PROPORTIONALITY

A DEFINITION

Definition Proportional

Two variables x and y are **proportional** if the ratio of the two variables is equal to a constant value k called the **coefficient of proportionality**.

$$\frac{y}{x} = k$$

Ex: Determine if the table is proportional

x	1	2	3
y	15	30	45

Answer: As all ratios are equal: $\frac{15}{1} = \frac{30}{2} = \frac{45}{3} = 15$, it is a proportional table.

B LINEARITY

Proposition Linearity

Two variables x and y are proportional with coefficient of proportionality k if and only if

$$y = kx.$$

Ex: For the relation $y = 2x$, we have the following table of values:

x	1	2	3
y	2	4	6

$\times 2$

$$2 = 2 \times 1, \quad 4 = 2 \times 2, \quad 6 = 2 \times 3$$

C METHODS FOR CALCULATING A FOURTH PROPORTIONAL

Method Calculating a Fourth Proportional

For her birthday, Su invites her friends to the cinema. She was supposed to pay 28 dollars for 4 tickets. Eventually, Su's parents decide to join and offer to pay.
Knowing that the price is proportional to the number of tickets, how much will Su's parents pay for 6 tickets?

- **Method 1: Coefficient of Proportionality**
  - The coefficient of proportionality is:

$$\begin{aligned} \text{Coefficient} &= \frac{\text{Price}}{\text{Number of Tickets}} \\ &= \frac{28}{4} \\ &= 7 \end{aligned}$$

$\div 7$

Number of Tickets	4	6
Price	28	

$\times 7$

  - For 6 tickets, the price is:

$$\begin{aligned} \text{Price} &= \text{Coefficient} \times \text{Number of Tickets} \\ &= 7 \times 6 \\ &= 42 \text{ dollars per ticket} \end{aligned}$$

Therefore, 6 tickets cost 42 dollars.

- **Method 2: Cross Multiplication in a Proportional Table**

Number of Tickets	4	6
Price	28	x

We apply cross multiplication:

$$4 \times x = 28 \times 6$$
$$x = 28 \times 6 \div 4$$
$$x = 42$$

Therefore, 6 tickets cost 42 dollars.

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{28}{4} \overset{\div 4}{=} \frac{7}{1} \overset{\times 6}{=} \frac{42}{6}$$

Thus, 6 tickets cost 42 dollars.

- **Method 4: Proportion Equation**

$$\frac{28}{4} = \frac{x}{6}$$
$$4 \times x = 28 \times 6 \quad \text{(cross multiplication)}$$
$$x = \frac{28 \times 6}{4}$$
$$x = 42$$

Therefore, 6 tickets cost 42 dollars.

- **Method 5: Unit Rate in Words**
  - 4 tickets cost 28 dollars, so 1 ticket costs $28 \div 4 = 7$ dollars.
  - 6 tickets cost $7 \times 6 = 42$ dollars.