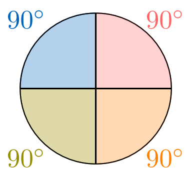
RADIANS AND THE UNIT CIRCLE

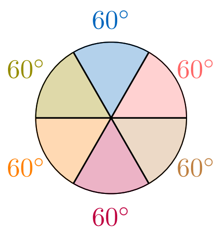
A RADIAN MEASURE

A.1 CONVERTING DEGREES TO RADIANS IN TERMS OF π



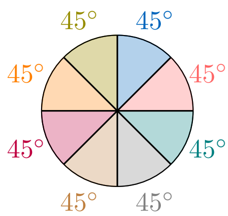
Ex 1: Convert to radians in terms of π:

$90° = \square$



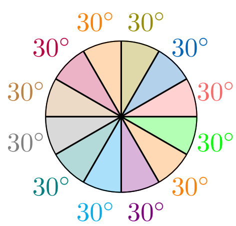
Ex 2: Convert to radians in terms of π:

$60° = \square$



Ex 3: Convert to radians in terms of π:

$45° = \square$



Ex 4: Convert to radians in terms of π:

$30° = \square$

A.2 CONVERTING DEGREES TO RADIANS

Ex 5: Convert to radians (rounded to 2 decimal places).

$46.5° \approx \square$ rad

Ex 6: Convert to radians (rounded to 2 decimal places).

$110° \approx \square$ rad

Ex 7: Convert to radians (rounded to 2 decimal places).

$43° \approx \square$ rad

Ex 8: Convert to radians (rounded to 2 decimal places).

$300° \approx \square$ rad

A.3 CONVERTING RADIANS TO DEGREES

Ex 9: Convert to degrees (rounded to the nearest integer).

$1.25 \text{ rad} \approx \square°$

Ex 10: Convert to degrees (rounded to the nearest integer).

$0.7 \text{ rad} \approx \square°$

Ex 11: Convert to degrees (rounded to the nearest integer).

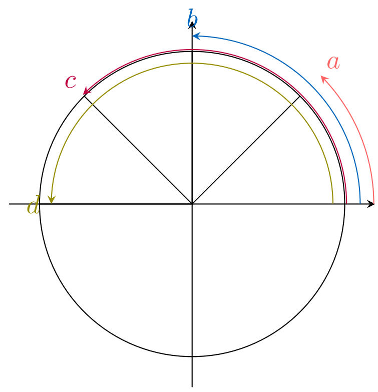
$4.5 \text{ rad} \approx \square°$

Ex 12: Convert to degrees (rounded to the nearest integer).

$2 \text{ rad} \approx \square°$

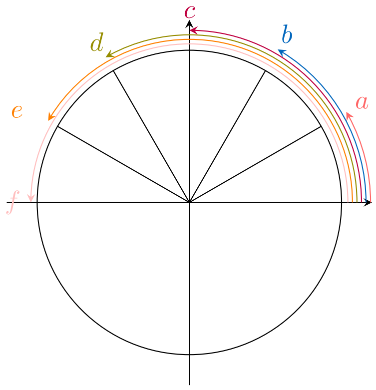
A.4 CONVERTING REFERENCE ANGLES TO RADIANS

Ex 13: Convert to radians in terms of π:



- $a = \square$
- $b = \square$
- $c = \square$
- $d = \square$

Ex 14: Convert to radians in terms of π:



- $a = \square$
- $b = \square$
- $c = \square$
- $d = \square$
- $e = \square$
- $f = \square$

A.5 CALCULATING TRIGONOMETRIC VALUES

Ex 15: Calculate (round to 2 decimal places)

$$\cos\left(\frac{\pi}{4}\right) \approx \square$$

Ex 16: Calculate (round to 2 decimal places)

$$\cos\left(\frac{\pi}{6}\right) \approx \square$$

Ex 17: Calculate (round to 1 decimal place)

$$\sin\left(\frac{5\pi}{6}\right) \approx \square$$

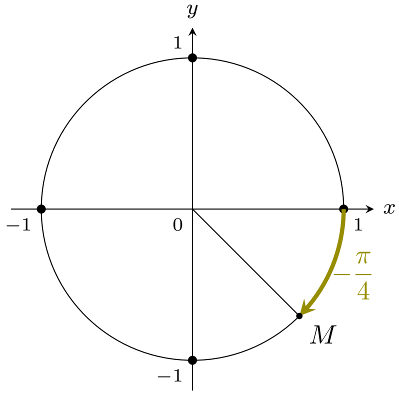
Ex 18: Calculate (round to 2 decimal places)

$$\sin\left(-\frac{\pi}{5}\right) \approx \square$$

B TRIGONOMETRY ON THE UNIT CIRCLE

B.1 EXPRESSING THE COORDINATES OF A POINT ON THE UNIT CIRCLE

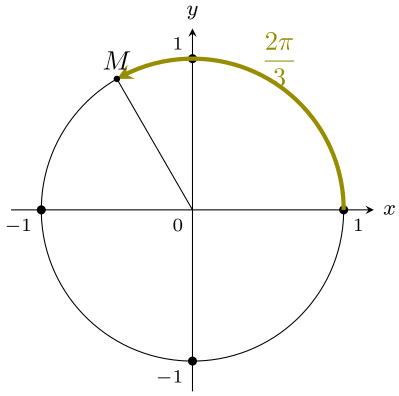
Ex 19:



Determine the coordinates of point M in terms of sine and cosine:

$$M(\square, \square).$$

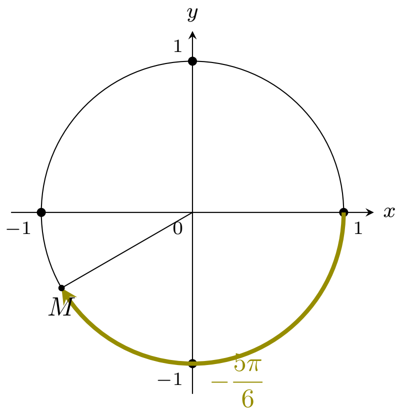
Ex 20:



Determine the coordinates of point M in terms of sine and cosine:

$$M(\square, \square).$$

Ex 21:



Determine the coordinates of point M in terms of sine and cosine:

$$M(\square, \square).$$

B.2 FINDING SINE AND COSINE VALUES ON THE UNIT CIRCLE

Ex 22: Find the values:

- $\cos\left(\frac{\pi}{2}\right) = \square$
- $\sin\left(\frac{\pi}{2}\right) = \square$

Ex 23: Find the values:

- $\cos(\pi) = \square$
- $\sin(\pi) = \square$

Ex 24: Find the values:

- $\cos\left(-\frac{\pi}{2}\right) = \square$
- $\sin\left(-\frac{\pi}{2}\right) = \square$

Ex 25: Find the values:

- $\cos(-\pi) = \square$
- $\sin(-\pi) = \square$

B.3 DETERMINING THE SIGN OF SINE AND COSINE

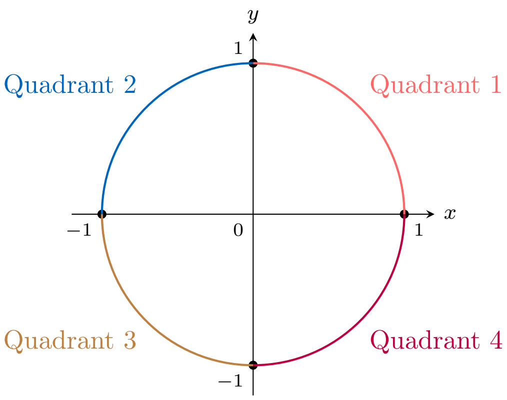
Ex 26: Determine the sign of $\cos\left(-\frac{\pi}{4}\right)$: ☐ + ☐ − .

Ex 27: Determine the sign of $\cos\left(\frac{2\pi}{3}\right)$: ☐ + ☐ − .

Ex 28: Determine the sign of $\sin\left(-\frac{\pi}{6}\right)$: ☐ + ☐ − .

Ex 29: Determine the sign of $\sin\left(\frac{5\pi}{6}\right)$: ☐ + ☐ − .

Ex 30:



- For Quadrant 1, $\cos\theta$ is ☐ + ☐ − and $\sin\theta$ is ☐ + ☐ − .
- For Quadrant 2, $\cos\theta$ is ☐ + ☐ − and $\sin\theta$ is ☐ + ☐ − .
- For Quadrant 3, $\cos\theta$ is ☐ + ☐ − and $\sin\theta$ is ☐ + ☐ − .
- For Quadrant 4, $\cos\theta$ is ☐ + ☐ − and $\sin\theta$ is ☐ + ☐ − .

C TRIGONOMETRIC IDENTITIES

C.1 EXPRESSING TRIGONOMETRIC VALUES IN TERMS OF REFERENCE ANGLES

Ex 31: Express $\cos\left(\frac{7\pi}{6}\right)$ in terms of sine or cosine of $\frac{\pi}{6}$ (use a unit circle):

$$\cos\left(\frac{7\pi}{6}\right) = \square$$

Ex 32: Express $\sin\left(\frac{5\pi}{6}\right)$ in terms of sine or cosine of $\frac{\pi}{6}$ (use a unit circle):

$$\sin\left(\frac{5\pi}{6}\right) = \square$$

Ex 33: Express $\sin\left(-\frac{\pi}{6}\right)$ in terms of sine or cosine of $\frac{\pi}{6}$ (use a unit circle):

$$\sin\left(-\frac{\pi}{6}\right) = \square$$

Ex 34: Express $\cos\left(\frac{13\pi}{6}\right)$ in terms of cosine or sine of $\frac{\pi}{6}$ (use a unit circle):

$$\cos\left(\frac{13\pi}{6}\right) = \square$$

Ex 35: Express $\cos\left(\frac{\pi}{3}\right)$ in terms of sine or cosine of $\frac{\pi}{6}$ (use a unit circle):

$$\cos\left(\frac{\pi}{3}\right) = \square$$

C.2 EXPLAINING TRIGONOMETRIC EQUALITIES

Ex 36: Explain why $\cos\left(\frac{13\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$.

Ex 37: Explain why $\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$.

Ex 38: Explain why $\sin\left(\frac{\pi}{4}\right) = -\sin\left(-\frac{\pi}{4}\right)$.

Ex 39: Explain why $\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$.

Ex 40: Explain why $\sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right)$.

C.3 FINDING EXACT TRIGONOMETRIC VALUES USING THE PYTHAGOREAN IDENTITY

Ex 41: Find the exact value of $\sin\theta$ if $\cos\theta = \frac{\sqrt{3}}{2}$ and $0 \leq \theta \leq \pi$.

$$\sin\theta = \boxed{}$$

Ex 42: Find the exact value of $\cos\theta$ if $\sin\theta = \frac{\sqrt{2}}{2}$ and $-\dfrac{\pi}{2} \leq \theta < \dfrac{\pi}{2}$.

$$\cos\theta = \boxed{}$$

Ex 43: Find the exact value of $\sin\theta$ if $\cos\theta = \frac{1}{2}$ and $-\pi \leq \theta < 0$.

$$\sin\theta = \boxed{}$$

Ex 44: Find the exact value of $\cos\theta$ if $\sin\theta = \dfrac{\sqrt{2}}{2}$ and $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$.

$$\cos\theta = \boxed{}$$

D MULTIPLES OF $\frac{\pi}{4}$

D.1 READING TRIGONOMETRIC VALUES FOR MULTIPLES OF $\pi/4$

Ex 45: Use a unit circle to find:

$$\cos\left(\frac{3\pi}{4}\right) = \boxed{}$$

Ex 46: Use a unit circle to find:

$$\sin\left(\frac{7\pi}{4}\right) = \boxed{}$$

Ex 47: Use a unit circle to find:

$$\sin\left(\frac{5\pi}{4}\right) = \boxed{}$$

Ex 48: Use a unit circle to find:

$$\cos\left(-\frac{\pi}{4}\right) = \boxed{}$$

E MULTIPLES OF $\frac{\pi}{6}$

E.1 READING TRIGONOMETRIC VALUES FOR MULTIPLES OF $\pi/6$

Ex 49: Use a unit circle to find:

$$\cos\left(\frac{2\pi}{3}\right) = \boxed{}$$

Ex 50: Use a unit circle to find:

$$\sin\left(\frac{5\pi}{6}\right) = \boxed{}$$

Ex 51: Use a unit circle to find:

$$\cos\left(\frac{7\pi}{6}\right) = \boxed{}$$

Ex 52: Use a unit circle to find:

$$\sin\left(-\frac{\pi}{3}\right) = \boxed{}$$

F TANGENT FUNCTION

F.1 EVALUATING THE TANGENT AT SPECIAL ANGLES

Ex 53: Find:

$$\tan\left(\frac{\pi}{4}\right) = \boxed{}$$

Ex 54: Find:

$$\tan\left(\frac{\pi}{3}\right) = \boxed{}$$

Ex 55: Find:

$$\tan\left(\frac{\pi}{6}\right) = \boxed{}$$

Ex 56: Find:

$$\tan\left(-\frac{\pi}{4}\right) = \boxed{}$$

Ex 57: Find:

$$\tan(0) = \boxed{}$$

F.2 PROVING TANGENT PROPERTIES

Ex 58: Prove the identity: $\tan(-\theta) = -\tan\theta$.

Ex 59: Prove the identity: $\tan(\theta + \pi) = \tan\theta$.

Ex 60: Prove the identity: $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan\theta}$.

Ex 61: Prove the identity: $\tan(\pi - \theta) = -\tan\theta$.

G ANGLE SUM AND DIFFERENCE IDENTITIES

G.1 SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

Ex 62: Simplify:

$$\cos(2x)\cos(x) - \sin(2x)\sin(x) = \boxed{}$$

Ex 63: Simplify:

$$\sin(2x)\cos(x) + \cos(2x)\sin(x) = \boxed{}$$

Ex 64: Simplify:

$$\cos(x)\cos(2x) + \sin(x)\sin(2x) = \boxed{}$$

Ex 65: Simplify:

$$\sin(x)\cos(2x) - \cos(x)\sin(2x) = \boxed{}$$

G.2 PROVING SUM-TO-PRODUCT AND HALF-ANGLE IDENTITIES

Ex 66: Prove the sum-to-product identity:

$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$$

Ex 67: Prove the sum-to-product identity:

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$$

Ex 68: Prove the identity:

$$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)}$$

Ex 69: Prove the identity:

$$\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$$

Ex 70:

1. Show that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$.
2. Hence show that $\sin A\cos B=\frac{1}{2}\sin(A+B)+\frac{1}{2}\sin(A-B)$.
3. Hence write the following as sums:
   (a) $\sin 30^\circ \cos\theta$
   (b) $\sin 6\alpha\cos\alpha$

Ex 71:

1. Show that $\cos(A+B)+\cos(A-B)=2\cos A\cos B$.
2. Hence show that $\cos A\cos B=\frac{1}{2}\cos(A+B)+\frac{1}{2}\cos(A-B)$.
3. Hence write the following as sums:
   (a) $\cos 30^\circ \cos\theta$
   (b) $\cos 6\alpha\cos\alpha$

G.3 PROVING DOUBLE ANGLE IDENTITIES

Ex 72: Prove the double angle identity: $\sin 2\theta = 2\sin\theta\cos\theta$.

Ex 73: Use the compound angle identities to prove the double angle identity: $\cos 2\theta=\cos^2\theta-\sin^2\theta$.

Ex 74: Use the compound angle identities to prove the double angle identity: $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$.

G.4 DERIVING TRIPLE ANGLE IDENTITIES

Ex 75: Consider the identity for $\cos(A+B)$.

1. By setting $A=B=\theta$, derive the double-angle formula for cosine: $\cos 2\theta = 2\cos^2\theta - 1$.
2. Using the result from part (1) or otherwise, express $\cos 3\theta$ in terms of $\cos\theta$.

Ex 76: Consider the identity for $\sin(A + B)$.

1. By setting $A = B = \theta$, derive the double-angle formula for sine: $\sin 2\theta = 2 \sin\theta \cos\theta$.

2. Using the result from part (1) or otherwise, express $\sin 3\theta$ in terms of $\sin\theta$.