PROPORTIONALITY

A WHAT IS PROPORTIONALITY?

Discover: Imagine you are buying cookies. Each cookie costs \$2. The number of cookies is x and the total cost is y. We have:

1 cookie costs	$2 = 2 \times 1$
2 cookies cost	$4 = 2 \times 2$
3 cookies cost	$6 = 2 \times 3$
4 cookies cost	$8 = 2 \times 4$
x cookies cost	$y = 2 \times x$

x : number of cookies	1	2	3	4	x
y : total cost	2	4	6	8	$2 \times x$

$\times 2$

- **Ratio definition:** No matter how many cookies you buy, the ratio $\frac{y}{x}$ is always the same and equal to the price of one cookie:
$$\frac{8}{4} = \frac{6}{3} = \frac{4}{2} = \frac{y}{x} = 2.$$
- **Linearity definition:** The total cost can also be expressed with a formula (a linear function):
$$y = 2 \times x.$$

Definition Proportional

Two variables x and y are **proportional** if the ratio $\frac{y}{x}$ is constant, equal to a value k called the **coefficient of proportionality**:
$$\frac{y}{x} = k.$$
Equivalently, y is **proportional** to x if, for the same constant k,
$$y = k \times x.$$

Ex: Does this table represent a proportional relationship?

x	1	2	3
y	15	30	45

Answer: Yes. The table represents a proportional relationship because each ratio is equal:
$$\frac{15}{1} = \frac{30}{2} = \frac{45}{3} = 15.$$

B CALCULATING A FOURTH PROPORTIONAL

Method Calculating a Fourth Proportional

If 4 tickets cost \$28, how much do 6 tickets cost if each ticket costs the same?

- **Method 1: Using the Coefficient of Proportionality**
Find the unit price (price for 1 ticket):
$$\text{Unit price} = \frac{28}{4} = 7.$$
Now multiply by 6 for 6 tickets:
$$\text{Total for 6 tickets} = 7 \times 6 = 42.$$

- **Method 2: Proportion Equation**

$$\frac{28}{4} = \frac{x}{6}$$
$$4 \times x = 28 \times 6 \quad \text{(cross-multiplication)}$$
$$x = \frac{28 \times 6}{4}$$
$$x = 42$$

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{28}{4} \overset{\div 4}{=} \frac{7}{1} \overset{\times 6}{=} \frac{42}{6}$$

- **Method 4: Cross-multiplication (Product in Cross)**

Number of Tickets	4 $\div$ / $\times$ 6	
Price	28	$6 \times 28 \div 4 = 42$

So, 6 tickets cost 42 dollars.