RATIOS

A DEFINITION

A.1 EXPRESSING RATIOS IN DIFFERENT FORMS

Ex 1: The ratio $\frac{3}{5}$ to $\frac{2}{5}$ is $\frac{3}{2}$.

Answer: The ratio 3 to 2 can be expressed as 3:2 or $\frac{3}{2}$.

Ex 2: The ratio 5 to 4 is 5:4.

Answer: The ratio 5 to 4 can be expressed as 5:4 or $\frac{5}{4}$.

- **Ex 3:** The ratio 7 to 3 is 7:3.
- Answer: The ratio 7 to 3 can be expressed as 7:3 or $\frac{7}{3}$.
- **Ex 4:** The ratio 8 to 5 is 8:5.
- Answer: The ratio 8 to 5 can be expressed as 8:5 or $\frac{8}{5}$.
- **Ex 5:** The ratio 10 to 6 is 10: 6.

Answer: The ratio 10 to 6 can be expressed as 10:6 or $\frac{10}{6}$.

B PART-PART AND PART-WHOLE RATIOS

B.1 FINDING RATIOS IN PART-PART

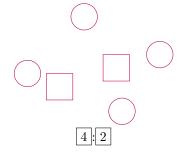
Ex 6: What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There are 3 boys.
- The ratio of girls to boys is 2:3 or $\frac{2}{3}$.

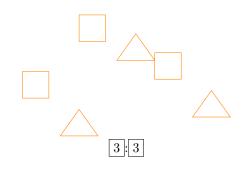
Ex 7: What is the ratio of circles to rectangles?



Answer:

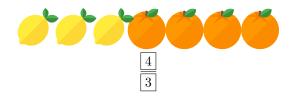
- There are 4 circles.
- There are 2 rectangles.

- The ratio of circles to rectangles is 4:2 or $\frac{4}{2}$.
- **Ex 8:** What is the ratio of squares to triangles?



Answer:

- There are 3 squares.
- There are 3 triangles.
- The ratio of squares to triangles is 3:3 or $\frac{3}{3}$.
- $\mathbf{Ex}~\mathbf{9:}$ What is the ratio of oranges to lemons?



Answer:

- There are 4 oranges.
- There are 3 lemons.
- The ratio of oranges to lemons is $\frac{4}{3}$, or 4:3.

Ex 10: What is the ratio of girls to boys?



Answer:

- There are 2 girls.
- There are 4 boys.
- The ratio of girls to boys is $\frac{2}{4}$, or 2 : 4.

B.2 FINDING RATIOS IN PART-WHOLE

Ex 11: What is the ratio of girls to kids?



Answer:

- There are 2 girls.
- There are 5 kids.
- The ratio of girls to kids is 2:5 or $\frac{2}{\xi}$.

Ex 12: What is the ratio of boys to kids?



Answer:

- There is 1 boy.
- There are 4 kids.
- The ratio of boys to kids is 1:4 or $\frac{1}{4}$.

Ex 13: Louis loves to play sports. In all, he has earned 5 swimming medals, 3 running medals, 6 cycling medals, and 2 triathlon medals.

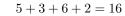
What is the ratio of Louis's swimming medals to all of his medals?

5:16

Answer:

• Louis has earned 5 swimming medals.

•



Louis has earned 16 total medals.

• The ratio of swimming medals to all of his medals is 5:16 or $\frac{5}{16}$.

Ex 14: Anna loves to read books. In all, she has read 12 mystery novels, 8 science fiction novels, 5 fantasy novels, and 3 historical novels.

What is the ratio of Anna's mystery novels to all of her books?

12:28

Answer:

• Anna has read 12 mystery novels.

Anna has read 28 books in total.

• The ratio of mystery novels to all of her books is 12:28 or $\frac{12}{28}$.

12 + 8 + 5 + 3 = 28

Ex 15: The table shows the number of different types of birds that are swimming at a lake.

Bird	Number
Seagulls	1
Ducks	9
Geese	7
Swans	2

What is the ratio of swans to total birds?

2:19

Answer:

- There are 2 swans.
- There are 1 + 9 + 7 + 2 = 19 birds.
- The ratio of swans to total birds is 2:19 or $\frac{2}{10}$.

Ex 16: The table shows the number of different types of fruits in a basket.

Fruit	Number
Apples	3
Oranges	5
Bananas	4
Grapes	6

What is the ratio of apples to total fruits?

3:18

Answer:

- There are 3 apples.
- There are 3 + 5 + 4 + 6 = 18 fruits.
- The ratio of apples to total fruits is 3:18 or $\frac{3}{18}$.

Ex 17: The table shows the number of different types of vehicles in a parking lot.

Vehicle	Number
Cars	10
Bicycles	6
Motorcycles	4
Trucks	2

What is the ratio of trucks to total vehicles?

2 |: |22|

Answer:

- There are 2 trucks.
- There are 10 + 6 + 4 + 2 = 22 vehicles.
- The ratio of trucks to total vehicles is 2:22 or $\frac{2}{22}$.



C EQUAL RATIOS

C.1 MULTIPLYING THE RATIOS

Ex 18: Multiply the ratio by 2:

$$3:5=6:10$$

Answer:

•
$$\frac{3}{5} = \frac{6}{10}$$
$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2}$$
$$= \frac{6}{10}$$

•
$$3:5=6:10.$$

Ex 19: Multiply the ratio by 3:

$$4:7 = 12:21$$

Answer:

•
$$\frac{4}{7} = \frac{12}{21}$$

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3}$$

$$= \frac{12}{21}$$

•
$$4:7=12:21.$$

Ex 20: Multiply the ratio by 4:

$$5:3 = 20:12$$

Answer:

•
$$\frac{5}{3} = \frac{20}{12}$$
$$\frac{5}{3} = \frac{5 \times 4}{3 \times 4}$$
$$= \frac{20}{12}$$

• 5:3=20:12.

Ex 21: Multiply the ratio by 5:

$$: 5 = 10 : 25$$

2

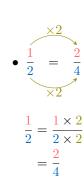
Answer:

• 2:5=10:25.

C.2 FINDING THE MISSING VALUE

Ex 22:

Answer:



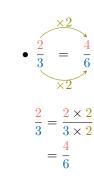
1:2=2:4

Ex 23:

Answer:

• 1:2=2:4.

2:3 = 4:6

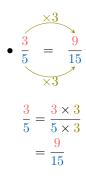


• 2:3=4:6.

Ex 24:

3:5=9:15

Answer:

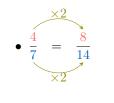


•
$$3:5=9:15.$$

Ex 25:

$$4:7 = 8:14$$

Answer:



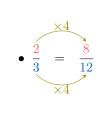
$$\frac{4}{7} = \frac{4 \times 2}{7 \times 2}$$
$$= \frac{8}{14}$$

•
$$4:7=8:14$$

Ex 26:

$$2:3=8:12$$

Answer:



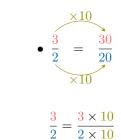
$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4}$$
$$= \frac{8}{12}$$

•
$$2:3=8:12$$

Ex 27:

3:2 = 30:20

Answer:



 $\frac{3}{2} = \frac{3 \times 10}{2 \times 10}$ $= \frac{30}{20}$

• 3:2=30:20.

D PROPORTION

D.1 IDENTIFYING THE PROPORTION

MCQ 28: Two vinaigrettes are being prepared:

- Vinaigrette A is made with 2 mL of oil and 1 mL of vinegar.
- Vinaigrette B is made with 4 mL of oil and 2 mL of vinegar.

Will these two vinaigrettes taste the same?

 \boxtimes Yes

 \square No

Answer:



• Since both ratios are equal, there is a proportion. So, the vinaigrettes will taste the same.

MCQ 29: On the cement package, it is indicated: 2 kilos of cement for 3 liters of water.

A worker prepares a mixture with 4 kilos of cement and 6 liters of water.

Did he follow the recommended proportions?

 \boxtimes Yes

 \square No

Answer:



• Since the two ratios are equal, the worker followed the recommended proportions.

MCQ 30: Two smoothie recipes are being prepared:

- Smoothie A is made with 3 cups of fruit and 2 cups of yogurt.
- Smoothie B is made with 6 cups of fruit and 4 cups of yogurt.

Will these two smoothies taste the same?

 \boxtimes Yes

 \Box No

Answer:



• Since both ratios are equal, the proportions are the same. Therefore, the smoothies will taste the same.



MCQ 31: A gardener uses a fertilizer mix:

- The recommended mix is 5 grams of fertilizer per 2 liters of water.
- The gardener prepares a mixture with 10 grams of fertilizer and 4 liters of water.

Did the gardener follow the recommended proportions?

 \Box Yes

□ No

Answer:



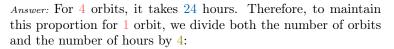
• Since both ratios are equals, the gardener followed the recommended proportions.

E UNITARY METHOD

E.1 BRINGING TO THE UNIT

Ex 32: A satellite makes 4 orbits around the Earth in 24 hours. How many hours does it take to complete one orbit?

6 hours





Thus, to make 1 orbit, it takes 6 hours.

Ex 33: A car travels 500 kilometers in 5 hours. How many kilometers does it travel in 1 hour?

100 kilometers

Answer: For 5 hours, the car travels 500 kilometers. Therefore, to maintain this proportion for 1 hour, we divide both the number of kilometers and the number of hours by 5:



Thus, to travel 1 hour, the car covers 100 kilometers.

Ex 34: A factory produces 720 widgets in 8 hours. How many widgets does it produce in 1 hour?

90 widgets

Answer: For 8 hours, the factory produces 720 widgets. Therefore, to maintain this proportion for 1 hour, we divide both the number of widgets and the number of hours by 8:



Thus, in 1 hour, the factory produces 90 widgets.

Ex 35: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 1 loaf of bread?

0.5 kilograms

Answer: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:



Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

E.2 CALCULATING FROM THE UNIT

Ex 36: To make 1 chocolate cake, 4 eggs are needed. How many eggs are needed to make 2 cakes?

8 eggs

Answer: For 1 cake, it takes 4 eggs. Therefore, to maintain this proportion for 2 cakes, you multiply both the number of cakes and the number of eggs by 2:



Thus, to make 2 chocolate cakes, you need 8 eggs.

Ex 37: The price of 1 kilogram of apples is \$2.5. What is the price for 3 kilograms of apples?

\$ 7.5

Answer: The price for 1 kilogram of apples is 2.5. Therefore, to maintain this proportion for 3 kilograms, you multiply the price by 3:



Thus, the price for 3 kilograms of apples is 7.5.

Ex 38: To build 1 bookshelf, 10 wooden planks are needed. How many wooden planks are needed to build 3 bookshelves?

30 wooden planks

Answer: For 1 bookshelf, it takes 10 wooden planks. Therefore, to maintain this proportion for 3 bookshelves, you multiply both the number of bookshelves and the number of wooden planks by 3:





Thus, to build 3 bookshelves, you need 30 wooden planks.

Ex 39: To paint 1 m^2 , 0.2 liters of paint are needed. How many liters of paint are needed to paint 3 m^2 ?

Answer: For 1 m^2 , it takes 0.2 liters of paint. Therefore, to maintain this proportion for 3 m^2 , you multiply both the area and the number of liters of paint by 3:



Thus, to paint 3 m^2 , you need 0.6 liters of paint.

E.3 CONVERTING TO AND FROM THE UNIT

Ex 40: To make a special juice mix, you need 5 apples for every 15 oranges. How many oranges do you need if you have 3 apples?

9 oranges

Answer:

• To the unit: For 5 apples, you need 15 oranges. Therefore, to maintain this proportion for 1 apple, we divide both the number of oranges and the number of apples by 5:

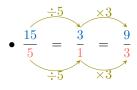


Thus, for 1 apple, you need 3 oranges.

• From the unit: For 1 apple, you need 3 oranges. Therefore, to maintain this proportion for 3 apples, we multiply both the number of oranges and the number of apples by 3:



Thus, for 3 apples, you need 9 oranges.



Ex 41: A baker uses 2 kilograms of flour to make 4 loaves of bread. How many kilograms of flour does it take to make 3 loaf of bread?

1.5 kilograms

Answer:

• To the unit: For 4 loaves, the baker uses 2 kilograms of flour. Therefore, to maintain this proportion for 1 loaf, we divide both the number of kilograms and the number of loaves by 4:

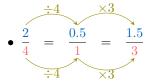


Thus, to make 1 loaf of bread, it takes 0.5 kilograms of flour.

• From the unit: For 1 loaves, the baker uses 0.5 kilograms of flour. Therefore, to maintain this proportion for 3 loaf, we multiply both the number of kilograms and the number of loaves by 3:



Thus, to make 3 loaf of bread, it takes 1.5 kilograms of flour.



Ex 42: An artist mixes 3 liters of red paint with 6 liters of blue paint to create a purple shade. How many liters of red paint are needed to mix with 9 liters of blue paint to maintain the same shade of purple?



Answer:

• To the unit: For 6 liters of blue paint, the artist uses 3 liters of red paint. Therefore, to maintain this proportion for 1 liter of blue paint, we divide both the number of liters of red paint and blue paint by 6:



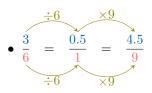
Thus, to mix with 1 liter of blue paint, it takes 0.5 liters of red paint.

• From the unit: For 1 liter of blue paint, the artist uses 0.5 liters of red paint. Therefore, to maintain this proportion for 9 liters of blue paint, we multiply both the number of liters of red paint and blue paint by 9:



(°<u>+</u>°)

Thus, to mix with 9 liters of blue paint, it takes 4.5 liters of **Ex 45**: red paint.



Ex 43: To make a magic potion, you need 10 drops of dragon's blood for every 5 drops of phoenix tears. How many drops of phoenix tears do you need if you have 4 drops of dragon's blood?

Answer:

• To the unit: For 10 drops of dragon's blood, you need 5 drops of phoenix tears. Therefore, to maintain this proportion for $1\ \mathrm{drop}$ of dragon's blood, we divide both the number of drops of phoenix tears and the number of drops of dragon's blood by 10:

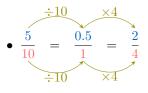


Thus, for 1 drop of dragon's blood, you need 0.5 drops of phoenix tears.

• From the unit: For 1 drop of dragon's blood, you need 0.5 drops of phoenix tears. Therefore, to maintain this proportion for 4 drops of dragon's blood, we multiply both the number of drops of phoenix tears and the number of drops of dragon's blood by 4:



Thus, for 4 drops of dragon's blood, you need 2 drops of phoenix tears.

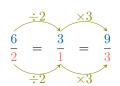


E.4 SOLVING NUMERATOR

Ex 44:

$$\frac{6}{2} = \frac{9}{3}$$

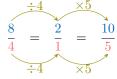
Answer:



Answer:

Ex 46:

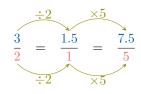
Answer:

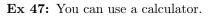


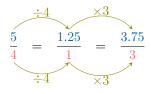
10

 $\overline{5}$









 $\frac{5}{4} = \frac{3.75}{3}$

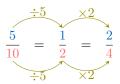
E.5 SOLVING DENOMINATOR

Ex 48:

Answer:

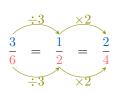
Answer:

 $\frac{5}{10} =$ $\frac{2}{4}$



Ex 49:

Answer:



(*<u>+</u>)

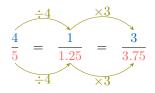
 $\frac{3}{6} =$

4

Ex 50: You can use a calculator.

$$\frac{4}{5} = \frac{3}{3.75}$$

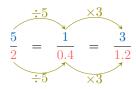
Answer:



Ex 51: You can use a calculator.

 $\frac{5}{2} = \frac{3}{\boxed{1.2}}$

Answer:



F CROSS-MULTIPLICATION METHOD

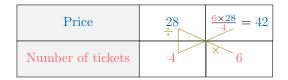
F.1 FINDING A QUANTITY

Ex 52: 4 tickets cost 28 dollars. Find the cost of 6 tickets (you can use a calculator).

42 dollars

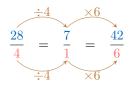
Answer:

• Method 1: Cross Multiplication in a Table



So, 6 tickets cost 42 dollars.

• Method 2: Unitary Rate with Equivalent Ratios



So, 6 tickets cost 42 dollars.

• Method 3: Cross Multiplication Equation

$$\frac{\frac{28}{4}}{\frac{28}{6}} = \frac{x}{6}$$

$$4 \times x = 28 \times 6 \quad \text{(cross multiplication)}$$

$$x = \frac{28 \times 6}{4}$$

$$x = 42$$

So, 6 tickets cost 42 dollars.

• Method 4: Unit Rate in Words

- 4 tickets cost 28 dollars, so 1 ticket costs $28 \div 4 = 7$ dollars.
- -6 tickets cost $7 \times 6 = 42$ dollars.

Ex 53: A recipe requires 200 grams of flour to make 8 cookies. How much flour is needed to make 12 cookies (you can use a calculator).

300 grams

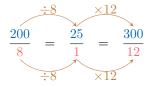
Answer:

• Method 1: Cross Multiplication in a Table

Flour (grams)	200 ÷	$\frac{12 \times 200}{8} = 300$
Number of cookies	8	× 12

So, 12 cookies need 300 grams of flour.

• Method 2: Unitary Rate with Equivalent Ratios



So, 12 cookies need 300 grams of flour.

• Method 3: Cross Multiplication Equation

$$\frac{200}{8} = \frac{x}{12}$$

$$8 \times x = 200 \times 12 \quad (\text{cross multiplication})$$

$$x = \frac{200 \times 12}{8}$$

$$x = 300$$

So, 12 cookies need 300 grams of flour.

• Method 4: Unit Rate in Words

- 8 cookies need 200 grams of flour, so 1 cookie needs $200 \div 8 = 25$ grams of flour.
- 12 cookies need $25 \times 12 = 300$ grams of flour.

Ex 54: To make a certain shade of paint, you need 1.5 liters of blue paint for every 3 liters of base paint. How much blue paint is needed if you have 4.5 liters of base paint (you can use a calculator).

2.25 liters

Answer:

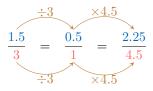
• Method 1: Cross Multiplication in a Table

Blue paint (liters)	1.5	$\frac{4.5 \times 1.5}{3} = 2.25$
Base paint (liters)	3	× 4.5



So, you need 2.25 liters of blue paint.

• Method 2: Unitary Rate with Equivalent Ratios



So, you need 2.25 liters of blue paint.

• Method 3: Cross Multiplication Equation

$$\frac{1.5}{3} = \frac{x}{4.5}$$

$$3 \times x = 1.5 \times 4.5 \quad \text{(cross multiplication)}$$

$$x = \frac{1.5 \times 4.5}{3}$$

$$x = 2.25$$

So, you need 2.25 liters of blue paint.

• Method 4: Unit Rate in Words

- 3 liters of base paint need 1.5 liters of blue paint, so 1 liter of base paint needs $1.5 \div 3 = 0.5$ liters of blue paint.
- -4.5 liters of base paint need $0.5 \times 4.5 = 2.25$ liters of blue paint.

Ex 55: A car travels 120 kilometers on 7.5 liters of fuel. Assuming the car's fuel consumption rate is constant, how much fuel is needed to travel 200 kilometers (you can use a calculator).

12.5 liters

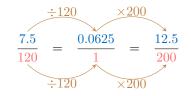
Answer:

• Method 1: Cross Multiplication in a Table

Fuel (liters)	7.5	$\frac{200 \times 7.5}{120} = 12.5$
Distance (kilometers)	120	× 200

So, you need 12.5 liters of fuel.

• Method 2: Unitary Rate with Equivalent Ratios



So, you need 12.5 liters of fuel.

• Method 3: Cross Multiplication Equation

$$\frac{7.5}{120} = \frac{x}{200}$$

$$120 \times x = 7.5 \times 200 \quad \text{(cross multiplication)}$$

$$x = \frac{7.5 \times 200}{120}$$

$$x = 12.5$$

So, you need 12.5 liters of fuel.

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• Method 4: Unit Rate in Words

- 120 kilometers need 7.5 liters of fuel, so 1 kilometer needs $7.5 \div 120 = 0.0625$ liters of fuel.

-200 kilometers need $0.0625 \times 200 = 12.5$ liters of fuel.

Ex 56: On a map, 4 centimeters represents a real distance of 10 kilometers. If two cities are 6 centimeters apart on the map, what is the actual distance between them in kilometers (you can use a calculator).

15 kilometers

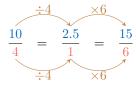
Answer:

• Method 1: Cross Multiplication in a Table

Real Distance (km)	10 ÷	$\frac{6 \times 10}{4} = 15$
Map Distance (cm)	4	\times_6

So, the actual distance is 15 kilometers.

• Method 2: Unitary Rate with Equivalent Ratios



So, the actual distance is 15 kilometers.

• Method 3: Cross Multiplication Equation

$$\frac{10}{4} = \frac{x}{6}$$

$$4 \times x = 10 \times 6 \quad (\text{cross multiplication})$$

$$x = \frac{10 \times 6}{4}$$

$$x = 15$$

So, the actual distance is 15 kilometers.

• Method 4: Unit Rate in Words

- 4 centimeters on the map represent 10 kilometers in reality, so 1 centimeter represents $10 \div 4 = 2.5$ kilometers.
- -6 centimeters on the map represent $2.5 \times 6 = 15$ kilometers in reality.