RATIOS

A DEFINITION

A.1 EXPRESSING RATIOS IN DIFFERENT FORMS

Ex 1: The ratio 3 to 2 is $\boxed{3}$: $\boxed{2}$.

Solution: The ratio 3 to 2 can be expressed as $3 : 2$ or $\dfrac{3}{2}$.

Ex 2: The ratio 5 to 4 is $\boxed{5}$: $\boxed{4}$.

Solution: The ratio 5 to 4 can be expressed as $5 : 4$ or $\dfrac{5}{4}$.

Ex 3: The ratio 7 to 3 is $\boxed{7}$: $\boxed{3}$.

Solution: The ratio 7 to 3 can be expressed as $7 : 3$ or $\dfrac{7}{3}$.

Ex 4: The ratio 8 to 5 is $\boxed{8}$: $\boxed{5}$.

Solution: The ratio 8 to 5 can be expressed as $8 : 5$ or $\dfrac{8}{5}$.

Ex 5: The ratio 10 to 6 is $\boxed{10}$: $\boxed{6}$.

Solution: The ratio 10 to 6 can be expressed as $10 : 6$ or $\dfrac{10}{6}$.

B PART-PART AND PART-WHOLE RATIOS
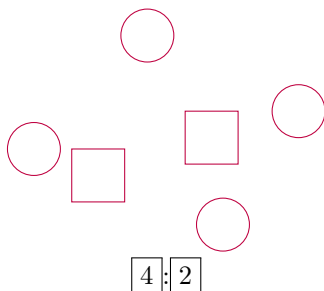
B.1 FINDING RATIOS IN PART-PART

Ex 6: What is the ratio of girls to boys?

$\boxed{2}$: $\boxed{3}$

Solution:

- There are 2 girls.
- There are 3 boys.
- The ratio of girls to boys is $2 : 3$ or $\dfrac{2}{3}$.

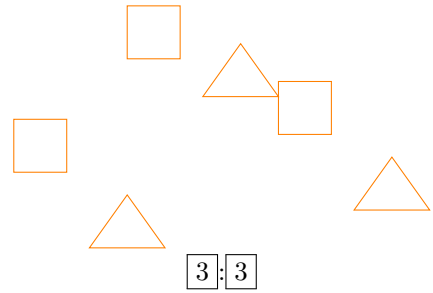**Ex 7:** What is the ratio of circles to rectangles?

$\boxed{4}$: $\boxed{2}$

Solution:

- There are 4 circles.
- There are 2 rectangles.

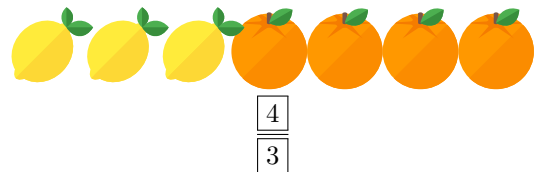- The ratio of circles to rectangles is $4 : 2$ or $\dfrac{4}{2}$.

Ex 8: What is the ratio of squares to triangles?

$\boxed{3}$: $\boxed{3}$

Solution:

- There are 3 squares.
- There are 3 triangles.
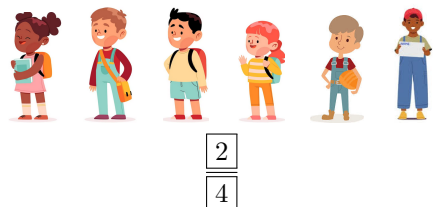- The ratio of squares to triangles is $3 : 3$ or $\dfrac{3}{3}$.

Ex 9: What is the ratio of oranges to lemons?

$\boxed{\dfrac{4}{3}}$

Solution:

- There are 4 oranges.
- There are 3 lemons.
- The ratio of oranges to lemons is $\dfrac{4}{3}$, or $4 : 3$.

Ex 10: What is the ratio of girls to boys?

$\boxed{\dfrac{2}{4}}$

Solution:

- There are 2 girls.
- There are 4 boys.
- The ratio of girls to boys is $\dfrac{2}{4}$, or $2 : 4$.

B.2 FINDING RATIOS IN PART-WHOLE

Ex 11: What is the ratio of girls to kids?

$\boxed{2} : \boxed{5}$

Solution:

- There are 2 girls.

- There are 5 kids.

- The ratio of girls to kids is $2 : 5$ or $\dfrac{2}{5}$.

Ex 12: What is the ratio of boys to kids?

$\boxed{1} : \boxed{4}$

Solution:

- There is 1 boy.

- There are 4 kids.

- The ratio of boys to kids is $1 : 4$ or $\dfrac{1}{4}$.

Ex 13: Louis loves to play sports. In all, he has earned 5 swimming medals, 3 running medals, 6 cycling medals, and 2 triathlon medals.
What is the ratio of Louis's swimming medals to all of his medals?

$\boxed{5} : \boxed{16}$

Solution:

- Louis has earned 5 swimming medals.

-
$$5 + 3 + 6 + 2 = 16$$

  Louis has earned 16 total medals.

- The ratio of swimming medals to all of his medals is $5 : 16$ or $\dfrac{5}{16}$.

Ex 14: Anna loves to read books. In all, she has read 12 mystery novels, 8 science fiction novels, 5 fantasy novels, and 3 historical novels.
What is the ratio of Anna's mystery novels to all of her books?

$\boxed{12} : \boxed{28}$

Solution:

- Anna has read 12 mystery novels.

-
$$12 + 8 + 5 + 3 = 28$$

  Anna has read 28 books in total.

- The ratio of mystery novels to all of her books is $12 : 28$ or $\dfrac{12}{28}$.

Ex 15: The table shows the number of different types of birds that are swimming at a lake.

Bird	Number
Seagulls	1
Ducks	9
Geese	7
Swans	2

What is the ratio of swans to total birds?

$\boxed{2} : \boxed{19}$

Solution:

- There are 2 swans.

- There are $1 + 9 + 7 + 2 = 19$ birds.

- The ratio of swans to total birds is $2 : 19$ or $\dfrac{2}{19}$.

Ex 16: The table shows the number of different types of fruits in a basket.

Fruit	Number
Apples	3
Oranges	5
Bananas	4
Grapes	6

What is the ratio of apples to total fruits?

$\boxed{3} : \boxed{18}$

Solution:

- There are 3 apples.

- There are $3 + 5 + 4 + 6 = 18$ fruits.

- The ratio of apples to total fruits is $3 : 18$ or $\dfrac{3}{18}$.

Ex 17: The table shows the number of different types of vehicles in a parking lot.

Vehicle	Number
Cars	10
Bicycles	6
Motorcycles	4
Trucks	2

What is the ratio of trucks to total vehicles?

$\boxed{2} : \boxed{22}$

Solution:

- There are 2 trucks.

- There are $10 + 6 + 4 + 2 = 22$ vehicles.

- The ratio of trucks to total vehicles is $2 : 22$ or $\dfrac{2}{22}$.

C EQUAL RATIOS

C.1 MULTIPLYING THE RATIOS

Ex 18: Multiply the ratio by 2:

$$3 : 5 = \boxed{6} : \boxed{10}$$

Solution:

- $$\frac{3}{5} \; \overset{\times 2}{\underset{\times 2}{=}} \; \frac{6}{10}$$

- $$\frac{3}{5} = \frac{3 \times 2}{5 \times 2}$$
  $$= \frac{6}{10}$$

- $3 : 5 = 6 : 10$.

Ex 19: Multiply the ratio by 3:

$$4 : 7 = \boxed{12} : \boxed{21}$$

Solution:

- $$\frac{4}{7} \; \overset{\times 3}{\underset{\times 3}{=}} \; \frac{12}{21}$$

- $$\frac{4}{7} = \frac{4 \times 3}{7 \times 3}$$
  $$= \frac{12}{21}$$

- $4 : 7 = 12 : 21$.

Ex 20: Multiply the ratio by 4:

$$5 : 3 = \boxed{20} : \boxed{12}$$

Solution:

- $$\frac{5}{3} \; \overset{\times 4}{\underset{\times 4}{=}} \; \frac{20}{12}$$

- $$\frac{5}{3} = \frac{5 \times 4}{3 \times 4}$$
  $$= \frac{20}{12}$$

- $5 : 3 = 20 : 12$.

Ex 21: Multiply the ratio by 5:

$$2 : 5 = \boxed{10} : \boxed{25}$$

Solution:

- $$\frac{2}{5} \; \overset{\times 5}{\underset{\times 5}{=}} \; \frac{10}{25}$$

- $$\frac{2}{5} = \frac{2 \times 5}{5 \times 5}$$
  $$= \frac{10}{25}$$

- $2 : 5 = 10 : 25$.

C.2 FINDING THE MISSING VALUE

Ex 22:

$$1 : 2 = 2 : \boxed{4}$$

Solution:

- $$\frac{1}{2} \; \overset{\times 2}{\underset{\times 2}{=}} \; \frac{2}{4}$$

- $$\frac{1}{2} = \frac{1 \times 2}{2 \times 2}$$
  $$= \frac{2}{4}$$

- $1 : 2 = 2 : 4$.

Ex 23:

$$2 : 3 = \boxed{4} : 6$$

Solution:

- $$\frac{2}{3} \; \overset{\times 2}{\underset{\times 2}{=}} \; \frac{4}{6}$$

- $$\frac{2}{3} = \frac{2 \times 2}{3 \times 2}$$
  $$= \frac{4}{6}$$

- $2 : 3 = 4 : 6$.

Ex 24:

$$3 : 5 = 9 : \boxed{15}$$

Solution:

- $$\frac{3}{5} \; \overset{\times 3}{\underset{\times 3}{=}} \; \frac{9}{15}$$

- $$\frac{3}{5} = \frac{3 \times 3}{5 \times 3}$$
  $$= \frac{9}{15}$$

- $3 : 5 = 9 : 15$.

Ex 25:

$$4 : 7 = \boxed{8} : 14$$

Solution:

- $$\frac{4}{7} \overset{\times 2}{\underset{\times 2}{=}} \frac{8}{14}$$

- $$\frac{4}{7} = \frac{4 \times 2}{7 \times 2}$$
$$= \frac{8}{14}$$

- $4 : 7 = 8 : 14$.

Ex 26:

$$2 : 3 = 8 : \boxed{12}$$

Solution:

- $$\frac{2}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{8}{12}$$

- $$\frac{2}{3} = \frac{2 \times 4}{3 \times 4}$$
$$= \frac{8}{12}$$

- $2 : 3 = 8 : 12$.

Ex 27:

$$3 : 2 = \boxed{30} : 20$$

Solution:

- $$\frac{3}{2} \overset{\times 10}{\underset{\times 10}{=}} \frac{30}{20}$$

- $$\frac{3}{2} = \frac{3 \times 10}{2 \times 10}$$
$$= \frac{30}{20}$$

- $3 : 2 = 30 : 20$.

C.3 SIMPLIFYING RATIOS

Ex 28:

$$2 : 4 = 1 : \boxed{2}$$

Solution:

- $$\frac{2}{4} \overset{\div 2}{\underset{\div 2}{=}} \frac{1}{2}$$

- $$\frac{2}{4} = \frac{1 \times \cancel{2}}{2 \times \cancel{2}}$$
$$= \frac{1}{2}$$

- $2 : 4 = 1 : 2$

Ex 29:

$$4 : 6 = 2 : \boxed{3}$$

Solution:

- $$\frac{4}{6} \overset{\div 2}{\underset{\div 2}{=}} \frac{2}{3}$$

- $$\frac{4}{6} = \frac{2 \times \cancel{2}}{3 \times \cancel{2}}$$
$$= \frac{2}{3}$$

- $4 : 6 = 2 : 3$

Ex 30:

$$5 : 10 = 1 : \boxed{2}$$

Solution:

- $$\frac{5}{10} \overset{\div 5}{\underset{\div 5}{=}} \frac{1}{2}$$

- $$\frac{5}{10} = \frac{1 \times \cancel{5}}{2 \times \cancel{5}}$$
$$= \frac{1}{2}$$

- $5 : 10 = 1 : 2$

Ex 31:

$$14 : 7 = \boxed{2} : 1$$

Solution:

- $$\frac{14}{7} \xrightarrow{\div 7} \frac{2}{1}$$ (with $\div 7$ below)

-
$$\frac{14}{7} = \frac{2 \times 7}{1 \times 7}$$
$$= \frac{2}{1}$$

- $14 : 7 = 2 : 1$

Ex 32:

$$15 : 9 = \boxed{5} : 3$$

Solution:

- $$\frac{15}{9} \xrightarrow{\div 3} \frac{5}{3}$$ (with $\div 3$ below)

-
$$\frac{15}{9} = \frac{5 \times 3}{3 \times 3}$$
$$= \frac{5}{3}$$

- $15 : 9 = 5 : 3$

Ex 33:

$$18 : 12 = \boxed{3} : 2$$

Solution:

- $$\frac{18}{12} \xrightarrow{\div 6} \frac{3}{2}$$ (with $\div 6$ below)

-
$$\frac{18}{12} = \frac{3 \times 6}{2 \times 6}$$
$$= \frac{3}{2}$$

- $18 : 12 = 3 : 2$

C.4 FINDING EQUAL RATIO

MCQ 34: Select one ratio that is equal $1 : 2$.

- ☐ $1 : 3$
- ☒ $2 : 4$
- ☐ $4 : 2$
- ☐ $3 : 4$

Solution:

- $$\frac{1}{2} \xrightarrow{\times 2} \frac{2}{4}$$ (with $\times 2$ below)

-
$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2}$$
$$= \frac{2}{4}$$

- $1 : 2 = 2 : 4$.

MCQ 35: Select one ratio that is equal to $3 : 2$.

- ☐ $2 : 3$
- ☐ $4 : 3$
- ☐ $3 : 4$
- ☒ $6 : 4$

Solution:

- $$\frac{3}{2} \xrightarrow{\times 2} \frac{6}{4}$$ (with $\times 2$ below)

-
$$\frac{3}{2} = \frac{3 \times 2}{2 \times 2}$$
$$= \frac{6}{4}$$

- $3 : 2 = 6 : 4$.

MCQ 36: Select one ratio that is equal to $4 : 3$.

- ☐ $3 : 4$
- ☐ $9 : 6$
- ☒ $12 : 9$
- ☐ $4 : 9$

Solution:

- $$\frac{4}{3} \xrightarrow{\times 3} \frac{12}{9}$$ (with $\times 3$ below)

-
$$\frac{4}{3} = \frac{4 \times 3}{3 \times 3}$$
$$= \frac{12}{9}$$

- $4 : 3 = 12 : 9$.

MCQ 37: Select one ratio that is equal to $3 : 4$.

- ☒ $75 : 100$
- ☐ $4 : 3$
- ☐ $9 : 12$
- ☐ $30 : 50$

Solution:

- $$\frac{3}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{75}{100}$$

- $$\frac{3}{4} = \frac{3 \times 25}{4 \times 25}$$
$$= \frac{75}{100}$$

- $3 : 4 = 75 : 100$.

D PROPORTION

D.1 IDENTIFYING THE PROPORTION

MCQ 38: Two vinaigrettes are being prepared:

- Vinaigrette A is made with 20 mL of oil and 30 mL of vinegar.

- Vinaigrette B is made with 10 mL of oil and 15 mL of vinegar.

Will these two vinaigrettes taste the same?

☒ Yes

☐ No

Solution:

- For Vinaigrette A:

$$\frac{20}{30} \overset{\div 10}{\underset{\div 10}{=}} \frac{2}{3}$$

- For Vinaigrette B:

$$\frac{10}{15} \overset{\div 5}{\underset{\div 5}{=}} \frac{2}{3}$$

Since both ratios are equal, there is a proportion. So, the vinaigrettes will taste the same.

MCQ 39: On the cement package, it is indicated: 2 kilos of cement for 3 liters of water.
A worker prepares a mixture with 4 kilos of cement and 6 liters of water.
Did he follow the recommended proportions?

☒ Yes

☐ No

Solution:

- $$\frac{4}{6} \overset{\div 2}{\underset{\div 2}{=}} \frac{2}{3}$$

- Since the two ratios are equal, the worker followed the recommended proportions.

MCQ 40: In architecture, the golden ratio is often used to create aesthetically pleasing designs. The golden ratio is approximately 1 : 1.618.
An architect designs a rectangle with a length of 3.236 m and a width of 2 m.
Did the architect use the golden ratio in his design (you can use a calculator)?

☒ Yes

☐ No

Solution:

- $$\frac{3.236}{2} \overset{\div 2}{\underset{\div 2}{=}} \frac{1.618}{1}$$

- Since the ratio is equal to the golden ratio, the architect used the golden ratio in his design.

MCQ 41: In a cake recipe, the ratio of flour to sugar is 3 : 2. If a baker uses 9 cups of flour, how many cups of sugar should they use to keep the recipe's proportions?

☐ 4 cups

☐ 5 cups

☒ 6 cups

☐ 7 cups

Solution:

- $$\frac{3}{2} \overset{\times 3}{\underset{\times 3}{=}} \frac{9}{6}$$

- Since the ratio 9 : 6 is equal to 3 : 2, the baker should use 6 cups of sugar.

D.2 FINDING A QUANTITY

Ex 42: In the class, there are 20 girls for 10 boys.
For each boy, there are $\boxed{2}$ girls.

Solution:

- $$\frac{20}{10} \overset{\div 10}{\underset{\div 10}{=}} \frac{2}{1}$$

- $$\frac{20}{10} = \frac{2 \times 10}{1 \times 10}$$
$$= \frac{2}{1}$$

- Since $20 : 10 = 2 : 1$, for 1 boy, there are 2 girls.

Ex 43: To make orange juice, you need 2 oranges and 1 lemon. Su has 14 oranges. How many lemons does she need?

$\boxed{7}$ lemons.

Solution:

- $$\xrightarrow{\times 7}$$
  $$\frac{2}{1} = \frac{14}{7}$$
  $$\xleftarrow{\times 7}$$

- $$\frac{2}{1} = \frac{2 \times 7}{1 \times 7}$$
  $$= \frac{14}{7}$$

- Since $2 : 1 = 14 : 7$, for 14 oranges, Su needs 7 lemons.

Ex 44: In a library, there are 30 fiction books for 15 non-fiction books.
For each non-fiction book, there are $\boxed{2}$ fiction books.

Solution:

- $$\xrightarrow{\div 15}$$
  $$\frac{30}{15} = \frac{2}{1}$$
  $$\xleftarrow{\div 15}$$

- $$\frac{30}{15} = \frac{2 \times \cancel{15}}{1 \times \cancel{15}}$$
  $$= \frac{2}{1}$$

- Since $30 : 15 = 2 : 1$, for 1 non-fiction book, there are 2 fiction books.

E UNITARY METHOD

E.1 BRINGING TO THE UNIT

Ex 45: A satellite makes 4 orbits around the Earth in 24 hours. How many hours does it take to complete one orbit?

$\boxed{6}$ hours

Solution: For 4 orbits, it takes 24 hours. Therefore, to maintain this proportion for 1 orbit, we divide both the number of orbits and the number of hours by 4:

$$\xrightarrow{\div 4}$$
$$\frac{24}{4} = \frac{6}{1}$$
$$\xleftarrow{\div 4}$$

Thus, to make 1 orbit, it takes 6 hours.

Ex 46: A car travels 500 kilometers in 5 hours. How many kilometers does it travel in 1 hour?

$\boxed{100}$ kilometers

Solution: For 5 hours, the car travels 500 kilometers. Therefore, to maintain this proportion for 1 hour, we divide both the number of kilometers and the number of hours by 5:

$$\xrightarrow{\div 5}$$
$$\frac{500}{5} = \frac{100}{1}$$
$$\xleftarrow{\div 5}$$

Thus, to travel 1 hour, the car covers 100 kilometers.

Ex 47: A factory produces 720 widgets in 8 hours. How many widgets does it produce in 1 hour?

$\boxed{90}$ widgets

Solution: For 8 hours, the factory produces 720 widgets. Therefore, to maintain this proportion for 1 hour, we divide both the number of widgets and the number of hours by 8:

$$\xrightarrow{\div 8}$$
$$\frac{720}{8} = \frac{90}{1}$$
$$\xleftarrow{\div 8}$$

Thus, in 1 hour, the factory produces 90 widgets.

Ex 48: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 1 loaf of bread?

$\boxed{0.5}$ kilograms

Solution: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:

$$\xrightarrow{\div 4}$$
$$\frac{2}{4} = \frac{0.5}{1}$$
$$\xleftarrow{\div 4}$$

Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

E.2 CALCULATING FROM THE UNIT

Ex 49: To make 1 chocolate cake, 4 eggs are needed. How many eggs are needed to make 2 cakes?

$\boxed{8}$ eggs

Solution: For 1 cake, it takes 4 eggs. Therefore, to maintain this proportion for 2 cakes, you multiply both the number of cakes and the number of eggs by 2:

$$\xrightarrow{\times 2}$$
$$\frac{4}{1} = \frac{8}{2}$$
$$\xleftarrow{\times 2}$$

Thus, to make 2 chocolate cakes, you need 8 eggs.

Ex 50: The price of 1 kilogram of apples is \$2.5. What is the price for 3 kilograms of apples?

\$ $\boxed{7.5}$

Solution: The price for 1 kilogram of apples is \$2.5. Therefore, to maintain this proportion for 3 kilograms, you multiply the price by 3:

$$\overset{\times 3}{\frac{2.5}{1} = \frac{7.5}{3}}$$
$$\underset{\times 3}{}$$

Thus, the price for 3 kilograms of apples is \$7.5.

Ex 51: To build 1 bookshelf, 10 wooden planks are needed. How many wooden planks are needed to build 3 bookshelves?

$$\boxed{30} \text{ wooden planks}$$

Solution: For 1 bookshelf, it takes 10 wooden planks. Therefore, to maintain this proportion for 3 bookshelves, you multiply both the number of bookshelves and the number of wooden planks by 3:

$$\overset{\times 3}{\frac{10}{1} = \frac{30}{3}}$$
$$\underset{\times 3}{}$$

Thus, to build 3 bookshelves, you need 30 wooden planks.

Ex 52: To paint 1 m², 0.2 liters of paint are needed. How many liters of paint are needed to paint 3 m²?

$$\boxed{0.6} \text{ liters of paint}$$

Solution: For 1 m², it takes 0.2 liters of paint. Therefore, to maintain this proportion for 3 m², you multiply both the area and the number of liters of paint by 3:

$$\overset{\times 3}{\frac{0.2}{1} = \frac{0.6}{3}}$$
$$\underset{\times 3}{}$$

Thus, to paint 3 m², you need 0.6 liters of paint.

E.3 CONVERTING TO AND FROM THE UNIT

Ex 53: To make a special juice mix, you need 5 apples for every 15 oranges. How many oranges do you need if you have 3 apples?

$$\boxed{9} \text{ oranges}$$

Solution:

- **To the unit**: For 5 apples, you need 15 oranges. Therefore, to maintain this proportion for 1 apple, we divide both the number of oranges and the number of apples by 5:

$$\overset{\div 5}{\frac{15}{5} = \frac{3}{1}}$$
$$\underset{\div 5}{}$$

  Thus, for 1 apple, you need 3 oranges.

- **From the unit**: For 1 apple, you need 3 oranges. Therefore, to maintain this proportion for 3 apples, we multiply both the number of oranges and the number of apples by 3:

$$\overset{\times 3}{\frac{3}{1} = \frac{9}{3}}$$
$$\underset{\times 3}{}$$

Thus, for 3 apples, you need 9 oranges.

$$\overset{\div 5 \quad \times 3}{\frac{15}{5} = \frac{3}{1} = \frac{9}{3}}$$
$$\underset{\div 5 \quad \times 3}{}$$

Ex 54: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 3 loaf of bread?

$$\boxed{1.5} \text{ kilograms}$$

Solution:

- **To the unit**: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:

$$\overset{\div 4}{\frac{2}{4} = \frac{0.5}{1}}$$
$$\underset{\div 4}{}$$

  Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

- **From the unit**: For 1 loaves, the baker uses 0.5 kilograms of flour. Therefore, to maintain this proportion for 3 loaf, we multiply both the number of kilograms and the number of loaves by 3:

$$\overset{\times 3}{\frac{0.5}{1} = \frac{1.5}{3}}$$
$$\underset{\times 3}{}$$

  Thus, to make 3 loaf of bread, it takes 1.5 kilograms of flour.

$$\overset{\div 4 \quad \times 3}{\frac{2}{4} = \frac{0.5}{1} = \frac{1.5}{3}}$$
$$\underset{\div 4 \quad \times 3}{}$$

Ex 55: An artist mixes 3 liters of red paint with 6 liters of blue paint to create a purple shade. How many liters of red paint are needed to mix with 9 liters of blue paint to maintain the same shade of purple?

$$\boxed{4.5} \text{ liters}$$

Solution:

- **To the unit**: For 6 liters of blue paint, the artist uses 3 liters of red paint. Therefore, to maintain this proportion for 1 liter of blue paint, we divide both the number of liters of red paint and blue paint by 6:

$$\overset{\div 6}{\frac{3}{6} = \frac{0.5}{1}}$$
$$\underset{\div 6}{}$$

Thus, to mix with 1 liter of blue paint, it takes 0.5 liters of red paint.

- **From the unit**: For 1 liter of blue paint, the artist uses 0.5 liters of red paint. Therefore, to maintain this proportion for 9 liters of blue paint, we multiply both the number of liters of red paint and blue paint by 9:

$$\overset{\times 9}{\underset{\times 9}{\frac{0.5}{1} = \frac{4.5}{9}}}$$

Thus, to mix with 9 liters of blue paint, it takes 4.5 liters of red paint.

$$\bullet \quad \overset{\div 6}{\underset{\div 6}{\frac{3}{6}}} = \overset{\times 9}{\underset{\times 9}{\frac{0.5}{1}}} = \frac{4.5}{9}$$

Ex 56: To make a magic potion, you need 10 drops of dragon's blood for every 5 drops of phoenix tears. How many drops of phoenix tears do you need if you have 4 drops of dragon's blood?

$$\boxed{2} \text{ drops of phoenix tears}$$

Solution:

- **To the unit**: For 10 drops of dragon's blood, you need 5 drops of phoenix tears. Therefore, to maintain this proportion for 1 drop of dragon's blood, we divide both the number of drops of phoenix tears and the number of drops of dragon's blood by 10:

$$\overset{\div 10}{\underset{\div 10}{\frac{5}{10} = \frac{0.5}{1}}}$$

Thus, for 1 drop of dragon's blood, you need 0.5 drops of phoenix tears.

- **From the unit**: For 1 drop of dragon's blood, you need 0.5 drops of phoenix tears. Therefore, to maintain this proportion for 4 drops of dragon's blood, we multiply both the number of drops of phoenix tears and the number of drops of dragon's blood by 4:

$$\overset{\times 4}{\underset{\times 4}{\frac{0.5}{1} = \frac{2}{4}}}$$

Thus, for 4 drops of dragon's blood, you need 2 drops of phoenix tears.

$$\bullet \quad \overset{\div 10}{\underset{\div 10}{\frac{5}{10}}} = \overset{\times 4}{\underset{\times 4}{\frac{0.5}{1}}} = \frac{2}{4}$$

E.4 SOLVING NUMERATOR

Ex 57:

$$\frac{6}{2} = \frac{x}{3}$$

$$x = \boxed{9}$$

Solution:

$$\overset{\div 2}{\underset{\div 2}{\frac{6}{2}}} = \overset{\times 3}{\underset{\times 3}{\frac{3}{1}}} = \frac{9}{3}$$
$$x = 9$$

Ex 58:

$$\frac{8}{4} = \frac{x}{5}$$

$$x = \boxed{10}$$

Solution:

$$\overset{\div 4}{\underset{\div 4}{\frac{8}{4}}} = \overset{\times 5}{\underset{\times 5}{\frac{2}{1}}} = \frac{10}{5}$$
$$x = 10$$

Ex 59:

$$\frac{3}{2} = \frac{x}{5}$$

$$x = \boxed{7.5}$$

Solution:

$$\overset{\div 2}{\underset{\div 2}{\frac{3}{2}}} = \overset{\times 5}{\underset{\times 5}{\frac{1.5}{1}}} = \frac{7.5}{5}$$
$$x = 7.5$$

Ex 60:

$$\frac{5}{4} = \frac{x}{3}$$

$$x = \boxed{3.75}$$

Solution:

$$\overset{\div 4}{\underset{\div 4}{\frac{5}{4}}} = \overset{\times 3}{\underset{\times 3}{\frac{1.25}{1}}} = \frac{3.75}{3}$$
$$x = 3.75$$

E.5 SOLVING DENOMINATOR

Ex 61:

$$\frac{5}{10} = \frac{2}{x}$$

$$x = \boxed{4}$$

Solution:

$$x = 4$$

Ex 62:

$$\frac{3}{6} = \frac{2}{x}$$

$$x = \boxed{4}$$

Solution:

$$x = 4$$

Ex 63:

$$\frac{4}{5} = \frac{3}{x}$$

$$x = \boxed{3.75}$$

Solution:

$$x = 3.75$$

Ex 64:

$$\frac{5}{2} = \frac{3}{x}$$

$$x = \boxed{1.2}$$

Solution:

$$x = 1.2$$

F CROSS-MULTIPLICATION METHOD

F.1 FINDING A QUANTITY

Ex 65: 5 apples cost 10 dollars.
Find the cost of 8 apples.

$$\boxed{16} \text{ dollars}$$

Solution:

- **Set up the proportion:** The cost of 5 apples is to 10 dollars and the cost of 8 apples is to x dollars:

$$\frac{10}{5} = \frac{x}{8}$$

- **Solve for x:**

$$\frac{10}{5} = \frac{x}{8}$$
$$5 \times x = 10 \times 8 \quad \text{(cross multiplication)}$$
$$x = \frac{10 \times 8}{5} \quad \text{(dividing both sides by 5)}$$
$$x = 16$$

- Therefore, 8 apples cost 16 dollars.

- Other method (unitary method):

Ex 66: 6 oranges cost 12 dollars.
Find the cost of 9 oranges.

$$\boxed{18} \text{ dollars}$$

Solution:

- **Set up the proportion:** The cost of 6 oranges is to 12 dollars as the cost of 9 oranges is to x dollars:

$$\frac{12}{6} = \frac{x}{9}$$

- **Solve for x:**

$$\frac{12}{6} = \frac{x}{9}$$
$$6 \times x = 12 \times 9 \quad \text{(cross multiplication)}$$
$$x = \frac{12 \times 9}{6} \quad \text{(dividing both sides by 6)}$$
$$x = 18$$

- Therefore, 9 oranges cost 18 dollars.

- Other method (unitary method):

Ex 67: 6 oranges cost 12 dollars.
Find the cost of 9 oranges.

$$\boxed{18}\ \text{dollars}$$

Solution:

- **Set up the proportion:** The cost of 6 oranges is to 12 dollars as the cost of 9 oranges is to x dollars:

$$\frac{12}{6} = \frac{x}{9}$$

- **Solve for x:**

$$\frac{12}{6} = \frac{x}{9}$$
$$6 \times x = 12 \times 9 \quad \text{(cross multiplication)}$$
$$x = \frac{12 \times 9}{6} \quad \text{(dividing both sides by 6)}$$
$$x = 18$$

- Therefore, 9 oranges cost 18 dollars.

- Other method (unitary method):

Ex 68: A recipe requires 200 grams of flour to make 8 cookies. How much flour is needed to make 12 cookies (you can use a calculator)?

$$\boxed{300}\ \text{grams}$$

Solution:

- **Set up the proportion:** The amount of flour for 8 cookies is to 200 grams as the amount of flour for 12 cookies is to x grams:

$$\frac{200}{8} = \frac{x}{12}$$

- **Solve for x:**

$$\frac{200}{8} = \frac{x}{12}$$
$$8 \times x = 200 \times 12 \quad \text{(cross multiplication)}$$
$$x = \frac{200 \times 12}{8} \quad \text{(dividing both sides by 8)}$$
$$x = 300$$

- Therefore, 12 cookies require 300 grams of flour.

- Other method (unitary method):

Ex 69: To make a certain shade of paint, you need 1.5 liters of blue paint for every 3 liters of base paint.
How much blue paint is needed if you have 4.5 liters of base paint (you can use a calculator)?

$$\boxed{2.25}\ \text{liters}$$

Solution:

- **Set up the proportion:** The amount of blue paint for 3 liters of base paint is to 1.5 liters as the amount of blue paint for 4.5 liters of base paint is to x liters:

$$\frac{1.5}{3} = \frac{x}{4.5}$$

- **Solve for x:**

$$\frac{1.5}{3} = \frac{x}{4.5}$$
$$3 \times x = 1.5 \times 4.5 \quad \text{(cross multiplication)}$$
$$x = \frac{1.5 \times 4.5}{3} \quad \text{(dividing both sides by 3)}$$
$$x = 2.25$$

- Therefore, 4.5 liters of base paint require 2.25 liters of blue paint.

- Other method (unitary method):