A WHAT IS A RATIO?

A.1 EXPRESSING RATIOS IN DIFFERENT FORMS

Ex 1: The ratio of $\frac{3}{3}$ to $\frac{3}{2}$ is $\frac{3}{2}$ or $\frac{3}{2}$.

Answer: The ratio of 3 to 2 can be written as 3:2 or $\frac{3}{2}$.

Ex 2: The ratio of 4 to 5 is $\boxed{4}$: $\boxed{5}$ or $\boxed{\frac{4}{5}}$.

Answer: The ratio of 4 to 5 can be written as 4:5 or $\frac{4}{5}$.

Ex 3: The ratio of 7 to 3 is $\boxed{7}$: $\boxed{3}$ or $\boxed{\frac{7}{3}}$.

Answer: The ratio of 7 to 3 can be written as 7:3 or $\frac{7}{3}$.

Ex 4: The ratio of 6 to 9 is $\boxed{6}$ $\boxed{9}$ or $\boxed{\frac{6}{9}}$.

Answer: The ratio of 6 to 9 can be written as 6:9 or $\frac{6}{9}$.

Ex 5: What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There is 1 boy.
- The ratio of girls to boys is 2:1.

A.2 FINDING RATIOS IN PART-PART RELATIONSHIPS

Ex 6: What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There is 1 boy.
- The ratio of girls to boys is 2:1.

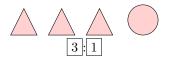
Ex 7: What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- \bullet There are 3 boys.
- The ratio of girls to boys is 2:3.

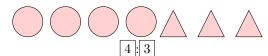
Ex 8: What is the ratio of triangles to circles?



Answer:

- There are 3 triangles.
- There is 1 circle.
- The ratio of triangles to circles is 3:1.

Ex 9: What is the ratio of circles to triangles?



Answer:

- There are 4 circles.
- There are 3 triangles.
- The ratio of circles to triangles is 4:3.

A.3 FINDING RATIOS IN WHOLE-PART RELATIONSHIPS

Ex 10: What is the ratio of girls to kids?



Answer:

- There are 2 girls.
- There are 3 kids.
- The ratio of girls to kids is $\frac{2}{3}$.

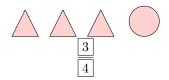
Ex 11: What is the ratio of girls to kids?



Answer:

- There are 2 girls.
- There are 5 kids.
- The ratio of girls to kids is $\frac{2}{5}$.

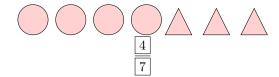
Ex 12: What is the ratio of triangles to shapes?



Answer:

- There are 3 triangles.
- There are 4 shapes.
- The ratio of triangles to shapes is $\frac{3}{4}$.

Ex 13: What is the ratio of circles to shapes?



Answer:

- There are 4 circles.
- There are 7 shapes.
- The ratio of circles to shapes is $\frac{4}{7}$.

Ex 14: Anna loves to read books. In all, she has read 12 mystery novels, 8 science fiction novels, 5 fantasy novels, and 3 historical novels

What is the ratio of Anna's mystery novels to all of her books?



Answer:

- Anna has read 12 mystery novels.
- The total number of books is 12 + 8 + 5 + 3 = 28.
- The ratio of mystery novels to all books is $\frac{12}{28}$ or $\frac{12}{28}$

Ex 15: The table shows the number of different types of birds swimming at a lake.

Bird	Number
Seagulls	1
Ducks	9
Geese	7
Swans	2

What is the ratio of swans to total birds?

Answer:

- There are 2 swans.
- The total number of birds is 1+9+7+2=19.
- The ratio of swans to total birds is 2:19 or $\frac{2}{19}$.

Ex 16: The table shows the number of different types of fruits in a basket.

Fruit	Number
Apples	3
Oranges	5
Bananas	4
Grapes	6

What is the ratio of apples to total fruits?

Answer:

- There are 3 apples.
- The total number of fruits is 3 + 5 + 4 + 6 = 18.
- The ratio of apples to total fruits is 3:18 or $\frac{3}{18}$.

Ex 17: The table shows the number of different types of vehicles in a parking lot.

Vehicle	Number
Cars	10
Bicycles	6
Motorcycles	4
Trucks	2

What is the ratio of trucks to total vehicles?

Answer:

- There are 2 trucks.
- The total number of vehicles is 10 + 6 + 4 + 2 = 22.
- The ratio of trucks to total vehicles is 2:22 or $\frac{2}{22}$.

Ex 18: Louis loves to play sports. In all, he has earned 5 swimming medals, 3 running medals, 6 cycling medals, and 2 triathlon medals.

What is the ratio of Louis's swimming medals to all of his medals?



Answer:

- Louis has earned 5 swimming medals.
- The total number of medals is 5+3+6+2=16.
- The ratio of swimming medals to all medals is 5:16 or $\frac{5}{16}$.



B EQUIVALENT RATIOS

B.1 SIMPLIFYING RATIOS

Ex 19: What is the ratio of red apples to all apples (write in simplified form)?



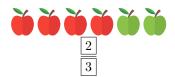
Answer:

- There are 2 red apples.
- There are 4 apples in total.
- The ratio of red apples to all apples is $\frac{2}{4}$.



• The simplified ratio is $\frac{1}{2}$ (half are red).

Ex 20: What is the ratio of red apples to all apples (write in simplified form)?



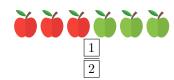
Answer:

- There are 4 red apples.
- There are 6 apples in total.
- The ratio of red apples to all apples is $\frac{4}{6}$.



• The simplified ratio is $\frac{2}{3}$.

Ex 21: What is the ratio of red apples to all apples (write in simplified form)?



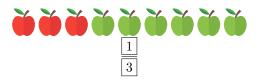
Answer:

- There are 3 red apples.
- \bullet There are 6 apples in total.
- The ratio of red apples to all apples is $\frac{3}{6}$.



• The simplified ratio is $\frac{1}{2}$ (half are red).

Ex 22: What is the ratio of red apples to all apples (write in simplified form)?



Answer:

- There are 3 red apples.
- There are 9 apples in total.
- The ratio of red apples to all apples is $\frac{3}{9}$.



• The simplified ratio is $\frac{1}{3}$.

B.2 MULTIPLYING THE RATIOS

Ex 23: Multiply the ratio by 2:

$$3:5=\boxed{6}:\boxed{10}$$

Answer:

$$\bullet \ \frac{3}{5} = \frac{6}{10}$$

 $\frac{\frac{3}{5}}{\frac{3}{5}} = \frac{\frac{3}{5} \times \frac{3}{5}}{\frac{6}{10}}$ $= \frac{\frac{6}{10}}{\frac{10}{5}}$

• 3:5=6:10.

Ex 24: Multiply the ratio by 3:

$$4:7 = \boxed{12}: \boxed{21}$$

Answer:



 $\frac{4}{7} = \frac{4 \times 7}{7 \times 7}$ $= \frac{12}{12}$

• 4:7=12:21.

Ex 25: Multiply the ratio by 4:

$$5:3=20:12$$

Answer:

$$\bullet \ \frac{5}{3} = \frac{20}{12}$$

•

$$\frac{5}{3} = \frac{5 \times 4}{3 \times 4}$$
$$= \frac{20}{12}$$

• 5:3=20:12.

Ex 26: Multiply the ratio by 5:

$$2:5=\boxed{10}$$
: $\boxed{25}$

Answer:



•

$$\frac{2}{5} = \frac{2 \times 5}{5 \times 5}$$
$$= \frac{10}{25}$$

• 2:5=10:25.

B.3 FINDING THE MISSING VALUE

Ex 27:

$$1:2=2:\boxed{4}$$

Answer:



•

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2}$$
$$= \frac{2}{4}$$

• 1:2=2:4.

Ex 28:

$$2:3=4:6$$

Answer:

$$\bullet \ \frac{2}{3} = \frac{4}{6}$$

•

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5}$$
$$= \frac{4}{6}$$

• 2:3=4:6.

Ex 29:

$$3:5=9:15$$

Answer:



•

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3}$$
$$= \frac{9}{15}$$

• 3:5=9:15.

Ex 30:

$$4:7 = 8:14$$

Answer:



 $\frac{4}{7} =$

• 4:7=8:14.

Ex 31:

$$2:3=8:\boxed{12}$$

Answer:



 $\frac{2}{3} = \frac{2}{3}$ $= \frac{8}{3}$

• 2:3=8:12.

Ex 32:

$$3:2=30:20$$

Answer:



$$\frac{3}{2} = \frac{3 \times 10}{2 \times 10} = \frac{30}{20}$$

• 3:2=30:20.

C PROPORTION

C.1 IDENTIFYING THE PROPORTION

MCQ 33: Two vinaigrettes are being prepared:

- Vinaigrette A is made with 2 mL of oil and 1 mL of vinegar.
- Vinaigrette B is made with 4 mL of oil and 2 mL of vinegar.

Will these two vinaigrettes taste the same?

⊠ Yes

 \square No

Answer:



• Since both ratios are equal, there is a proportion. So, the vinaigrettes will taste the same.

MCQ 34: On the cement package, it is indicated: 2 kilos of cement for 3 liters of water.

A worker prepares a mixture with 4 kilos of cement and 6 liters of water.

Did he follow the recommended proportions?

⊠ Yes

 \square No

Answer:

$$\bullet \ \frac{2}{3} = \frac{4}{6}$$

• Since the two ratios are equal, the worker followed the recommended proportions.

MCQ 35: Two smoothie recipes are being prepared:

- Smoothie A is made with 3 cups of fruit and 2 cups of yogurt.
- Smoothie B is made with 6 cups of fruit and 4 cups of yogurt.

Will these two smoothies taste the same?

□ No

Answer:



• Since both ratios are equal, the proportions are the same. Therefore, the smoothies will taste the same.

MCQ 36: A gardener uses a fertilizer mix:

- The recommended mix is 5 grams of fertilizer per 2 liters of water.
- The gardener prepares a mixture with 10 grams of fertilizer and 4 liters of water.

Did the gardener follow the recommended proportions?

⊠ Yes

□ No

Answer:



• Since both ratios are equal, the gardener followed the recommended proportions.

D UNITARY METHOD

D.1 BRINGING TO THE UNIT

Ex 37: A satellite makes 4 orbits around the Earth in 24 hours. How many hours does it take to complete one orbit?

6 hours

Answer: For 4 orbits, it takes 24 hours. Therefore, to maintain this proportion for 1 orbit, we divide both the number of orbits and the number of hours by 4:



Thus, to make 1 orbit, it takes 6 hours.

Ex 38: A car travels 500 kilometers in 5 hours. How many kilometers does it travel in 1 hour?

100 kilometers

Answer: For 5 hours, the car travels 500 kilometers. Therefore, to maintain this proportion for 1 hour, we divide both the number of kilometers and the number of hours by 5:



In 1 hour, the car covers 100 kilometers.

Ex 39: A factory produces 720 widgets in 8 hours. How many widgets does it produce in 1 hour?

Answer: For 8 hours, the factory produces 720 widgets. Therefore, to maintain this proportion for 1 hour, we divide both the number of widgets and the number of hours by 8:



Thus, in 1 hour, the factory produces 90 widgets.

Ex 40: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 1 loaf of bread?

Answer: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:

$$\frac{2}{4} = \frac{0.5}{1}$$

Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

D.2 CALCULATING FROM THE UNIT

Ex 41: To make 1 chocolate cake, 4 eggs are needed. How many eggs are needed to make 2 cakes?

Answer: For 1 cake, it takes 4 eggs. Therefore, to maintain this proportion for 2 cakes, you multiply both the number of cakes and the number of eggs by 2:

$$\frac{4}{1} = \frac{8}{2}$$

Thus, to make 2 chocolate cakes, you need 8 eggs.

Ex 42: The price of 1 kilogram of apples is \$2.5. What is the price for 3 kilograms of apples?

Answer: The price for 1 kilogram of apples is \$2.5. Therefore, to maintain this proportion for 3 kilograms, you multiply the price by 3:



Thus, the price for 3 kilograms of apples is \$7.5.

Ex 43: To build 1 bookshelf, 10 wooden planks are needed. How many wooden planks are needed to build 3 bookshelves?

Answer: For 1 bookshelf, it takes 10 wooden planks. Therefore, to maintain this proportion for 3 bookshelves, you multiply both the number of bookshelves and the number of wooden planks by 3:

$$\frac{10}{1} = \frac{30}{3}$$

Thus, to build 3 bookshelves, you need 30 wooden planks.

Ex 44: To paint 1 m^2 , 0.2 liters of paint are needed. How many liters of paint are needed to paint 3 m^2 ?

Answer: For 1 m^2 , it takes 0.2 liters of paint. Therefore, to maintain this proportion for 3 m^2 , you multiply both the area and the number of liters of paint by 3:



Thus, to paint 3 m², you need 0.6 liters of paint.

D.3 CONVERTING TO AND FROM THE UNIT

Ex 45: To make a special juice mix, you need 5 apples for every 15 oranges. How many oranges do you need if you have 3 apples?

Answer:

• To the unit: For 5 apples, you need 15 oranges. Therefore, to maintain this proportion for 1 apple, we divide both the number of oranges and the number of apples by 5:



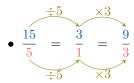
Thus, for 1 apple, you need 3 oranges.

• From the unit: For 1 apple, you need 3 oranges. Therefore, to maintain this proportion for 3 apples, we multiply both the number of oranges and the number of apples by 3:





Thus, for 3 apples, you need 9 oranges.



Ex 46: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 3 loaves of bread?

1.5 kilograms

Answer:

• To the unit: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:

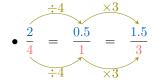


Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

• From the unit: For 1 loaf, the baker uses 0.5 kilograms of flour. Therefore, to maintain this proportion for 3 loaves, we multiply both the number of kilograms and the number of loaves by 3:

$$\frac{0.5}{1} = \frac{1.5}{3}$$

Thus, to make 3 loaf of bread, it takes 1.5 kilograms of flour.



Ex 47: An artist mixes 3 liters of red paint with 6 liters of blue paint to create a purple shade. How many liters of red paint are needed to mix with 9 liters of blue paint to maintain the same shade of purple?

4.5 liters

Answer:

• To the unit: For 6 liters of blue paint, the artist uses 3 liters of red paint. Therefore, to maintain this proportion for 1 liter of blue paint, we divide both the number of liters of red paint and blue paint by 6:

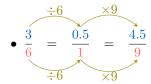


Thus, to mix with 1 liter of blue paint, it takes 0.5 liters of red paint.

• From the unit: For 1 liter of blue paint, the artist uses 0.5 liters of red paint. Therefore, to maintain this proportion for 9 liters of blue paint, we multiply both the number of liters of red paint and blue paint by 9:



Thus, to mix with 9 liters of blue paint, it takes 4.5 liters of red paint.



Ex 48: To make a magic potion, you need 10 drops of dragon's blood for every 5 drops of phoenix tears. How many drops of phoenix tears do you need if you have 4 drops of dragon's blood?

2 drops of phoenix tears

Answer:

• To the unit: For 10 drops of dragon's blood, you need 5 drops of phoenix tears. Therefore, to maintain this proportion for 1 drop of dragon's blood, we divide both the number of drops of phoenix tears and the number of drops of dragon's blood by 10:

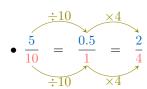


Thus, for 1 drop of dragon's blood, you need 0.5 drops of phoenix tears.

• From the unit: For 1 drop of dragon's blood, you need 0.5 drops of phoenix tears. Therefore, to maintain this proportion for 4 drops of dragon's blood, we multiply both the number of drops of phoenix tears and the number of drops of dragon's blood by 4:

$$\frac{0.5}{1} = \frac{2}{4}$$

Thus, for 4 drops of dragon's blood, you need 2 drops of phoenix tears.



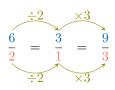
D.4 SOLVING NUMERATOR

D.5 SOLVING DENOMINATOR

Answer:

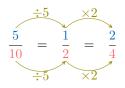
Ex 49:

$$\frac{6}{2} = \frac{\boxed{9}}{3}$$



Ex 53:

Answer:



Ex 50:

$$\frac{8}{4} = \boxed{\frac{10}{5}}$$

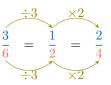
Ex 54:

$$\frac{3}{3} = \frac{2}{4}$$

Answer:

$$\underbrace{\frac{2}{8}}_{4} = \underbrace{\frac{2}{1}}_{5} = \underbrace{\frac{10}{5}}_{5}$$

Answer:

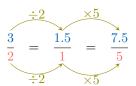


$$\frac{3}{2} = \frac{\boxed{7.5}}{5}$$

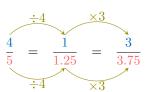
Ex 55:



Answer:



Answer:



Ex 52:

$$\frac{5}{4} = \frac{\boxed{3.75}}{2}$$

Ex 56:

$$\frac{5}{2} = \frac{3}{1.2}$$

Answer:

$$\frac{5}{4} = \frac{1.25}{1} = \frac{3.75}{3}$$

Answer:

$$\frac{5}{2} = \frac{1}{0.4} = \frac{3}{1.2}$$

E CROSS-MULTIPLICATION METHOD

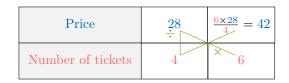
E.1 FINDING A QUANTITY

Ex 57: 4 tickets cost 28 dollars. Find the cost of 6 tickets.

42 dollars

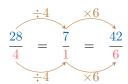
Answer:

• Method 1: Cross Multiplication in a Table



So, 6 tickets cost 42 dollars.

• Method 2: Unitary Rate with Equivalent Ratios



So, 6 tickets cost 42 dollars.

• Method 3: Cross Multiplication Equation

$$\frac{28}{4} = \frac{x}{6}$$

$$4 \times x = 28 \times 6 \quad \text{(cross multiplication)}$$

$$x = \frac{28 \times 6}{4}$$

$$x = 42$$

So, 6 tickets cost 42 dollars.

- Method 4: Unit Rate in Words
 - -4 tickets cost 28 dollars, so 1 ticket costs $28 \div 4 = 7$ dollars
 - -6 tickets cost $7 \times 6 = 42$ dollars.

Ex 58: A recipe requires 200 grams of flour to make 8 cookies. How much flour is needed to make 12 cookies?

300 grams

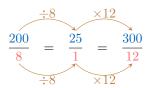
Answer:

• Method 1: Cross Multiplication in a Table

Flour (grams)	200 ÷	$\frac{12 \times 200}{8} = 300$
Number of cookies	8	× 12

So, 12 cookies need 300 grams of flour.

• Method 2: Unitary Rate with Equivalent Ratios



So, 12 cookies need 300 grams of flour.

• Method 3: Cross Multiplication Equation

$$\frac{200}{8} = \frac{x}{12}$$

$$8 \times x = 200 \times 12 \quad \text{(cross multiplication)}$$

$$x = \frac{200 \times 12}{8}$$

$$x = 300$$

So, 12 cookies need 300 grams of flour.

• Method 4: Unit Rate in Words

- 8 cookies need 200 grams of flour, so 1 cookie needs $200 \div 8 = 25$ grams of flour.
- 12 cookies need $25 \times 12 = 300$ grams of flour.

Ex 59: To make a certain shade of paint, you need 1.5 liters of blue paint for every 3 liters of base paint. How much blue paint is needed if you have 4.5 liters of base paint?

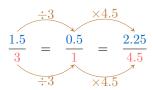
Answer:

• Method 1: Cross Multiplication in a Table

Blue paint (liters)	1.5 ÷	$\frac{4.5 \times 1.5}{3} = 2.25$
Base paint (liters)	3	× 4.5

So, you need 2.25 liters of blue paint.

• Method 2: Unitary Rate with Equivalent Ratios



So, you need 2.25 liters of blue paint.

• Method 3: Cross Multiplication Equation

$$\frac{1.5}{3} = \frac{x}{4.5}$$

$$3 \times x = 1.5 \times 4.5 \quad \text{(cross multiplication)}$$

$$x = \frac{1.5 \times 4.5}{3}$$

$$x = 2.25$$

So, you need 2.25 liters of blue paint.

• Method 4: Unit Rate in Words

- 3 liters of base paint need 1.5 liters of blue paint, so 1 liter of base paint needs $1.5 \div 3 = 0.5$ liters of blue paint.
- -4.5 liters of base paint need $0.5 \times 4.5 = 2.25$ liters of blue paint.

Ex 60: A car travels 120 kilometers on 7.5 liters of fuel. Assuming the car's fuel consumption rate is constant, how much fuel is needed to travel 200 kilometers?

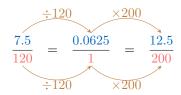
Answer:

• Method 1: Cross Multiplication in a Table

Fuel (liters)	7.5 ÷	$\frac{200 \times 7.5}{120} = 12.5$
Distance (kilometers)	120	× 200

So, you need 12.5 liters of fuel.

• Method 2: Unitary Rate with Equivalent Ratios



So, you need 12.5 liters of fuel.

• Method 3: Cross Multiplication Equation

$$\frac{7.5}{120} = \frac{x}{200}$$

$$120 \times x = 7.5 \times 200 \quad \text{(cross multiplication)}$$

$$x = \frac{7.5 \times 200}{120}$$

$$x = 12.5$$

So, you need 12.5 liters of fuel.

• Method 4: Unit Rate in Words

- 120 kilometers need 7.5 liters of fuel, so 1 kilometer needs $7.5 \div 120 = 0.0625$ liters of fuel.
- -200 kilometers need $0.0625 \times 200 = 12.5$ liters of fuel.

Ex 61: On a map, 4 centimeters represents a real distance of 10 kilometers. If two cities are 6 centimeters apart on the map, what is the actual distance between them in kilometers?

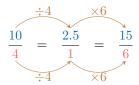
Answer:

• Method 1: Cross Multiplication in a Table



So, the actual distance is 15 kilometers.

• Method 2: Unitary Rate with Equivalent Ratios



So, the actual distance is 15 kilometers.

• Method 3: Cross Multiplication Equation

$$\frac{10}{4} = \frac{x}{6}$$

$$4 \times x = 10 \times 6 \quad \text{(cross multiplication)}$$

$$x = \frac{10 \times 6}{4}$$

$$x = 15$$

So, the actual distance is 15 kilometers.

• Method 4: Unit Rate in Words

- -4 centimeters on the map represent 10 kilometers in reality, so 1 centimeter represents $10 \div 4 = 2.5$ kilometers.
- 6 centimeters on the map represent $2.5 \times 6 = 15$ kilometers in reality.

10