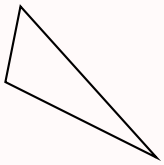
PROPERTIES OF TRIANGLES

A TYPES OF TRIANGLES

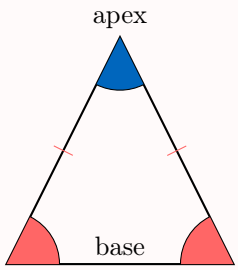
Definition Triangle

A **triangle** is a polygon with three sides. It has three vertices and three angles.



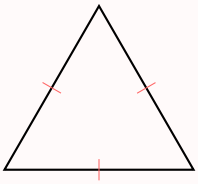
Definition Isosceles triangle

An **isosceles triangle** is a triangle in which two sides are equal in length. The third side is called the **base**, and the vertex opposite the base is called the **apex**.



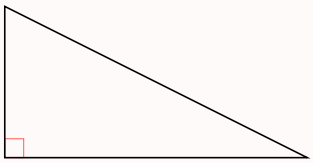
Definition Equilateral triangle

An **equilateral triangle** is a triangle whose three sides are equal in length. It is a special case of an isosceles triangle.



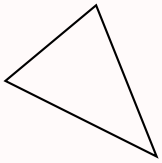
Definition Right-angled triangle

A **right-angled triangle** is a triangle with one right angle (90°). The side opposite the right angle is called the **hypotenuse**.



Definition Scalene triangle

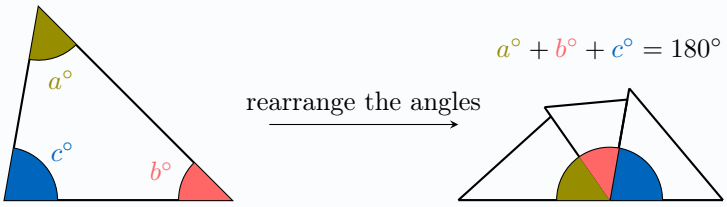
A **scalene triangle** is a triangle whose three sides have different lengths.



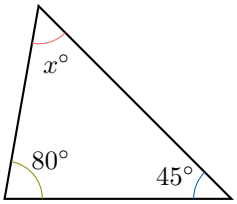
B ANGLES

Proposition Sum of the angles of a triangle

In any triangle, the sum of the three interior angles is 180°.



Ex: Find the angle $x°$.

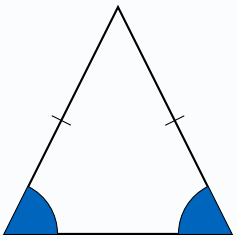


Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} x° + 45° + 80° &= 180° \\ x° + 125° &= 180° \\ x° &= 180° - 125° \\ x° &= 55° \end{aligned}$$

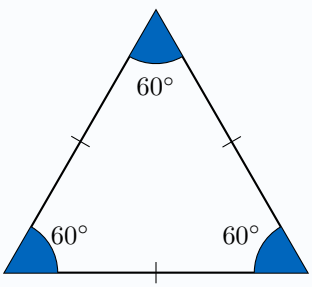
Proposition Angles in an isosceles triangle

In any isosceles triangle, the angles opposite the equal sides (base angles) are equal.



Proposition Angles in an equilateral triangle

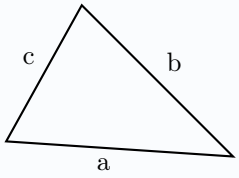
In any equilateral triangle, each angle measures 60°.



C TRIANGLE INEQUALITY THEOREM

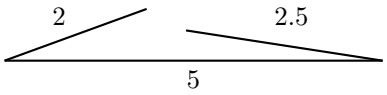
Theorem Triangle inequality theorem

In any triangle, the length of each side must be less than the sum of the lengths of the other two sides.



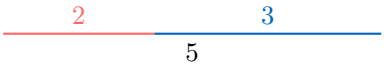
$$a < b + c$$
$$b < a + c$$
$$c < a + b$$

- If one side is longer than the sum of the other two sides, the sides cannot form a triangle because they do not meet to close the shape.

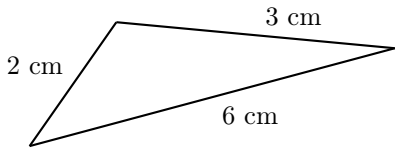


$5 > 2.5 + 2 = 4.5$

- If one side equals the sum of the other two sides, the result is a **degenerate triangle** (a straight line), which is not considered a triangle in this course.



Ex: Could these be the side lengths of a triangle?



Answer: The triangle inequality theorem states that each side must be less than the sum of the other two sides. Check all three sides:

- $2 < 6 + 3 = 9$ (holds)
- $3 < 6 + 2 = 8$ (holds)
- $6 \not< 3 + 2 = 5$ (does not hold)

Since not all inequalities hold, these side lengths cannot form a triangle.