REAL POLYNOMIALS

A DEFINITIONS

A.1 IDENTIFYING POLYNOMIAL PROPERTIES

Ex 1: For the polynomial $P(x) = 3x^5 - 7x^3 + 2x - 10$, state:

1. the degree is $\square$
2. the leading coefficient is $\square$
3. the constant term is $\square$

Ex 2: For the polynomial $P(x) = 7x - 4x^2 + 1$, state:

1. the degree is $\square$
2. the leading coefficient is $\square$
3. the constant term is $\square$

Ex 3: For the polynomial $Q(x) = -x^4 + 9x^2 - x$, state:

1. the degree is $\square$
2. the leading coefficient is $\square$
3. the coefficient of x^2 is $\square$
4. the constant term is $\square$

A.2 CLASSIFYING POLYNOMIALS BY DEGREE

MCQ 4: What is the correct classification for the polynomial $P(x) = 4 - 2x^3 + x$?

- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

MCQ 5: What is the correct classification for the polynomial $P(x) = 5x + x^2 - 1$?

- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

MCQ 6: What is the correct classification for the polynomial $P(x) = 6x^2 - x^3$?

- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

MCQ 7: What is the correct classification for the polynomial $P(x) = 3x - 5x^4 + 2$?

- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

MCQ 8: What is the correct classification for the polynomial $P(x) = 5 - 2x$?

- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

A.3 IDENTIFYING COEFFICIENTS

Ex 9: Find the values of a, b, and c given the polynomial identity:

$$ax^2 + bx + c = 5x^2 - 7$$

$$a = \square,\ b = \square,\ c = \square$$

Ex 10: Find the values of a, b, c, and d given the polynomial identity:

$$ax^3 + bx^2 + cx + d = 4 - 9x + 2x^3$$

$$a = \square,\ b = \square,\ c = \square,\ d = \square$$

Ex 11: Find the values of a, b, and c given the polynomial identity:

$$ax^4 + (b - 2)x^3 + 5x = -3x^4 + 7x^3 + cx$$

$$a = \square,\ b = \square,\ c = \square$$

B OPERATIONS WITH POLYNOMIALS

B.1 PERFORMING LINEAR OPERATIONS

Ex 12: For $P(x) = 4x^3 + 2x^2 - 5x + 1$ and $Q(x) = x^3 - 3x^2 + 7$, find:

$$P(x) + Q(x) = \square$$

Ex 13: For $P(x) = 4x^3 + 2x^2 - 5x + 1$ and $Q(x) = x^3 - 3x^2 + 7$, find:

$$P(x) - Q(x) = \square$$

Ex 14: For $P(x) = 2x^2 - x + 5$ and $Q(x) = x^3 - 3x^2 + 4$, find:

$$2P(x) - 3Q(x) = \square$$

Ex 15: For $P(x) = 2x^2 - 3x$ and $Q(x) = x^3 + x^2 - 1$, find:

$$2(P(x) + Q(x)) = \square$$

B.2 EXPANDING POLYNOMIALS

Ex 16: For $P(x) = x - 1$ and $Q(x) = x^2 + 2x + 1$, find:

$$P(x)Q(x) = \boxed{}$$

Ex 17: For $P(x) = x - 1$ and $Q(x) = x^2 + x + 1$, find:

$$P(x)Q(x) = \boxed{}$$

Ex 18: For $P(x) = x^2 - 3x + 1$, find:

$$(P(x))^2 = \boxed{}$$

Ex 19: For $P(x) = x - 1$, $Q(x) = x + 1$, and $R(x) = x + 2$, find:

$$P(x)Q(x)R(x) = \boxed{}$$

B.3 IDENTIFYING COEFFICIENTS

Ex 20: Find the values of a, b, and c given the polynomial identity:

$$(x - 1)(ax^2 + bx + c) = 2x^3 - 5x^2 + 8x - 5, \forall x \in \mathbb{R}$$

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

Ex 21: Find the values of a, b, and c given the polynomial identity:

$$(x^2 + 2x - 1)(ax^2 + bx + c) = 2x^4 + 3x^3 - x^2 + 7x - 3, \forall x \in \mathbb{R}$$

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

Ex 22: Find the values of a and b given the polynomial identity:

$$(2x + a)(x^2 + bx - 1) = 2x^3 + 5x^2 + x - 3, \forall x \in \mathbb{R}$$

$$a = \boxed{}, b = \boxed{}$$

Ex 23: Find the values of a, b, and c given the polynomial identity:

$$x^2 - 2x + 3 = a(x - b)^2 + c, \forall x \in \mathbb{R}$$

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

C THE DIVISION ALGORITHM

C.1 PERFORMING POLYNOMIAL DIVISION

Ex 24: Write the division with remainder of $x^2 + 3x + 5$ by $x + 1$:

$$x^2 + 3x + 5 = (x + 1) \times \boxed{} + \boxed{}$$

Ex 25: Write the division with remainder of $2x^2 + 5x - 4$ by $x + 3$:

$$2x^2 + 5x - 4 = (x + 3) \times \boxed{} + \boxed{}$$

Ex 26: Write the division with remainder of x^3 by $x^2 - 1$:

$$x^3 = (x^2 - 1) \times \boxed{} + \boxed{}$$

Ex 27: Write the division with remainder of $2x^3 - 2x - 1$ by $x^2 + 2x + 1$:

$$2x^3 - 2x - 1 = (x^2 + 2x + 1) \times \boxed{} + \boxed{}$$

C.2 VERIFYING DIVISIBILITY

MCQ 28: Is the polynomial $D(x) = x - 2$ a divisor of $P(x) = x^3 - 4x^2 + x + 6$?

☐ Yes

☐ No

MCQ 29: Is the polynomial $D(x) = x + 1$ a divisor of $P(x) = x^3 + 2x^2 - x - 5$?

☐ Yes

☐ No

MCQ 30: Is the polynomial $D(x) = x - 1$ a divisor of $P(x) = 2x^3 - x^2 - 7x + 6$?

☐ Yes

☐ No

C.3 FINDING COEFFICIENTS OF FACTORS

Ex 31: The polynomial $P(x) = 2x^3 + x^2 - 4x + 1$ can be written in the form $(x - 1)(ax^2 + bx + c)$, where a, b, and c are constants. Determine the values of a, b, and c.

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

Ex 32: The polynomial $P(x) = x^3 - x^2 - 5x + 2$ can be written in the form $(x + 2)(ax^2 + bx + c)$, where a, b, and c are constants. Determine the values of a, b, and c.

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

Ex 33: The polynomial $P(x) = 3x^3 - 11x^2 + 7x - 3$ can be written in the form $(x - 3)(ax^2 + bx + c)$, where a, b, and c are constants. Determine the values of a, b, and c.

$$a = \boxed{}, b = \boxed{}, c = \boxed{}$$

D THE REMAINDER AND FACTOR THEOREMS

D.1 APPLYING THE REMAINDER THEOREM

Ex 34: Use the Remainder Theorem to find the remainder when $2x^3 - 5x^2 + 3x + 7$ is divided by $x - 2$.

$$R(x) = \boxed{}$$

Ex 35: Use the Remainder theorem to find the remainder when $x^4 - 3x^3 + x - 4$ is divided by $x + 2$.

$$R(x) = \boxed{}$$

Ex 36: Use the Remainder Theorem to find the remainder when $x^3 - 2x^2 - 5x + 8$ is divided by $x + 1$.

$$R(x) = \boxed{}$$

D.2 VERIFYING DIVISIBILITY

MCQ 37: Is $(x-1)$ a factor of the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$?

☐ Yes

☐ No

MCQ 38: Is $(x+2)$ a factor of the polynomial $P(x) = x^3 + x^2 - 4x - 4$?

☐ Yes

☐ No

MCQ 39: Is $(x-3)$ a factor of the polynomial $P(x) = 2x^3 - 5x^2 - 4x + 5$?

☐ Yes

☐ No

D.3 FINDING UNKNOWN COEFFICIENTS

Ex 40: When the polynomial $P(x) = 2x^3 + ax^2 - 5x + 1$ is divided by $x+1$, the remainder is 7. Find the value of a.

$$a = \boxed{}$$

Ex 41: When the polynomial $P(x) = x^3 + 2x^2 + ax - 8$ is divided by $x - 3$, the remainder is 10. Find the value of a.

$$a = \boxed{}$$

Ex 42: Given that $(x+5)$ is a factor of $P(x) = x^3 + ax^2 - 11x + 30$, find the value of a.

$$a = \boxed{}$$

D.4 FACTORISING POLYNOMIALS GIVEN A FACTOR

Ex 43: Consider the polynomial $P(x) = x^3 + kx^2 - 3x + 6$.

1. Find the value of k given that $(x-2)$ is a factor of $P(x)$.

$$k = \boxed{}$$

2. Hence, fully factorise $P(x)$.

$$P(x) = \boxed{}$$

Ex 44: Consider the polynomial $P(x) = x^3 - 2x^2 + kx + 6$.

1. Find the value of k given that $(x+2)$ is a factor of $P(x)$.

$$k = \boxed{}$$

2. Hence, fully factorise $P(x)$.

$$P(x) = \boxed{}$$

E QUADRATIC EQUATIONS WITH COMPLEX ROOTS

E.1 SOLVING QUADRATIC EQUATIONS

Ex 45: Solve the equation $z^2 + 1 = 0$ for real numbers and for complex numbers.

Ex 46: Solve the equation $z^2 + 2 = 0$ for real numbers and for complex numbers.

Ex 47: Solve the equation $z^2 - 4z + 5 = 0$ for complex numbers.

Ex 48: Solve the equation $z^2 + 2z + 2 = 0$ for complex numbers.

E.2 FACTORING POLYNOMIALS

Ex 49: Let $P(x) = x^3 - 4x^2 - 7x + 10$.

1. Show that $(x - 1)$ is a factor of $P(x)$.
2. Hence, fully factorise $P(x)$ into a product of three linear factors.

Ex 50: Let $P(x) = x^3 - x^2 + x - 1$.

1. Show that $(x - 1)$ is a factor of $P(x)$.
2. Hence, fully factorise $P(x)$ into a product of linear factors over the complex numbers.

Ex 51: Let $P(x) = x^3 - 2x + 4$.

1. Show that $(x + 2)$ is a factor of $P(x)$.
2. Hence, fully factorise $P(x)$ into a product of linear factors over the complex numbers.

F THE FUNDAMENTAL THEOREM OF ALGEBRA

F.1 APPLYING THE CONJUGATE ROOT THEOREM

Ex 52: A polynomial $P(x)$ has real coefficients.
Given that $r_1 = 1 + i$ is a root of the equation $P(x) = 0$, find another root.

Another root is: ☐

Ex 53: A polynomial $P(x)$ has real coefficients.Given that $r_1 = -2 + 3i$ is a root of the equation $P(x) = 0$, find another root.

Another root is: $\boxed{}$

Ex 54: A polynomial $P(x)$ has real coefficients.Given that $r_1 = 5i$ is a root of the equation $P(x) = 0$, find another root.

Another root is: $\boxed{}$

G SUM AND PRODUCT OF ROOTS THEOREM

G.1 APPLYING VIETA'S FORMULAS

Ex 55: For the equation $3x^2 + 6x - 8 = 0$, find:

1. the sum of the roots. $\boxed{}$
2. the product of the roots. $\boxed{}$

Ex 56: For the equation $x^4 - 5x^3 + 2x - 1 = 0$, find:

1. the sum of the roots. $\boxed{}$
2. the product of the roots. $\boxed{}$

Ex 57: For the equation $2x^3 - 5x^2 + 7 = 0$, find:

1. the sum of the roots. $\boxed{}$
2. the product of the roots. $\boxed{}$

Ex 58: For the equation $-x^5 + 2x^4 - 3x^3 + x - 10 = 0$, find:

1. the sum of the roots. $\boxed{}$
2. the product of the roots. $\boxed{}$

G.2 FINDING ALL ROOTS OF A POLYNOMIAL

Ex 59: Given that $r_1 = 1 + i$ is a root of the polynomial $P(x) = x^3 - 4x^2 + 6x - 4$, find the remaining roots.

Ex 60: Given that $r_1 = 2 - i$ is a root of the polynomial $P(x) = x^3 - 3x^2 + x + 5$, find the remaining roots.

Ex 61: Consider the quartic polynomial $P(x) = x^4 - 6x^3 + 18x^2 - 30x + 25$.

1. It is given that $z_1 = 1 - 2i$ is a root of the equation $P(x) = 0$.
   (a) Since $P(x)$ has real coefficients, write down another complex root, z_2.
   (b) Find a real quadratic factor of $P(x)$ corresponding to the roots z_1 and z_2.
2. Hence, find the other two roots of the equation $P(x) = 0$.
3. Using the four roots of $P(x) = 0$, verify the product of the roots using Vieta's formulas.

Ex 62: Consider the quartic polynomial $P(x) = x^4 - 7x^3 + 14x^2 + 2x - 20$.

1. It is given that $z_1 = 3+i$ is a root of the equation $P(x) = 0$.
   (a) Since $P(x)$ has real coefficients, write down another complex root, z_2.
   (b) Find a real quadratic factor of $P(x)$ corresponding to the roots z_1 and z_2.
2. Hence, find the other two roots of the equation $P(x) = 0$.
3. Using the four roots of $P(x) = 0$, verify the sum of the roots using Vieta's formulas.