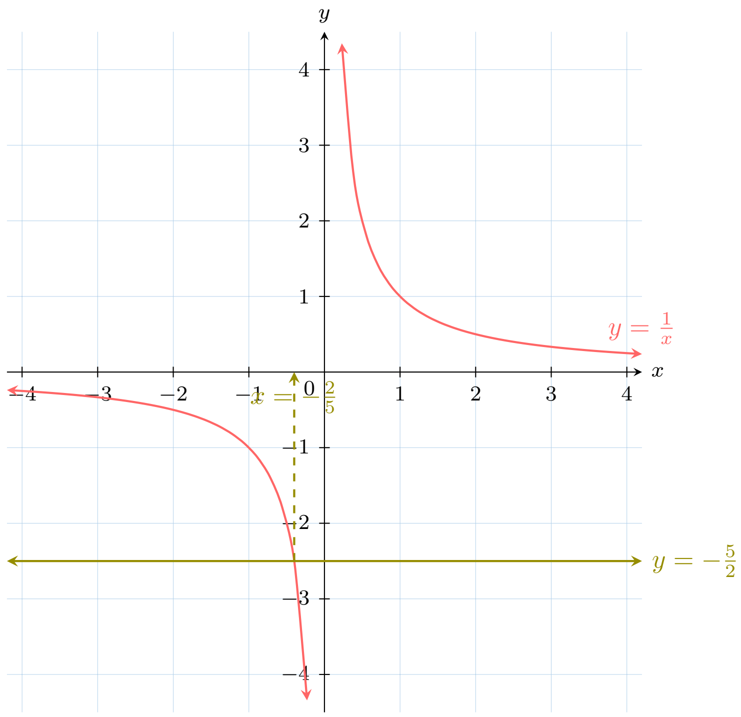
Ex 40: For $f(x) = \dfrac{1}{x}$, find x such that $f(x) = -\dfrac{5}{2}$.

$$x = \boxed{-\frac{2}{5}}.$$

Expected written answer:

$$\begin{aligned} & f(x) = -\frac{5}{2} \\ \Leftrightarrow & \frac{1}{x} = -\frac{5}{2} \\ \Leftrightarrow & x \times 5 = -1 \times 2 && \text{(cross-multiplication)} \\ \Leftrightarrow & x = -\frac{2}{5} && \text{(divide both sides by 5)} \end{aligned}$$

Answer explanation: Graphically, the **value of** x **that satisfies** $f(x) = -\frac{5}{2}$ is determined as follows:



D.3 COMPARING

Ex 41: Compare $\dfrac{1}{20}$ and $\dfrac{1}{100}$.

$$\frac{1}{20} \boxed{>} \frac{1}{100}$$

Expected written answer:

$$20 < 100$$
$$\frac{1}{20} > \frac{1}{100}$$

Answer explanation:

- $20 < 100$.
- Since the function $x \mapsto \frac{1}{x}$ is strictly decreasing on the interval $[0, +\infty)$, the reciprocal reverses the order of 20 and 100.
- Thus, we conclude:
$$\frac{1}{20} > \frac{1}{100}.$$

Ex 42: Compare $\dfrac{1}{\sqrt{2}}$ and $\dfrac{1}{2}$.

$$\frac{1}{\sqrt{2}} \boxed{>} \frac{1}{2}$$

Expected written answer:

$$\sqrt{2} > 2$$
$$\text{so } \frac{1}{\sqrt{2}} < \frac{1}{2}.$$

Answer explanation:

- $\sqrt{2} > 2$, so $\dfrac{1}{\sqrt{2}} < \dfrac{1}{2}$ because the reciprocal of a larger number is smaller.
- Thus, we conclude:
$$\frac{1}{\sqrt{2}} < \frac{1}{2}.$$

Ex 43: Compare $-\dfrac{1}{20}$ and $-\dfrac{1}{19}$.

$$-\frac{1}{20} \boxed{<} -\frac{1}{19}$$

Expected written answer:

$$-20 < -19$$
$$-\frac{1}{20} > -\frac{1}{19}$$

Answer explanation:

- $-20 < -19$.
- Since the function $x \mapsto \frac{1}{x}$ is strictly decreasing on the interval $(-\infty, 0]$, the reciprocal reverses the order of -20 and -19.
- Thus, we conclude:
$$-\frac{1}{20} > -\frac{1}{19}.$$

Ex 44: Compare $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{3}}$.

$$\frac{1}{\sqrt{2}} \boxed{>} \frac{1}{\sqrt{3}}$$

Expected written answer:

$$\sqrt{2} < \sqrt{3}$$
$$\text{so } \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}}.$$

Answer explanation:

- $\sqrt{2} < \sqrt{3}$, so $\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}}$ because the reciprocal of a smaller number is larger.
- Thus, we conclude:
$$\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}}.$$