PROPORTIONALITY

A DEFINITION

A.1 DETERMINING IF THE TABLE IS PROPORTIONAL

MCQ 1: Determine if the table is proportional

x	1	2	3
y	15	30	45

☒ Yes

☐ No

Answer: As all ratios are equal: $\frac{15}{1} = \frac{30}{2} = \frac{45}{3} = 15$, it is a proportional table.

MCQ 2: Determine if the table is proportional

x	2	4	6
y	3	7	9

☐ Yes

☒ No

Answer: The ratios are not equal: $\frac{3}{2} \neq \frac{7}{4}$, so it is not a proportional table.

MCQ 3: Determine if the table is proportional

x	2	4	6
y	3	6	9

☒ Yes

☐ No

Answer: As all ratios are equal: $\frac{3}{2} = \frac{6}{4} = \frac{9}{6} = 1.5$, it is a proportional table.

MCQ 4: Determine if the table is proportional

x	1	2	4
y	2	4	7

☐ Yes

☒ No

Answer: The ratios are not equal: $\frac{2}{1} \neq \frac{7}{4}$, so it is not a proportional table.

MCQ 5: Is this a proportional table for the price of oranges depending on the quantity?

Quantity (kg)	1	2	3
Price ($)	1.5	3	4.5

☒ Yes

☐ No

Answer: As all ratios are equal: $\frac{1.5}{1} = \frac{3}{2} = \frac{4.5}{3} = 1.5$, it is a proportional table.

MCQ 6: Is this a proportional table for the height of children depending on their age?

Age (years)	3	6	9
Height (cm)	90	120	150

☐ Yes

☒ No

Answer: The ratios are not equal: $\frac{90}{3} \neq \frac{120}{6}$, so it is not a proportional table.

A.2 FINDING THE UNIT RATE IN WORD PROBLEMS

Ex 7: If 2 kilograms of apples cost 5 dollars, what is the cost per kilogram?

$\boxed{2.5}$ dollars per kilogram

Answer: To find the proportionality coefficient, divide the total cost by the total weight:

$$\begin{aligned}\text{Cost per coefficient} &= \frac{\text{Total cost}}{\text{Total weight}} \\ &= \frac{5}{2} \\ &= 2.5 \text{ dollars per kilogram.}\end{aligned}$$

Ex 8: If a recipe requires 4 cups of flour to make 8 cupcakes, how many cups of flour are needed per cupcake?

$\boxed{0.5}$ cups of flour per cupcake

Answer: To find the amount of flour needed per cupcake, divide the total amount of flour by the total number of cupcakes:

$$\begin{aligned}\text{Flour per cupcake} &= \frac{\text{Total flour}}{\text{Total cupcakes}} \\ &= \frac{4}{8} \\ &= 0.5 \text{ cups of flour per cupcake.}\end{aligned}$$

Ex 9: If a car travels 150 kilometers in 3 hours, what is the average speed of the car in kilometers per hour?

$\boxed{50}$ kilometers per hour

Answer: To find the speed, divide the total distance by the total time:

$$\begin{aligned}\text{Speed} &= \frac{\text{Total distance}}{\text{Total time}} \\ &= \frac{150}{3} \\ &= 50 \text{ kilometers per hour.}\end{aligned}$$

Ex 10: If a factory produces 200 widgets in 4 hours, what is the production rate in widgets per hour?

$\boxed{50}$ widgets per hour

Answer: To find the production rate, divide the total number of widgets by the total time:

$$\begin{aligned}\text{Production rate} &= \frac{\text{Total widgets}}{\text{Total time}} \\ &= \frac{200}{4} \\ &= 50 \text{ widgets per hour.}\end{aligned}$$

A.3 FINDING THE COEFFICIENT OF PROPORTIONALITY IN PROPORTIONAL TABLE

Ex 11: In this proportional table,

x	1	2	3
y	15	30	45

calculate the coefficient of proportionality.

$\boxed{15}$

Answer:

- $\frac{15}{1} = 15$ or $\frac{30}{2} = 15$ or $\frac{45}{3} = 15$
- The coefficient of proportionality is 15.

Ex 12: In this proportional table,

x	2	4	6
y	3	6	9

calculate the coefficient of proportionality.

$\boxed{1.5}$

Answer:

- $\frac{3}{2} = 1.5$ or $\frac{6}{4} = 1.5$ or $\frac{9}{6} = 1.5$
- The coefficient of proportionality is 1.5.

Ex 13: John is making a large batch of his special lemonade. The table below shows the number of cups of lemon juice (x) needed for a certain number of liters of lemonade (y).

Cups of lemon juice (x)	2	4	6
Liters of lemonade (y)	3	6	9

Calculate the coefficient of proportionality.

$\boxed{1.5}$

Answer:

- $\frac{3}{2} = 1.5$ or $\frac{6}{4} = 1.5$ or $\frac{9}{6} = 1.5$
- The coefficient of proportionality is 1.5.

Ex 14: Emma is mixing ingredients for a special type of concrete. The table below shows the number of kilograms of cement (x) needed for a certain number of kilograms of concrete mix (y).

Kilograms of cement (x)	1	2	3
Kilograms of concrete mix (y)	200	400	600

Calculate the coefficient of proportionality.

$\boxed{200}$

Answer:

- $\frac{200}{1} = 200$ or $\frac{400}{2} = 200$ or $\frac{600}{3} = 200$
- The coefficient of proportionality is 200.

Ex 15: Mark is preparing a solution for an experiment. The table below shows the number of milliliters of chemical A (x) needed for a certain number of milliliters of the solution (y).

Milliliters of chemical A (x)	4	8	12
Milliliters of solution (y)	6	12	18

Calculate the coefficient of proportionality.

$\boxed{1.5}$

Answer:

- $\frac{6}{4} = 1.5$ or $\frac{12}{8} = 1.5$ or $\frac{18}{12} = 1.5$
- The coefficient of proportionality is 1.5.

B LINEARITY

B.1 CALCULATING TOTALS USING RATES

Ex 16: The price of gasoline is \$1.9 per liter. I fill up 30 liters. What is the total cost of the fill-up (you can use a calculator)?

$\boxed{57}$ dollars

Answer: For 30 liters, the price is:

$$\begin{aligned}\text{Price} &= \text{Price per liter} \times \text{Number of liters} \\ &= 1.9 \times 30 \\ &= 57 \text{ dollars}\end{aligned}$$

Ex 17: At the market, apples cost \$2.5 per kilogram. I buy 12 kilograms of apples.
What is the total cost of the apples (you can use a calculator)?

$\boxed{30}$ dollars

Answer: For 12 kilograms, the price is:

$$\begin{aligned}\text{Total Cost} &= \text{Price per kilogram} \times \text{Number of kilograms} \\ &= 2.5 \times 12 \\ &= 30 \text{ dollars}\end{aligned}$$

Ex 18: A particular plant grows at a constant rate of 2.5 centimeters per day.
Using this growth rate, how tall will the plant be after 12 days (you can use a calculator)?

$\boxed{30}$ centimeters

Answer:

$$\begin{aligned}\text{Height} &= \text{Growth rate per day} \times \text{Number of days}\\ &= 2.5 \times 12\\ &= 30 \text{ centimeters}\end{aligned}$$

Ex 19: A car rental company charges a constant rate of 0.75 dollars per kilometer driven.
If a customer drives the rental car for 120 kilometers, how much will the rental cost (you can use a calculator)?

$\boxed{90}$ dollars

Answer: Using the given rate:

$$\begin{aligned}\text{Rental Cost} &= \text{Cost per kilometer} \times \text{Number of kilometers}\\ &= 0.75 \times 120\\ &= 90 \text{ dollars}\end{aligned}$$

B.2 CALCULATING QUANTITIES USING RATES

Ex 20: The price of gasoline is \$1.9 per liter. I spend \$57 to fill up my tank.
How many liters of gasoline did I purchase (you can use a calculator)?

$\boxed{30}$ liters

Answer: To find the number of liters, we use:

$$\text{Price per liter} \times \text{Number of liters} = \text{Total Cost}$$

$$\begin{aligned}\text{Number of liters} &= \frac{\text{Total Cost}}{\text{Price per liter}}\\ &= \frac{57}{1.9}\\ &= 30 \text{ liters}\end{aligned}$$

Ex 21: At the market, apples cost \$2.5 per kilogram. I spend \$30 on apples.
How many kilograms of apples did I buy (you can use a calculator)?

$\boxed{12}$ kilograms

Answer: To find the number of kilograms, we use:

$$\text{Price per kilogram} \times \text{Number of kilograms} = \text{Total Cost}$$

$$\begin{aligned}\text{Number of kilograms} &= \frac{\text{Total Cost}}{\text{Price per kilogram}}\\ &= \frac{30}{2.5}\\ &= 12 \text{ kilograms}\end{aligned}$$

Ex 22: At a print shop, it costs \$0.15 to print one page. I spend \$30 on printing.
How many pages did I print (you can use a calculator)?

$\boxed{200}$ pages

Answer: To find the number of pages, we use:

$$\begin{aligned}\text{Number of pages} &= \frac{\text{Total Cost}}{\text{Price per page}}\\ &= \frac{30}{0.15}\\ &= 200 \text{ pages}\end{aligned}$$

Ex 23: A plant grows at a constant rate of 2.5 centimeters per day. It reaches a height of 30 centimeters.
How many days did it take for the plant to reach this height (you can use a calculator)?

$\boxed{12}$ days

Answer: To find the number of days, we use:

$$\text{Growth rate per day} \times \text{Number of days} = \text{Total Height}$$

$$\begin{aligned}\text{Number of days} &= \frac{\text{Total Height}}{\text{Growth rate per day}}\\ &= \frac{30}{2.5}\\ &= 12 \text{ days}\end{aligned}$$

C METHODS FOR CALCULATING A FOURTH PROPORTIONAL

C.1 SOLVING PROPORTIONAL RELATIONSHIPS IN REAL-WORLD CONTEXTS

Ex 24: For her birthday, Su invites her friends to the cinema. She was supposed to pay 28 dollars for 4 tickets. Eventually, Su's parents decide to join and offer to pay.
Knowing that the price is proportional to the number of tickets, how much will Su's parents pay for 6 tickets (you can use a calculator) ?

$\boxed{42}$ dollars

Answer:

- **Method 1: Coefficient of Proportionality**
  - The coefficient of proportionality is:

$$\begin{aligned}\text{Coefficient} &= \frac{\text{Price}}{\text{Number of Tickets}}\\ &= \frac{28}{4}\\ &= 7\end{aligned}$$

$\div 7$	Number of Tickets	4	6
	Price	28	

$\times 7$

  - For 6 tickets, the price is:

$$\begin{aligned}\text{Price} &= \text{Coefficient} \times \text{Number of Tickets}\\ &= 7 \times 6\\ &= 42\end{aligned}$$

Therefore, 6 tickets cost 42 dollars.

- **Method 2: Cross Multiplication in a Proportional Table**

Number of Tickets	4 ÷	× 6
Price	28	x

We apply cross multiplication:

$$\begin{aligned} 4 \times x &= 28 \times 6 \\ x &= 28 \times 6 \div 4 \\ x &= 42 \end{aligned}$$

Therefore, 6 tickets cost 42 dollars.

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{28}{4} \overset{\div 4}{\underset{\div 4}{=}} \frac{7}{1} \overset{\times 6}{\underset{\times 6}{=}} \frac{42}{6}$$

Thus, 6 tickets cost 42 dollars.

- **Method 4: Proportion Equation**

$$\begin{aligned} \frac{28}{4} &= \frac{x}{6} \\ 4 \times x &= 28 \times 6 \quad \text{(cross multiplication)} \\ x &= \frac{28 \times 6}{4} \\ x &= 42 \end{aligned}$$

Therefore, 6 tickets cost 42 dollars.

- **Method 5: Unit Rate in Words**
  - 4 tickets cost 28 dollars, so 1 ticket costs $28 \div 4 = 7$ dollars.
  - 6 tickets cost $7 \times 6 = 42$ dollars.

Ex 25: For a school trip, the teacher buys 15 bus tickets for \$90. Later, the principal joins and offers to pay for additional tickets. Knowing that the price is proportional to the number of tickets, how much will the principal pay for 8 tickets (you can use a calculator)?

$\boxed{48}$ dollars

Answer:

- **Method 1: Coefficient of Proportionality**
  - The coefficient of proportionality is:

$$\begin{aligned} \text{Coefficient} &= \frac{\text{Price}}{\text{Number of Tickets}} \\ &= \frac{90}{15} \\ &= 6 \end{aligned}$$

÷ 6	Number of Tickets	15	8	× 6
	Price	90		

  - For 8 tickets, the price is:

$$\begin{aligned} \text{Price} &= \text{Coefficient} \times \text{Number of Tickets} \\ &= 6 \times 8 \\ &= 48 \end{aligned}$$

Therefore, 8 tickets cost 48 dollars.

- **Method 2: Cross Multiplication in a Proportional Table**

Number of Tickets	15 ÷	× 8
Price	90	x

We apply cross multiplication:

$$\begin{aligned} 15 \times x &= 90 \times 8 \\ x &= \frac{90 \times 8}{15} \\ x &= 48 \end{aligned}$$

Therefore, 8 tickets cost 48 dollars.

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{90}{15} \overset{\div 15}{\underset{\div 15}{=}} \frac{6}{1} \overset{\times 8}{\underset{\times 8}{=}} \frac{48}{8}$$

Thus, 8 tickets cost 48 dollars.

- **Method 4: Proportion Equation**

$$\begin{aligned} \frac{90}{15} &= \frac{x}{8} \\ 15 \times x &= 90 \times 8 \quad \text{(cross multiplication)} \\ x &= \frac{90 \times 8}{15} \\ x &= 48 \end{aligned}$$

Therefore, 8 tickets cost 48 dollars.

- **Method 5: Unit Rate in Words**
  - 15 tickets cost 90 dollars, so 1 ticket costs $90 \div 15 = 6$ dollars.
  - 8 tickets cost $6 \times 8 = 48$ dollars.

Ex 26: In a factory, the amount of fruit juice produced is proportional to the amount of fruit used. The factory uses 10 kilograms of fruit to produce 25 liters of juice.
Knowing that the juice production is proportional to the amount of fruit, how many liters of juice will the factory produce with 14 kilograms of fruit (you can use a calculator)?

$\boxed{35}$ liters

Answer:

- **Method 1: Coefficient of Proportionality**

– The coefficient of proportionality is:

$$\begin{aligned}\text{Coefficient} &= \frac{\text{Juice Produced}}{\text{Fruit Used}} \\ &= \frac{25}{10} \\ &= 2.5\end{aligned}$$

÷ 2.5 ↑	Fruit Used (kg)	10	14	↓ × 2.5
	Juice Produced (L)	25	

– For 14 kilograms of fruit, the juice produced is:

$$\begin{aligned}\text{Juice Produced} &= \text{Coefficient} \times \text{Fruit Used} \\ &= 2.5 \times 14 \\ &= 35\end{aligned}$$

Therefore, 14 kilograms of fruit will produce 35 liters of juice.

- **Method 2: Cross Multiplication in a Proportional Table**

Fruit Used (kg)	10 ÷	× 14
Juice Produced (L)	25	x

We apply cross multiplication:

$$\begin{aligned}10 \times x &= 25 \times 14 \\ x &= \frac{25 \times 14}{10} \\ x &= 35\end{aligned}$$

Therefore, 14 kilograms of fruit will produce 35 liters of juice.

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{25}{10} \overset{\div 10}{\underset{\times 14}{=}} \frac{2.5}{1} \overset{\div 10}{\underset{\times 14}{=}} \frac{35}{14}$$

Thus, 14 kilograms of fruit will produce 35 liters of juice.

- **Method 4: Proportion Equation**

$$\begin{aligned}\frac{25}{10} &= \frac{x}{14} \\ 10 \times x &= 25 \times 14 \quad \text{(cross multiplication)} \\ x &= \frac{25 \times 14}{10} \\ x &= 35\end{aligned}$$

Therefore, 14 kilograms of fruit will produce 35 liters of juice.

- **Method 5: Unit Rate in Words**

– 10 kilograms of fruit produce 25 liters of juice, so 1 kilogram of fruit produces $25 \div 10 = 2.5$ liters.

– 14 kilograms of fruit will produce $2.5 \times 14 = 35$ liters of juice.

Ex 27: In a car's fuel consumption, the amount of gasoline used is proportional to the number of kilometers driven. The car uses 8.5 liters of gasoline to drive 100 kilometers.
Knowing that fuel consumption is proportional to the distance, how much gasoline will the car use to drive 175 kilometers (you can use a calculator)?

$\boxed{14.875}$ liters

Answer:

- **Method 1: Coefficient of Proportionality**

– The coefficient of proportionality is:

$$\begin{aligned}\text{Coefficient} &= \frac{\text{Gasoline (L)}}{\text{Distance (km)}} \\ &= \frac{8.5}{100} \\ &= 0.085\end{aligned}$$

÷ 0.085 ↑	Distance (km)	100	175	↓
	Gasoline (L)	8.5	

– For 175 kilometers, the gasoline consumption is:

$$\begin{aligned}\text{Gasoline (L)} &= \text{Coefficient} \times \text{Distance (km)} \\ &= 0.085 \times 175 \\ &= 14.875\end{aligned}$$

Therefore, 175 kilometers will require 14.875 liters of gasoline.

- **Method 2: Cross Multiplication in a Proportional Table**

Distance (km)	100 ÷	× 175
Gasoline (L)	8.5	x

We apply cross multiplication:

$$\begin{aligned}100 \times x &= 8.5 \times 175 \\ x &= \frac{8.5 \times 175}{100} \\ x &= 14.875\end{aligned}$$

Therefore, 175 kilometers will require 14.875 liters of gasoline.

- **Method 3: Unit Rate with Equivalent Ratios**

$$\frac{8.5}{100} \overset{\div 100}{\underset{\times 175}{=}} \frac{0.085}{1} \overset{\div 100}{\underset{\times 175}{=}} \frac{14.875}{175}$$

Thus, 175 kilometers will require 14.875 liters of gasoline.

- **Method 4: Proportion Equation**

$$\frac{8.5}{100} = \frac{x}{175}$$
$$100 \times x = 8.5 \times 175 \quad \text{(cross multiplication)}$$
$$x = \frac{8.5 \times 175}{100}$$
$$x = 14.875$$

Therefore, 175 kilometers will require 14.875 liters of gasoline.

- **Method 5: Unit Rate in Words**
  - 100 kilometers require 8.5 liters of gasoline, so 1 kilometer requires $8.5 \div 100 = 0.085$ liters.
  - 175 kilometers will require $0.085 \times 175 = 14.875$ liters of gasoline.

C.2 FINDING MISSING VALUES IN PROPORTIONAL TABLE

Ex 28: In a classroom, the number of notebooks each student gets is proportional to the number of students.
Find the missing values in the table of notebooks distribution.

Number of students	5	15	**25**
Number of notebooks	**10**	30	50

Answer: To find the missing values, we first need to determine the proportionality coefficient using the known values.
The proportionality coefficient is calculated as follows:

$$\frac{\text{Notebooks}}{\text{Students}} = \frac{30}{15}$$
$$= 2$$

÷ 2				× 2
Number of students	5	15		
Number of notebooks		30	50	

- For a number of students of 5:

$$\text{Number of notebooks} = 2 \times 5$$
$$= 10$$

- For a number of notebooks of 50:

$$\text{Number of students} = 50 \div 2$$
$$= 25$$

Ex 29: In an apartment building, the maintenance fees paid are proportional to the floor area of the property for each owner.
Find the missing values in the table of fees for some owners.

Floor area in m^2	3	10	**15**
Fees ($)	**39**	130	195

Answer: To find the missing values, we first need to determine the proportionality coefficient using the known values.
The proportionality coefficient is calculated as follows:

$$\frac{\text{Fees}}{\text{Area}} = \frac{130}{10}$$
$$= 13$$

÷ 13				× 13
Area (m^2)	3	10		
Fees ($)		130	195	

- For an area of 3 m^2:

$$\text{Fees} = 13 \times 3$$
$$= 39$$

- For fees of 195:

$$\text{Area} = 195 \div 13$$
$$= 15$$

Ex 30: In a factory, the amount of fruit juice produced is proportional to the amount of fruits used.
Find the missing values in the table of juice production for some batches.

Amount of fruits (kg)	5	20	**30**
Amount of juice (liters)	**15**	60	90

Answer: To find the missing values, we first need to determine the proportionality coefficient using the known values.
The proportionality coefficient is calculated as follows:

$$\frac{\text{Juice}}{\text{Fruits}} = \frac{60}{20}$$
$$= 3$$

÷ 3				× 3
Amount of fruits (kg)	5	20		
Amount of juice (liters)		60	90	

- For an amount of fruits of 5 kg:

$$\text{Amount of juice} = 3 \times 5$$
$$= 15$$

- For an amount of juice of 90 liters:

$$\text{Amount of fruits} = 90 \div 3$$
$$= 30$$

Ex 31: In a bakery, the amount of dough needed is proportional to the number of loaves of bread produced.
Find the missing values in the table of dough requirements for some batches.

Number of loaves	4	12	**20**
Amount of dough (kg)	**2**	6	10

Answer: To find the missing values, we first need to determine the proportionality coefficient using the known values.
The proportionality coefficient is calculated as follows:

$$\begin{aligned}\frac{\text{Dough}}{\text{Loaves}} &= \frac{6}{12}\\ &= 0.5\end{aligned}$$

$\div 0.5$

Number of loaves	4	12	
Amount of dough (kg)		6	10

$\times 0.5$

- For a number of loaves of 4:

$$\begin{aligned}\text{Amount of dough} &= 0.5 \times 4\\ &= 2\end{aligned}$$

- For an amount of dough of 10 kg:

$$\begin{aligned}\text{Number of loaves} &= 10 \div 0.5\\ &= 20\end{aligned}$$