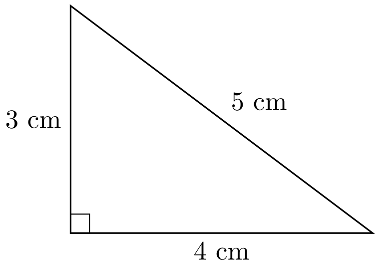
PYTHAGOREAN THEOREM

A RIGHT-ANGLED TRIANGLE

A.1 CALCULATING SQUARED SIDE LENGTHS

Ex 1:

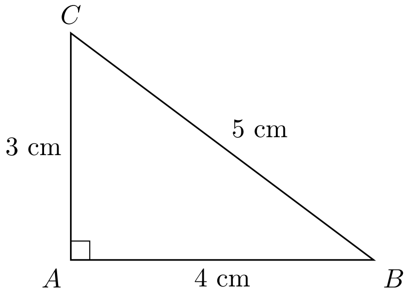


Find the length of the hypotenuse.

$\boxed{5}$ cm

Answer: The hypotenuse is the side opposite the right angle. Its length is 5 cm.

Ex 2:



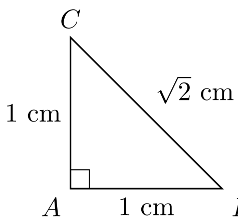
Let a be the length of one leg, b the length of the other leg, and c the length of the hypotenuse. Calculate a^2+b^2 and c^2.

$$a^2+b^2=\boxed{25}\text{ cm}^2 \text{ and } c^2=\boxed{25}\text{ cm}^2$$

Answer: The legs are $AB = 4$ cm and $AC = 3$ cm, and the hypotenuse is $BC = 5$ cm:

$$a^2+b^2 = 4^2+3^2 = 16+9 = 25\,\text{cm}^2$$
$$c^2 = 5^2 = 25\,\text{cm}^2$$

Ex 3:



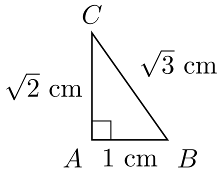
Let a be the length of one leg, b the length of the other leg, and c the length of the hypotenuse. Calculate a^2+b^2 and c^2.

$$a^2+b^2=\boxed{2}\text{ cm}^2 \text{ and } c^2=\boxed{2}\text{ cm}^2$$

Answer: The legs are $AB = 1$ cm and $AC = 1$ cm, and the hypotenuse is $BC = \sqrt{2}$ cm:

$$a^2+b^2 = 1^2+1^2 = 1+1 = 2\,\text{cm}^2$$
$$c^2 = (\sqrt{2})^2 = 2\,\text{cm}^2$$

Ex 4:



Let a be the length of one leg, b the length of the other leg, and c the length of the hypotenuse. Calculate a^2+b^2 and c^2.

$$a^2+b^2=\boxed{3}\text{ cm}^2 \text{ and } c^2=\boxed{3}\text{ cm}^2$$

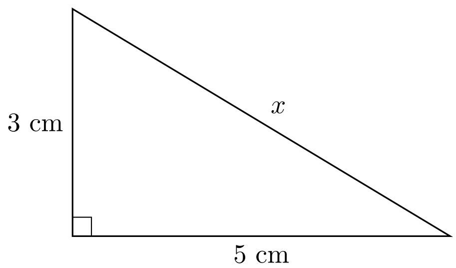
Answer: The legs are $AB = 1$ cm and $AC = \sqrt{2}$ cm, and the hypotenuse is $BC = \sqrt{3}$ cm:

$$a^2+b^2 = 1^2+(\sqrt{2})^2 = 1+2 = 3\,\text{cm}^2$$
$$c^2 = (\sqrt{3})^2 = 3\,\text{cm}^2$$

B PYTHAGOREAN THEOREM

B.1 FINDING THE LENGTH OF THE HYPOTENUSE

Ex 5:



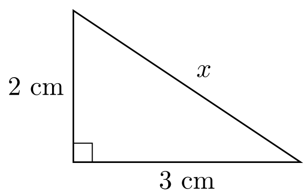
Find x.

$x \approx \boxed{5.8}$ cm (round to 1 decimal place)

Answer:

$$\begin{aligned} x^2 &= 3^2+5^2 && \text{(Pythagorean Theorem)}\\ x^2 &= 9+25 \\ x^2 &= 34 \\ x &= \sqrt{34} && \text{(the length is positive)}\\ x &\approx 5.8\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

Ex 6:



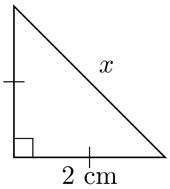
Find x.

$x \approx \boxed{3.6}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 &= 2^2 + 3^2 && \text{(Pythagorean Theorem)}\\
x^2 &= 4 + 9\\
x^2 &= 13\\
x &= \sqrt{13} && \text{(the length is positive)}\\
x &\approx 3.6\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

Ex 7:



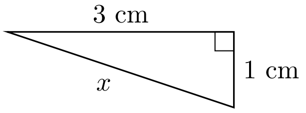
Find x.

$x \approx \boxed{2.8}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 &= 2^2 + 2^2 && \text{(Pythagorean Theorem)}\\
x^2 &= 4 + 4\\
x^2 &= 8\\
x &= \sqrt{8} && \text{(the length is positive)}\\
x &\approx 2.8\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

Ex 8:



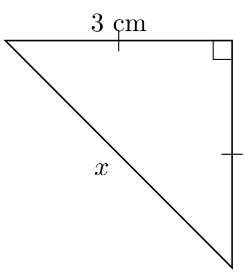
Find x.

$x \approx \boxed{3.2}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 &= 1^2 + 3^2 && \text{(Pythagorean Theorem)}\\
x^2 &= 1 + 9\\
x^2 &= 10\\
x &= \sqrt{10} && \text{(the length is positive)}\\
x &\approx 3.2\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

Ex 9:



Find x.

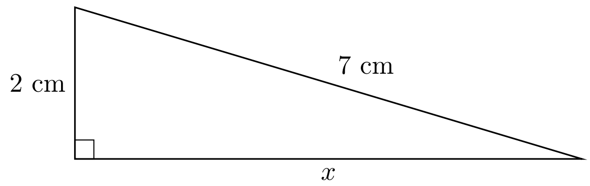
$x \approx \boxed{4.2}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 &= 3^2 + 3^2 && \text{(Pythagorean Theorem)}\\
x^2 &= 9 + 9\\
x^2 &= 18\\
x &= \sqrt{18} && \text{(the length is positive)}\\
x &\approx 4.2\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

B.2 FINDING THE LENGTH OF A LEG

Ex 10:



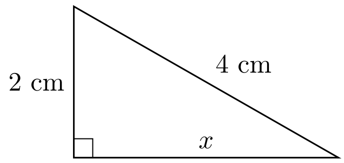
Find x.

$x \approx \boxed{6.7}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 + 2^2 &= 7^2 && \text{(Pythagorean Theorem)}\\
x^2 + 4 &= 49\\
x^2 &= 49 - 4\\
x^2 &= 45\\
x &= \sqrt{45} && \text{(the length is positive)}\\
x &\approx 6.7\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

Ex 11:



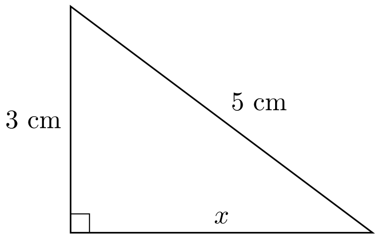
Find x.

$x \approx \boxed{3.5}$ cm (round to 1 decimal place)

Answer:

$$
\begin{aligned}
x^2 + 2^2 &= 4^2 && \text{(Pythagorean Theorem)}\\
x^2 + 4 &= 16\\
x^2 &= 16 - 4\\
x^2 &= 12\\
x &= \sqrt{12} && \text{(the length is positive)}\\
x &\approx 3.5\,\text{cm} && \text{(rounded to the nearest tenth)}
\end{aligned}
$$

Ex 12:



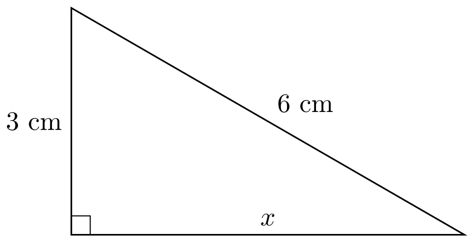
Find x.

$x =$ $\boxed{4}$ cm

Answer:

$$\begin{aligned} x^2+3^2 &= 5^2 && \text{(Pythagorean Theorem)}\\ x^2+9 &= 25\\ x^2 &= 25-9\\ x^2 &= 16\\ x &= \sqrt{16} && \text{(the length is positive)}\\ x &= 4\,\text{cm} \end{aligned}$$

Ex 13:



Find x.

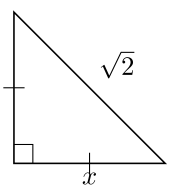
$x \approx$ $\boxed{5.2}$ cm (round to 1 decimal place)

Answer:

$$\begin{aligned} x^2+3^2 &= 6^2 && \text{(Pythagorean Theorem)}\\ x^2+9 &= 36\\ x^2 &= 36-9\\ x^2 &= 27\\ x &= \sqrt{27} && \text{(the length is positive)}\\ x &\approx 5.2\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

B.3 APPLYING THE PYTHAGOREAN THEOREM

Ex 14:



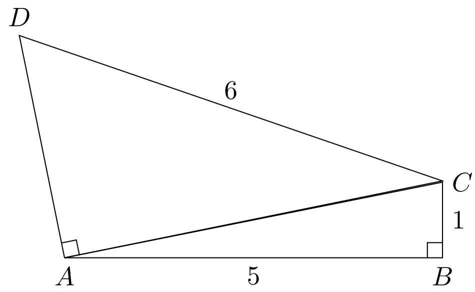
Find x.

$x =$ $\boxed{1}$

Answer:

$$\begin{aligned} x^2+x^2 &= (\sqrt{2})^2 && \text{(Pythagorean Theorem)}\\ 2x^2 &= 2\\ x^2 &= 1\\ x &= \sqrt{1} && \text{(the length is positive)}\\ x &= 1 \end{aligned}$$

Ex 15:



Find the length of AD.

$AD \approx$ $\boxed{3.2}$ cm (round to 1 decimal place)

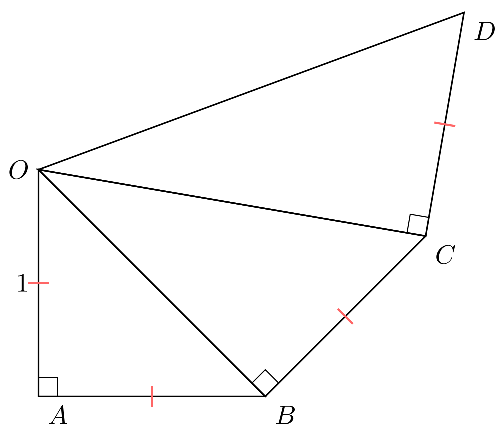
Answer: First, we apply the Pythagorean theorem to $\triangle ABC$:

$$\begin{aligned} AC^2 &= AB^2+BC^2\\ AC^2 &= 5^2+1^2\\ AC^2 &= 25+1\\ AC^2 &= 26\\ AC &= \sqrt{26} \end{aligned}$$

Then, we apply the Pythagorean theorem to $\triangle ACD$:

$$\begin{aligned} AC^2+AD^2 &= DC^2\\ (\sqrt{26})^2+AD^2 &= 6^2\\ 26+AD^2 &= 36\\ AD^2 &= 36-26\\ AD^2 &= 10\\ AD &= \sqrt{10} && \text{(the length is positive)}\\ AD &\approx 3.2\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

Ex 16:



Find OD.

$OD =$ $\boxed{2}$

Answer: First, apply the Pythagorean theorem to $\triangle ABO$:

$$\begin{aligned} BO^2 &= AO^2 + AB^2 \\ BO^2 &= 1^2 + 1^2 \\ BO^2 &= 2 \\ BO &= \sqrt{2} \end{aligned}$$

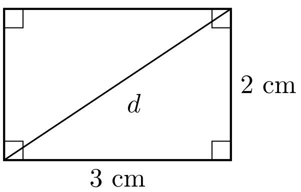
Then, to $\triangle BCO$:

$$\begin{aligned} CO^2 &= BO^2 + BC^2 \\ CO^2 &= (\sqrt{2})^2 + 1^2 \\ CO^2 &= 2 + 1 \\ CO^2 &= 3 \\ CO &= \sqrt{3} \end{aligned}$$

Finally, to $\triangle CDO$:

$$\begin{aligned} OD^2 &= CO^2 + CD^2 \\ OD^2 &= (\sqrt{3})^2 + 1^2 \\ OD^2 &= 3 + 1 \\ OD^2 &= 4 \\ OD &= \sqrt{4} \\ OD &= 2 \end{aligned}$$

Ex 17:



Find d.

$d \approx$ $\boxed{3.6}$ cm (round to 1 decimal place)

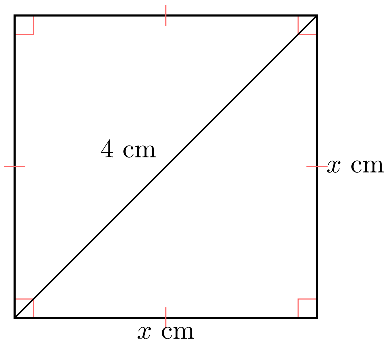
Answer:

$$\begin{aligned} d^2 &= 3^2 + 2^2 && \text{(Pythagorean Theorem)} \\ d^2 &= 9 + 4 \\ d^2 &= 13 \\ d &= \sqrt{13} && \text{(the length is positive)} \\ d &\approx 3.6\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

Ex 18: A square has a diagonal of length 4 cm. Find the length of the square's sides.

$x \approx$ $\boxed{2.8}$ cm (round to 1 decimal place)

Answer:

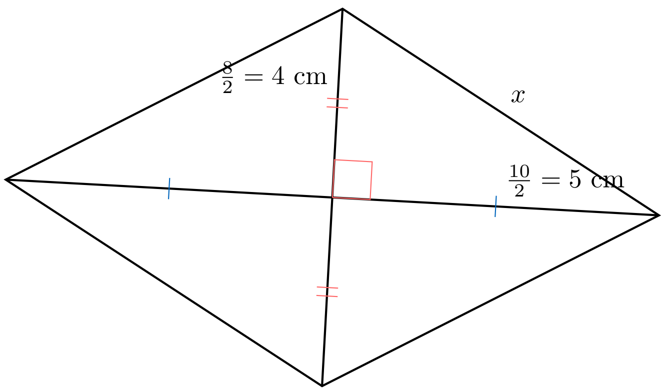


$$\begin{aligned} x^2 + x^2 &= 4^2 && \text{(Pythagorean Theorem)} \\ 2x^2 &= 16 \\ x^2 &= 8 \\ x &= \sqrt{8} && \text{(the length is positive)} \\ x &\approx 2.8\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

Ex 19: A rhombus has diagonals of length 8 cm and 10 cm. Find the length of its sides.

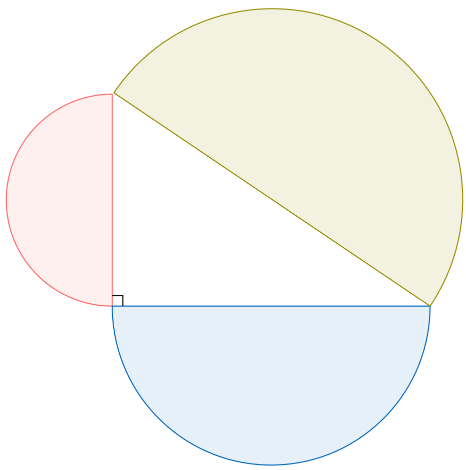
$x \approx$ $\boxed{6.4}$ cm (round to 1 decimal place)

Answer:



$$\begin{aligned} x^2 &= 4^2 + 5^2 && \text{(Pythagorean Theorem)} \\ x^2 &= 16 + 25 \\ x^2 &= 41 \\ x &= \sqrt{41} && \text{(the length is positive)} \\ x &\approx 6.4\,\text{cm} && \text{(rounded to the nearest tenth)} \end{aligned}$$

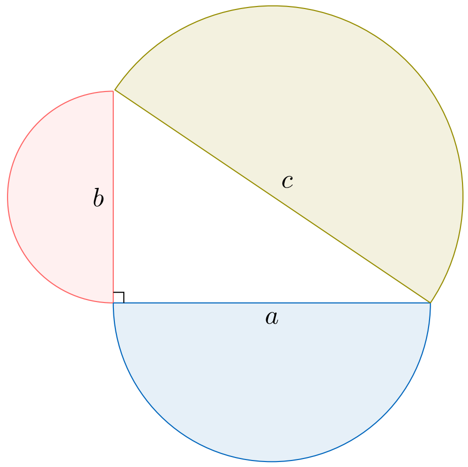
MCQ 20: State whether the sum of the areas of the blue and red half-circles equals the area of the green half-circle.



☒ True

☐ False

Answer: Let a and b be the lengths of the legs and c the length of the hypotenuse.



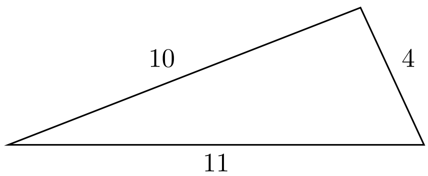
$$a^2 + b^2 = c^2 \qquad \text{(Pythagorean Theorem)}$$
$$\frac{\pi(a/2)^2}{2} + \frac{\pi(b/2)^2}{2} = \frac{\pi(c/2)^2}{2} \qquad \text{(multiply both sides by } \frac{\pi}{8}\text{)}$$

Thus, the sum of the areas of the blue and red half-circles equals the area of the green half-circle. The statement is **True**.

C VERIFYING RIGHT-ANGLED TRIANGLES

C.1 VERIFYING RIGHT-ANGLED TRIANGLES

Ex 21: Is this a right-angled triangle? Justify.

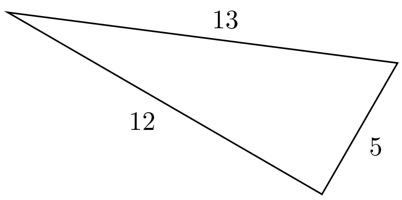


Answer: The two shorter sides have lengths 4 and 10:

$$4^2 + 10^2 = 16 + 100 = 116$$
$$11^2 = 121$$

Since $4^2 + 10^2 \neq 11^2$, the triangle is **not** right-angled by the contrapositive of the Pythagorean theorem.

MCQ 22: Is this a right-angled triangle?



Choose one answer:

☒ True

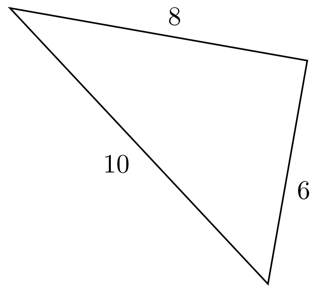
☐ False

Answer: The two shorter sides have lengths 5 and 12:

- $5^2 + 12^2 = 25 + 144 = 169$
- $13^2 = 169$

So $5^2 + 12^2 = 13^2$.
Therefore, the triangle **is** right-angled by the converse of the Pythagorean theorem.

MCQ 23: Is this a right-angled triangle?



Choose one answer:

☒ True

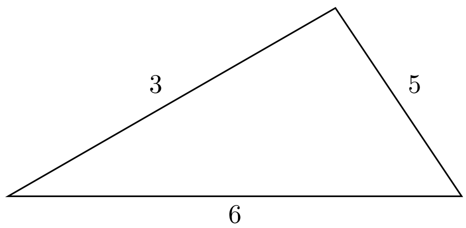
☐ False

Answer: The two shorter sides have lengths 6 and 8:

- $6^2 + 8^2 = 36 + 64 = 100$
- $10^2 = 100$

So $6^2 + 8^2 = 10^2$.
Therefore, the triangle **is** right-angled by the converse of the Pythagorean theorem.

MCQ 24: Is this a right-angled triangle?



Choose one answer:

☐ True

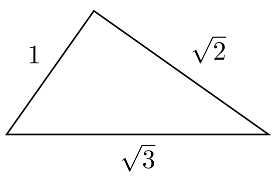
☒ False

Answer: The two shorter sides have lengths 3 and 5:

- $3^2 + 5^2 = 9 + 25 = 34$
- $6^2 = 36$

So $3^2 + 5^2 \neq 6^2$.
Therefore, the triangle is **not** right-angled by the contrapositive of the Pythagorean theorem.

MCQ 25: Is this a right-angled triangle?



Choose one answer:

☒ True

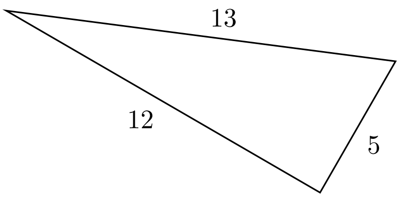
☐ False

Answer: The two shorter sides have lengths 1 and $\sqrt{2}$:

- $1^2 + (\sqrt{2})^2 = 1 + 2 = 3$
- $(\sqrt{3})^2 = 3$

So $1^2 + (\sqrt{2})^2 = (\sqrt{3})^2$.
Therefore, the triangle **is** right-angled by the converse of the Pythagorean theorem.

Ex 26: Is this a right-angled triangle? Justify.

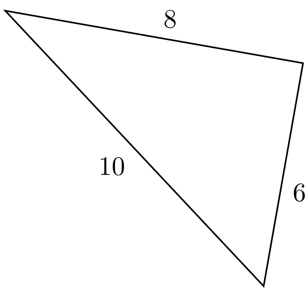


Answer: The two shorter sides have lengths 5 and 12:

$$5^2 + 12^2 = 25 + 144 = 169$$
$$13^2 = 169$$

Since $5^2 + 12^2 = 13^2$, the triangle is right-angled by the converse of the Pythagorean theorem.

Ex 27: Is this a right-angled triangle? Justify.

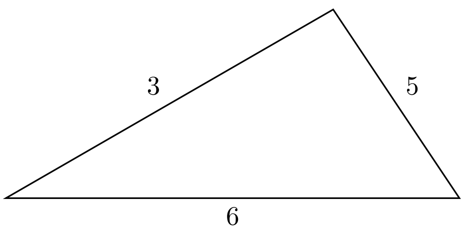


Answer: The two shorter sides have lengths 6 and 8:

$$6^2 + 8^2 = 36 + 64 = 100$$
$$10^2 = 100$$

Since $6^2 + 8^2 = 10^2$, the triangle is right-angled by the converse of the Pythagorean theorem.

Ex 28: Is this a right-angled triangle? Justify.

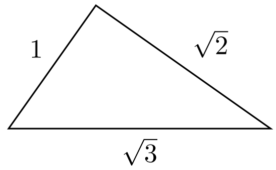


Answer: The two shorter sides have lengths 3 and 5:

$$3^2 + 5^2 = 9 + 25 = 34$$
$$6^2 = 36$$

Since $3^2 + 5^2 \neq 6^2$, the triangle is not right-angled by the contrapositive of the Pythagorean theorem.

Ex 29: Is this a right-angled triangle? Justify.



Answer: The two shorter sides have lengths 1 and $\sqrt{2}$:

$$1^2 + (\sqrt{2})^2 = 1 + 2 = 3$$
$$(\sqrt{3})^2 = 3$$

Since $1^2 + (\sqrt{2})^2 = (\sqrt{3})^2$, the triangle is right-angled by the converse of the Pythagorean theorem.