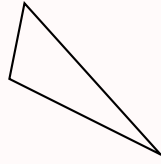# PROPERTIES OF TRIANGLES

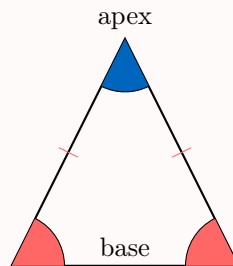## A TYPES OF TRIANGLES

Definition Triangle

A **triangle** is a polygon with three sides. It has three vertices and three angles.

Definition Isosceles triangle

An **isosceles triangle** is a triangle in which two sides are equal in length. The third side is called the **base**, and the vertex opposite the base is called the **apex**.
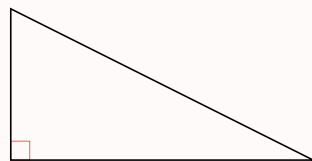

Definition Equilateral triangle

An **equilateral triangle** is a triangle whose three sides are equal in length. It is a special case of an isosceles triangle.
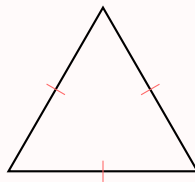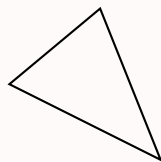

Definition Right-angled triangle

A **right-angled triangle** is a triangle with one right angle (90°). The side opposite the right angle is called the **hypotenuse**.
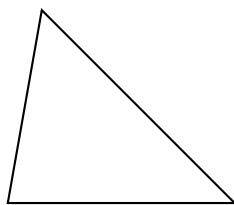

Definition Scalene triangle

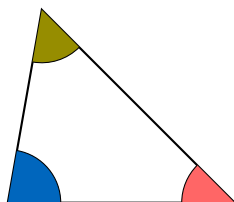A **scalene triangle** is a triangle whose three sides have different lengths.
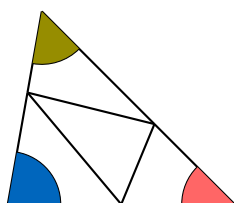

B ANGLES

Discover:

1. Draw any triangle.

2. Mark each angle of the triangle with a different color.

3. Cut out each corner (angle) of the triangle by cutting along the colored regions around each vertex.


4. Arrange the cut-out angles so that they are adjacent, with their vertices meeting at a single point and without overlapping.

5. Observe that the three angles now form a **straight angle** (an angle measuring 180°). Since a straight angle measures 180°, the sum of the three angles of the triangle is 180°.

Proposition **Sum of the angles of a triangle**

In any triangle, the sum of the three interior angles is 180°.
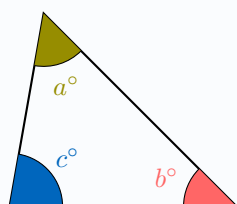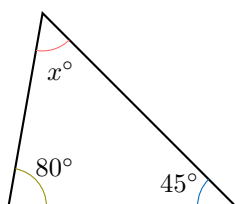

$$a° + b° + c° = 180°$$

rearrange the angles

Ex: Find the angle $x°$.

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$x° + 45° + 80° = 180°$$
$$x° + 125° = 180°$$
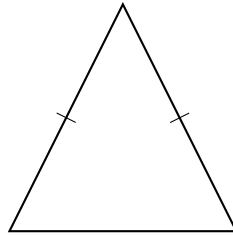$$x° = 180° - 125°$$
$$x° = 55°$$

Proposition **Angles in an isosceles triangle**

In any isosceles triangle, the angles opposite the equal sides (base angles) are equal.
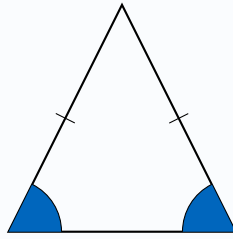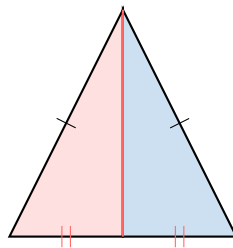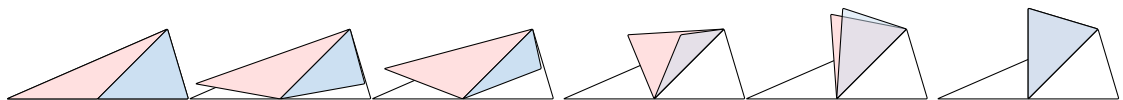
Proof

1. Draw any isosceles triangle with two equal sides.

2. Draw the **median** from the apex to the midpoint of the base. This divides the triangle into two halves. Shade each half differently.
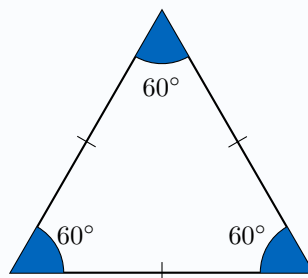

3. Cut out the isosceles triangle and fold it along the median. The median is an axis of symmetry because the two equal sides line up exactly.

4. Observe that the two halves coincide when folded. This shows that the base angles are equal because of the symmetry.
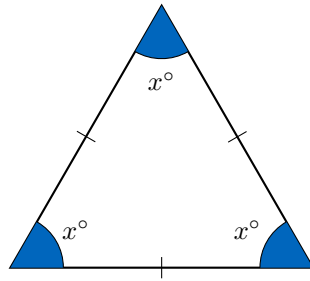
Proposition **Angles in an equilateral triangle**

In any equilateral triangle, each angle measures 60°.

Proof

Consider an equilateral triangle with all sides equal. Since the angles opposite equal sides are equal, all three angles have the same measure, say $x°$.

By the angle sum property of a triangle:

$$x° + x° + x° = 180°$$
$$3x° = 180°$$
$$x° = 180° \div 3$$
$$x° = 60°$$

Thus, each angle in an equilateral triangle measures $60°$.
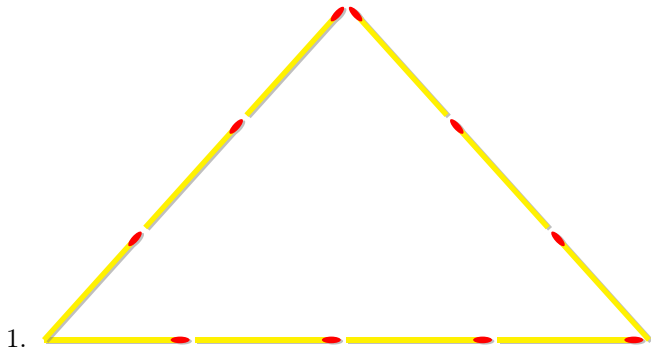
C TRIANGLE INEQUALITY THEOREM

Discover:

1. Align 4 toothpicks end-to-end to form one side of a triangle.

Then construct a triangle with the other two sides each having a length of 3 toothpicks. Make a small sketch to illustrate the solution.

2. Using 9 toothpicks, is it possible to construct a triangle with one side having a length of 5 toothpicks?

Answer:

1.

2. (a) Place 5 toothpicks end-to-end to form one side of the triangle.

   (b) This leaves 4 toothpicks to form the other two sides.

   (c) Suppose the second side uses 1 toothpick. Then, the third side uses the remaining 3 toothpicks.

   (d) Try to form a triangle with sides of 5, 1, and 3 toothpicks. When you attempt to connect the endpoints of the 5-toothpick side with the 1-toothpick and 3-toothpick sides, the 1-toothpick side is too short to reach the endpoint of the 3-toothpick side, preventing the sides from forming a closed shape.

   (e) Try another combination, such as using 2 toothpicks for each of the other two sides. Again, when you attempt to connect the endpoints of the 5-toothpick side with two 2-toothpick sides, the sides do not meet to form a closed shape because the 5-toothpick side is too long compared to the other two. No combination of the 4 remaining toothpicks (for example, 1 and 3, or 2 and 2) allows the sides to connect properly.

   (f) Therefore, with 9 toothpicks, it is **not** possible to construct a triangle with one side of length 5 toothpicks.
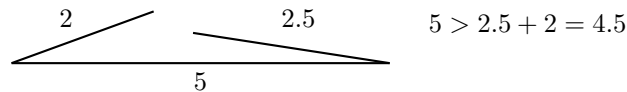

Theorem **Triangle inequality theorem**

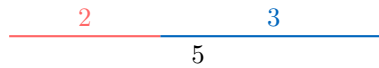In any triangle, the length of each side must be less than the sum of the lengths of the other two sides.
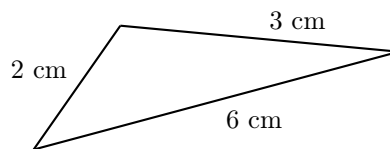
$$a < b + c$$
$$b < a + c$$
$$c < a + b$$
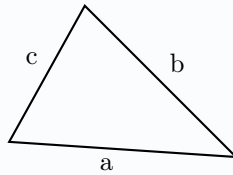
- If one side is longer than the sum of the other two sides, the sides cannot form a triangle because they do not meet to close the shape.

$$5 > 2.5 + 2 = 4.5$$

- If one side equals the sum of the other two sides, the result is a **degenerate triangle** (a straight line), which is not considered a triangle in this course.

Ex: Could these be the side lengths of a triangle?

Answer: The triangle inequality theorem states that each side must be less than the sum of the other two sides. Check all three sides:

- $2 < 6 + 3 = 9$ (holds)

- $3 < 6 + 2 = 8$ (holds)

- $6 \nless 3 + 2 = 5$ (does not hold)

Since not all inequalities hold, these side lengths cannot form a triangle.