PROPORTIONALITY

A WHAT IS PROPORTIONALITY?

A.1 RECOGNIZING A PROPORTIONAL TABLE

MCQ 1: Is this table proportional?

x	1	2	3
y	15	30	45

☒ Yes

☐ No

Answer: As all ratios are equal: $\frac{15}{1} = \frac{30}{2} = \frac{45}{3} = 15$, it is a proportional table.

MCQ 2: Is this table proportional?

x	2	4	6
y	3	7	9

☐ Yes

☒ No

Answer: The ratios are not equal: $\frac{3}{2} \neq \frac{7}{4}$, so it is not a proportional table.

MCQ 3: Is this table proportional?

x	2	4	6
y	3	6	9

☒ Yes

☐ No

Answer: As all ratios are equal: $\frac{3}{2} = \frac{6}{4} = \frac{9}{6} = 1.5$, it is a proportional table.

MCQ 4: Is this table proportional?

x	1	2	4
y	2	4	7

☐ Yes

☒ No

Answer: The ratios are not equal: $\frac{2}{1} \neq \frac{7}{4}$, so it is not a proportional table.

A.2 TESTING PROPORTIONAL RELATIONSHIPS IN WORD PROBLEMS

MCQ 5: Is the price of oranges proportional to the quantity?

Quantity (kg)	1	2	3
Price ($)	1.5	3	4.5

☒ Yes, the price is proportional to the quantity.

☐ No, the price is not proportional to the quantity.

Answer: As all ratios are equal: $\frac{1.5}{1} = \frac{3}{2} = \frac{4.5}{3} = 1.5$, the price is proportional to the quantity.

MCQ 6: Is the height of children proportional to their age?

Age (years)	3	6	9
Height (cm)	90	120	150

☐ Yes, the height is proportional to the age.

☒ No, the height is not proportional to the age.

Answer: The ratios are not equal: $\frac{90}{3} \neq \frac{120}{6}$, so the height is not proportional to the age.

MCQ 7: Is the distance traveled proportional to the time spent walking?

Time (hours)	1	2	3
Distance (km)	5	10	15

☒ Yes, the distance is proportional to the time.

☐ No, the distance is not proportional to the time.

Answer: As all ratios are equal: $\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = 5$, the distance is proportional to the time.

MCQ 8: Is the number of pages read proportional to the number of days?

Days	2	4	6
Pages	12	20	32

☐ Yes, the number of pages is proportional to the number of days.

☒ No, the number of pages is not proportional to the number of days.

Answer: The ratios are not equal: $\frac{12}{2} \neq \frac{20}{4}$, so the number of pages is not proportional to the number of days.

A.3 CALCULATING THE COEFFICIENT OF PROPORTIONALITY IN PROPORTIONAL TABLES

Ex 9: In this proportional table,

x	1	2	3
y	15	30	45

calculate the coefficient of proportionality.

15

Answer:

- $\frac{15}{1} = 15$, $\frac{30}{2} = 15$, or $\frac{45}{3} = 15$
- The coefficient of proportionality is 15.

Ex 10: In this proportional table,

x	2.5	5	7.5
y	6.25	12.5	18.75

calculate the coefficient of proportionality.

2.5

Answer:

- $\frac{6.25}{2.5} = 2.5$, $\frac{12.5}{5} = 2.5$, or $\frac{18.75}{7.5} = 2.5$
- The coefficient of proportionality is 2.5.

Ex 11: In this proportional table,

x	2	4	6
y	5	10	15

calculate the coefficient of proportionality.

2.5

Answer:

- $\frac{5}{2} = 2.5$, $\frac{10}{4} = 2.5$, or $\frac{15}{6} = 2.5$
- The coefficient of proportionality is 2.5.

Ex 12: In this proportional table,

x	2.5	5	7.5
y	10	20	30

calculate the coefficient of proportionality.

4

Answer:

- $\frac{10}{2.5} = 4$, $\frac{20}{5} = 4$, or $\frac{30}{7.5} = 4$
- The coefficient of proportionality is 4.

A.4 CALCULATING THE UNIT RATE IN PROPORTIONAL CONTEXTS

Ex 13: Larbi is making a large batch of his special lemonade. The table below shows the number of cups of lemon juice needed for a certain number of liters of lemonade.

Lemon juice (cups)	2	4	6
Lemonade (Liter)	3	6	9

Calculate the number of liters of lemonade per cup of lemon juice.

1.5

Answer:

- $\frac{3}{2} = 1.5$, $\frac{6}{4} = 1.5$, or $\frac{9}{6} = 1.5$
- The number of liters of lemonade per cup of lemon juice is 1.5.

Ex 14: Emma is preparing a special mix for concrete. The table below shows the number of kilograms of cement needed for a certain number of liters of water.

Cement (kg)	4	8	12
Water (L)	6	12	18

Calculate the number of liters of water per kilogram of cement.

1.5

Answer:

- $\frac{6}{4} = 1.5$, $\frac{12}{8} = 1.5$, or $\frac{18}{12} = 1.5$
- The number of liters of water per kilogram of cement is 1.5.

Ex 15: Alex is preparing a fruit salad for a party. The table below shows the number of apples used to make a certain number of servings of fruit salad.

Apples (number)	2	4	6
Servings	5	10	15

Calculate the number of servings per apple.

2.5

Answer:

- $\frac{5}{2} = 2.5$, $\frac{10}{4} = 2.5$, or $\frac{15}{6} = 2.5$
- The number of servings per apple is 2.5.

Ex 16: Maya is downloading videos for a school project. The table below shows the number of hours spent downloading and the total number of gigabytes downloaded.

Time (hours)	1.2	2.4	3.6
Data (GB)	6	12	18

Calculate the number of gigabytes downloaded per hour.

5

Answer:

- $\frac{6}{1.2} = 5$, $\frac{12}{2.4} = 5$, or $\frac{18}{3.6} = 5$
- The number of gigabytes per hour is 5.

A.5 CALCULATING THE UNIT RATE IN PROPORTIONAL CONTEXTS

Ex 17: If 2 kilograms of apples cost 5 dollars, what is the cost per kilogram?

$\boxed{2.5}$ dollars per kilogram

Answer: To find the cost per kilogram, divide the total cost by the total weight:

$$\begin{aligned}\text{Cost per kilogram} &= \frac{\text{Total cost}}{\text{Total weight}} \\ &= \frac{5}{2} \\ &= 2.5 \text{ dollars per kilogram.}\end{aligned}$$

Ex 18: If a recipe requires 4 cups of flour to make 8 cupcakes, how many cups of flour are needed per cupcake?

$\boxed{0.5}$ cups of flour per cupcake

Answer: To find the amount of flour needed per cupcake, divide the total amount of flour by the total number of cupcakes:

$$\begin{aligned}\text{Flour per cupcake} &= \frac{\text{Total flour}}{\text{Total cupcakes}} \\ &= \frac{4}{8} \\ &= 0.5 \text{ cups of flour per cupcake.}\end{aligned}$$

Ex 19: If a car travels 150 kilometers in 3 hours, what is the average speed of the car in kilometers per hour?

$\boxed{50}$ kilometers per hour

Answer: To find the average speed, divide the total distance by the total time:

$$\begin{aligned}\text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} \\ &= \frac{150}{3} \\ &= 50 \text{ kilometers per hour.}\end{aligned}$$

Ex 20: If a factory produces 200 widgets in 4 hours, what is the production rate in widgets per hour?

$\boxed{50}$ widgets per hour

Answer: To find the production rate, divide the total number of widgets by the total time:

$$\begin{aligned}\text{Production rate} &= \frac{\text{Total widgets}}{\text{Total time}} \\ &= \frac{200}{4} \\ &= 50 \text{ widgets per hour.}\end{aligned}$$

A.6 USING UNIT RATES TO CALCULATE A TOTAL

Ex 21: The price of gasoline is \$1.90 per liter. I fill up 30 liters.
What is the total cost of the fill-up?

$\boxed{57}$ dollars

Answer: For 30 liters, the total price is:

$$\begin{aligned}\text{Total price} &= \text{Price per liter} \times \text{Number of liters} \\ &= 1.90 \times 30 \\ &= 57 \text{ dollars}\end{aligned}$$

Ex 22: A printer can print 18 pages per minute. How many pages can it print in 7 minutes?

$\boxed{126}$ pages

Answer: In 7 minutes, the printer can print:

$$\begin{aligned}\text{Total pages} &= \text{Pages per minute} \times \text{Minutes} \\ &= 18 \times 7 \\ &= 126 \text{ pages}\end{aligned}$$

Ex 23: One kilogram of apples costs \$3.20. I buy 6 kilograms.
What is the total cost?

$\boxed{19.2}$ dollars

Answer: For 6 kilograms, the total price is:

$$\begin{aligned}\text{Total price} &= \text{Price per kilogram} \times \text{Number of kilograms} \\ &= 3.20 \times 6 \\ &= 19.2 \text{ dollars}\end{aligned}$$

Ex 24: A factory produces 250 bottles per hour. How many bottles are produced in 9 hours?

$\boxed{2250}$ bottles

Answer: In 9 hours, the factory produces:

$$\begin{aligned}\text{Total bottles} &= \text{Bottles per hour} \times \text{Hours} \\ &= 250 \times 9 \\ &= 2250 \text{ bottles}\end{aligned}$$

A.7 USING UNIT RATES TO CALCULATE A MISSING QUANTITY

Ex 25: The price of gasoline is \$1.9 per liter. You spend \$57 to fill up your tank.
How many liters of gasoline did you purchase?

$\boxed{30}$ liters

Answer: To find the number of liters, we use:

$$\begin{aligned}\text{Price per liter} \times \text{Number of liters} &= \text{Total Cost}\\ \text{Number of liters} &= \frac{\text{Total Cost}}{\text{Price per liter}}\\ &= \frac{57}{1.9}\\ &= 30 \text{ liters}\end{aligned}$$

Ex 26: A gardener plants 48 flowers per row. If she plants a total of 336 flowers, how many rows did she plant?

$\boxed{7}$ rows

Answer: To find the number of rows, we use:

$$\begin{aligned}\text{Flowers per row} \times \text{Number of rows} &= \text{Total flowers}\\ \text{Number of rows} &= \frac{\text{Total flowers}}{\text{Flowers per row}}\\ &= \frac{336}{48}\\ &= 7 \text{ rows}\end{aligned}$$

Ex 27: A baker bakes 36 cookies per tray. She baked 288 cookies in total. How many trays did she use?

$\boxed{8}$ trays

Answer: To find the number of trays, we use:

$$\begin{aligned}\text{Cookies per tray} \times \text{Number of trays} &= \text{Total cookies}\\ \text{Number of trays} &= \frac{\text{Total cookies}}{\text{Cookies per tray}}\\ &= \frac{288}{36}\\ &= 8 \text{ trays}\end{aligned}$$

Ex 28: Sarah completes 24 math exercises per hour. She managed to finish 168 exercises today.
How many hours did she spend working on math?

$\boxed{7}$ hours

Answer: To find the number of hours, we use:

$$\begin{aligned}\text{Exercises per hour} \times \text{Number of hours} &= \text{Total exercises}\\ \text{Number of hours} &= \frac{\text{Total exercises}}{\text{Exercises per hour}}\\ &= \frac{168}{24}\\ &= 7 \text{ hours}\end{aligned}$$

B CALCULATING A FOURTH PROPORTIONAL

B.1 CALCULATING A FOURTH PROPORTIONAL

Ex 29: A train travels 180 km in 3 hours at a constant speed. How far will it travel in 5 hours at the same speed?

$\boxed{300}$ km

Answer:

- **Method 1: Unit Rate**
  Find the distance for 1 hour:

$$\text{Distance per hour} = \frac{180}{3} = 60$$

  Now multiply by 5 for 5 hours:

$$\text{Total for 5 hours} = 60 \times 5 = 300$$

- **Method 2: Proportion Equation**

$$\begin{aligned}\frac{180}{3} &= \frac{x}{5}\\ 3 \times x &= 180 \times 5 \quad \text{(cross multiplication)}\\ x &= \frac{180 \times 5}{3}\\ x &= 300\end{aligned}$$

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{180}{3} \overset{\div 3}{=} \frac{60}{1} \overset{\times 5}{=} \frac{300}{5}$$

- **Method 4: Product in Cross**

Time (hours)	$\overset{\div}{3}$	$\overset{\times}{5}$
Distance (km)	180	$5 \times 180 \div 3 = 300$

So, the train travels 300 km in 5 hours.

Ex 30: You buy 21 liters of gasoline. The total cost is \$39.90. How much will you pay for 35 liters at the same price per liter?

$\boxed{66.5}$ dollars

Answer:

- **Method 1: Unit Rate**
  Find the price per liter:

$$\text{Price per liter} = \frac{39.90}{21} = 1.90$$

  Now multiply by 35 for 35 liters:

$$\text{Total for 35 liters} = 1.90 \times 35 = 66.5$$

- **Method 2: Proportion Equation**

$$\begin{aligned}\frac{39.90}{21} &= \frac{x}{35}\\ 21 \times x &= 39.90 \times 35 \quad \text{(cross multiplication)}\\ x &= \frac{39.90 \times 35}{21}\\ x &= 66.5\end{aligned}$$

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{39.90}{21} \overset{\div 21}{=} \frac{1.90}{1} \overset{\times 35}{=} \frac{66.5}{35}$$

- **Method 4: Product in Cross**

Liters	21 ÷	35 ×
Total price	39.90	$35 \times 39.90 \div 21 = 66.5$

So, 35 liters cost 66.5 dollars.

Ex 31: A printer prints 72 pages in 4 minutes at a constant speed. How many pages will it print in 15 minutes at the same speed?

$$\boxed{270} \text{ pages}$$

Answer:

- **Method 1: Unit Rate**
  Find the number of pages per minute:

$$\text{Pages per minute} = \frac{72}{4} = 18$$

  Now multiply by 15 for 15 minutes:

$$\text{Total for 15 minutes} = 18 \times 15 = 270$$

- **Method 2: Proportion Equation**

$$\begin{aligned} \frac{72}{4} &= \frac{x}{15} \\ 4 \times x &= 72 \times 15 \quad \text{(cross multiplication)} \\ x &= \frac{72 \times 15}{4} \\ x &= 270 \end{aligned}$$

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{72}{4} \overset{\div 4}{=} \frac{18}{1} \overset{\times 15}{=} \frac{270}{15}$$

- **Method 4: Product in Cross**

Time (minutes)	4 ÷	15 ×
Pages	72	$15 \times 72 \div 4 = 270$

So, the printer will print 270 pages in 15 minutes.

B.2 FINDING MISSING VALUES IN A PROPORTIONAL TABLE

Ex 32: In a classroom, the total number of notebooks is proportional to the number of students. Find the missing values in the table of notebooks distribution.

Number of students	5	15	$\boxed{25}$
Number of notebooks	$\boxed{10}$	30	50

Answer: To find the missing values, we first determine the proportionality coefficient:

$$\frac{\text{Notebooks}}{\text{Students}} = \frac{30}{15} = 2$$

÷ 2	Number of students	5	15		× 2
	Number of notebooks		30	50	

- For 5 students:

$$\text{Number of notebooks} = 2 \times 5 = 10$$

- For 50 notebooks:

$$\text{Number of students} = \frac{50}{2} = 25$$

Ex 33: In an apartment building, the maintenance fees paid are proportional to the floor area of the property for each owner. Find the missing values in the table of fees for some owners.

Floor area (m^2)	3	10	$\boxed{15}$
Fees ($)	$\boxed{39}$	130	195

Answer: The proportionality coefficient is:

$$\frac{\text{Fees}}{\text{Area}} = \frac{130}{10} = 13$$

÷ 13	Floor area (m^2)	3	10		× 13
	Fees ($)		130	195	

- For an area of 3 m^2:

$$\text{Fees} = 13 \times 3 = 39$$

- For fees of 195:

$$\text{Area} = \frac{195}{13} = 15$$

Ex 34: In a factory, the amount of fruit juice produced is proportional to the amount of fruits used. Find the missing values in the table of juice production for some batches.

Amount of fruits (kg)	5	20	$\boxed{30}$
Amount of juice (liters)	$\boxed{15}$	60	90

Answer: The proportionality coefficient is:

$$\frac{\text{Juice}}{\text{Fruits}} = \frac{60}{20} = 3$$

$\div 3$	Amount of fruits (kg)	5	20	
	Amount of juice (liters)		60	90

$\times 3$

- For 5 kg of fruit:

$$\text{Juice} = 3 \times 5 = 15$$

- For 90 liters of juice:

$$\text{Fruits} = \frac{90}{3} = 30$$

Ex 35: In a bakery, the amount of dough needed is proportional to the number of loaves of bread produced. Find the missing values in the table of dough requirements for some batches.

Number of loaves	4	12	20
Amount of dough (kg)	2	6	10

Answer: The proportionality coefficient is:

$$\frac{\text{Dough}}{\text{Loaves}} = \frac{6}{12} = 0.5$$

$\div 0.5$	Number of loaves	4	12	
	Amount of dough (kg)		6	10

$\times 0.5$

- For 4 loaves:

$$\text{Dough} = 0.5 \times 4 = 2$$

- For 10 kg of dough:

$$\text{Loaves} = \frac{10}{0.5} = 20$$