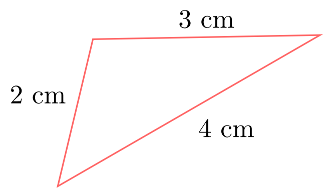
PROPERTIES OF TRIANGLES

A TYPES OF TRIANGLES

A.1 CLASSIFYING TRIANGLES BY SIDE LENGTHS

MCQ 1: Classify the triangle:



Choose one answer:

☒ Scalene

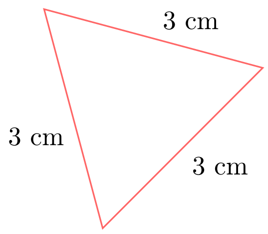
☐ Isosceles

☐ Equilateral

☐ Right-angled triangle

Answer: The triangle is scalene because its sides are 4 cm, 3 cm, and 2 cm, which are all different lengths.

MCQ 2: Classify the triangle:



Choose one answer:

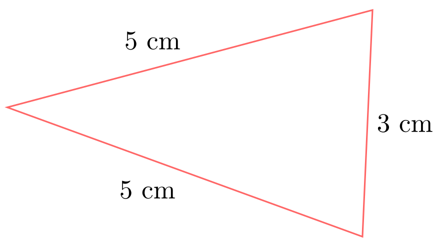
☐ Scalene

☒ Equilateral

☐ Right-angled triangle

Answer: The triangle is equilateral because all three sides are 3 cm long. It is also isosceles because an equilateral triangle has at least two equal sides.

MCQ 3: Classify the triangle:



Choose one answer:

☐ Scalene

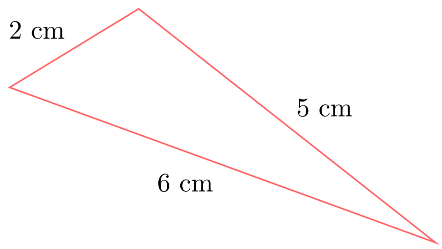
☒ Isosceles

☐ Equilateral

☐ Right-angled triangle

Answer: The triangle is isosceles because two sides are 5 cm and one side is 3 cm, so exactly two sides have the same length.

MCQ 4: Classify the triangle:



Choose one answer:

☒ Scalene

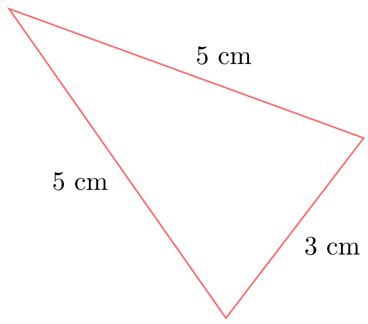
☐ Isosceles

☐ Equilateral

☐ Right-angled triangle

Answer: The triangle is scalene because its sides are 6 cm, 5 cm, and 2 cm, which are all different lengths.

MCQ 5: Classify the triangle:



Choose one answer:

☐ Scalene

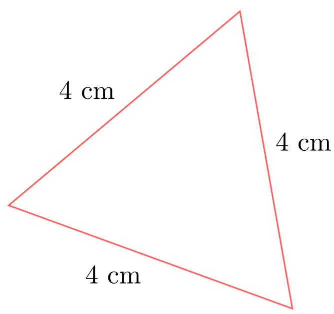
☒ Isosceles

☐ Equilateral

☐ Right-angled triangle

Answer: The triangle is isosceles because two sides are 5 cm and one side is 3 cm, so exactly two sides have the same length.

MCQ 6: Classify the triangle:



Choose one answer:

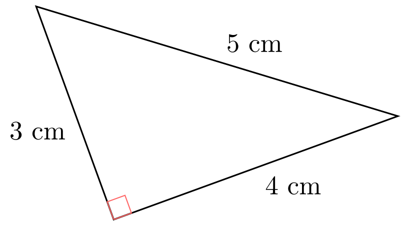
☐ Scalene

☒ Equilateral

☐ Right-angled triangle

Answer: The triangle is equilateral because all three sides are 4 cm long.

MCQ 7: Classify the triangle:



Choose one answer:

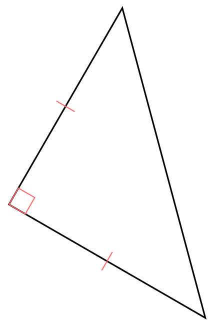
☐ Isosceles

☐ Equilateral

☒ Right-angle

Answer: The triangle is right-angled.

MCQ 8: Classify the triangle:



Choose one or two answers:

☐ Scalene

☒ Isosceles

☐ Equilateral

☒ Right-angle

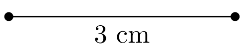
Answer: The triangle is right-angled and isosceles.

A.2 CONSTRUCTING TRIANGLES WITH A RULER AND COMPASS

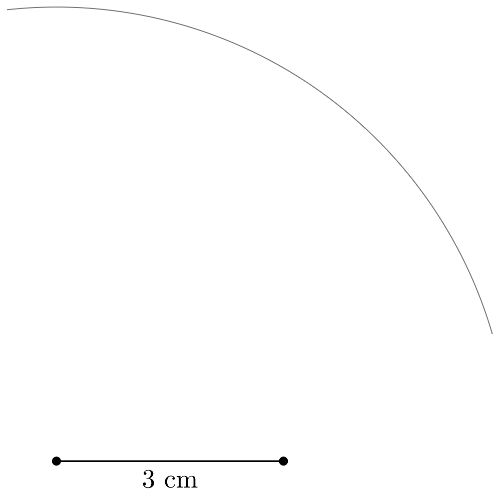
Ex 9: Construct a triangle ABC with $AB = 3$ cm, $AC = 6$ cm, and $BC = 5$ cm, leaving the construction marks visible, using a ruler and a compass.

Answer: To construct a triangle ABC with $AB = 3$ cm, $AC = 6$ cm, and $BC = 5$ cm:

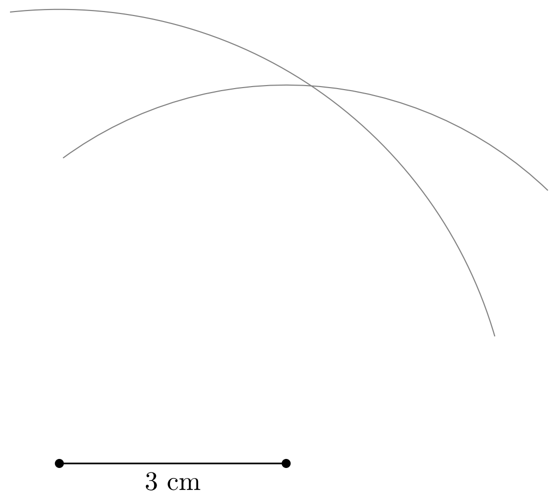
1. Draw the segment $\overline{AB}$ of length 3 cm using your ruler.



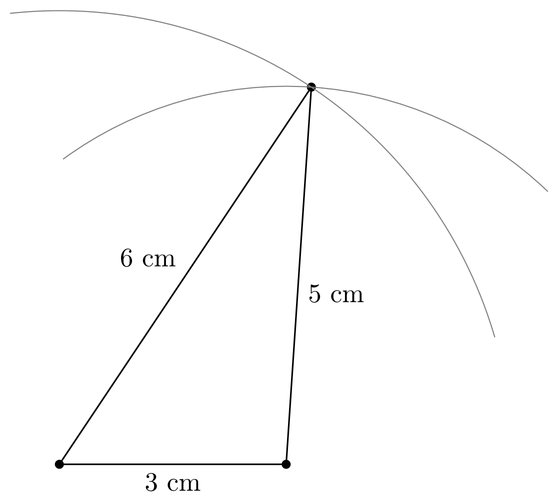
2. Draw an arc with center A and radius 6 cm using your compass.



3. Draw an arc with center B and radius 5 cm using your compass.



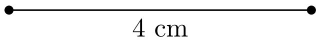
4. Mark the point C at the intersection of the two arcs, then draw the segments $\overline{AC}$ and $\overline{BC}$ using your ruler.



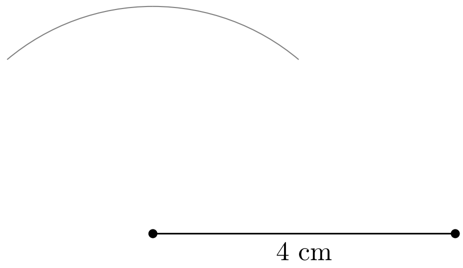
Ex 10: Construct a triangle ABC with $AB = 4$ cm, $AC = 3$ cm, and $BC = 5$ cm, leaving the construction marks visible, using a ruler and a compass.

Answer: To construct a triangle ABC with $AB = 4$ cm, $AC = 3$ cm, and $BC = 5$ cm:

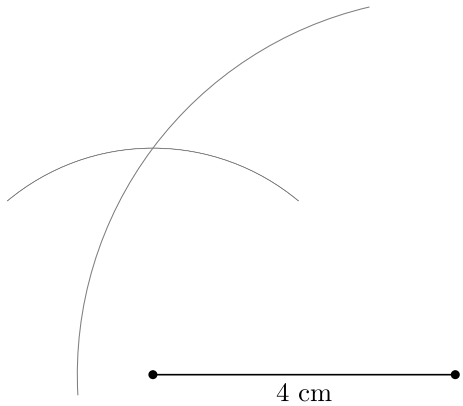
1. Draw the segment $\overline{AB}$ of length 4 cm using your ruler.



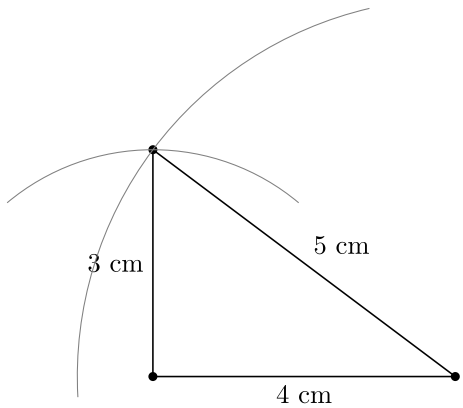
2. Draw an arc with center A and radius 3 cm using your compass.



3. Draw an arc with center B and radius 5 cm using your compass.



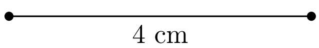
4. Mark the point C at the intersection of the two arcs, then draw the segments $\overline{AC}$ and $\overline{BC}$ using your ruler.



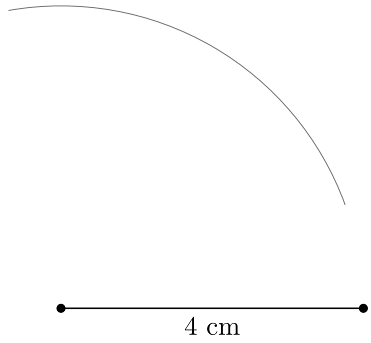
Ex 11: Construct an equilateral triangle ABC with $AB = 4$ cm, leaving the construction marks visible, using a ruler and a compass.

Answer: To construct an equilateral triangle ABC with $AB = 4$ cm:

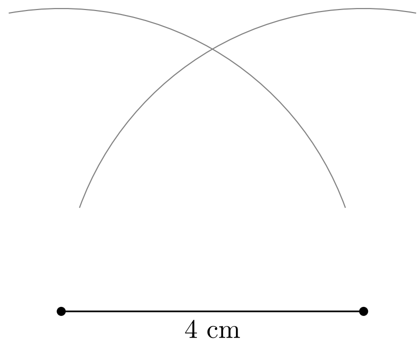
1. Draw the segment $\overline{AB}$ of length 4 cm using your ruler.



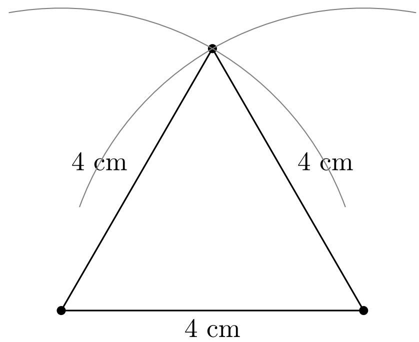
2. Draw an arc with center A and radius 4 cm using your compass.



3. Draw an arc with center B and radius 4 cm using your compass.



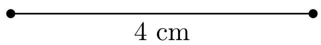
4. Mark the point C at the intersection of the two arcs, then draw the segments $\overline{AC}$ and $\overline{BC}$ using your ruler.



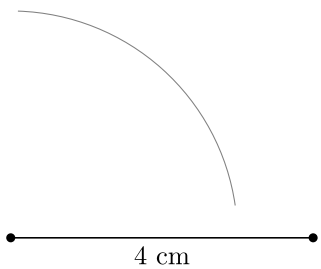
Ex 12: Construct an isosceles triangle ABC with $AB = 4$ cm, $AC = 3$ cm, and $BC = 3$ cm, leaving the construction marks visible, using a ruler and a compass.

Answer: To construct an isosceles triangle ABC with $AB = 4$ cm, $AC = 3$ cm, and $BC = 3$ cm:

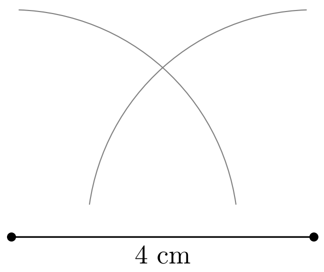
1. Draw the segment $\overline{AB}$ of length 4 cm using your ruler.



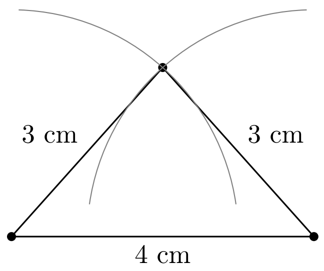
2. Draw an arc with center A and radius 3 cm using your compass.



3. Draw an arc with center B and radius 3 cm using your compass.



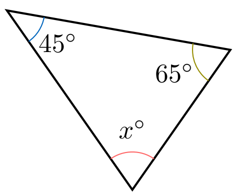
4. Mark the point C at the intersection of the two arcs, then draw the segments $\overline{AC}$ and $\overline{BC}$ using your ruler.



B ANGLES

B.1 FINDING AN UNKNOWN ANGLE

Ex 13: Find the unknown angle in the triangle below:

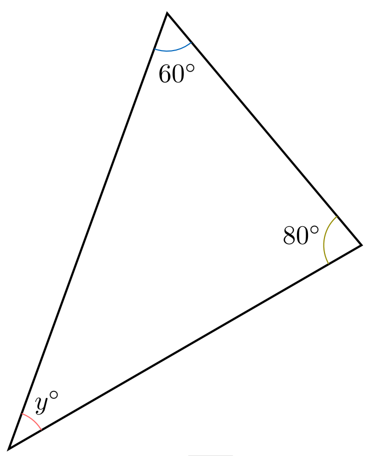


$x^\circ = \boxed{70}^\circ$

Answer: As the sum of the angles of a triangle is 180°

$$\begin{aligned} x^\circ + 45^\circ + 65^\circ &= 180^\circ \\ \therefore x^\circ + 110^\circ &= 180^\circ \\ \therefore x^\circ &= 180^\circ - 110^\circ \\ \therefore x^\circ &= 70^\circ \end{aligned}$$

Ex 14: Find the unknown angle in the triangle below:

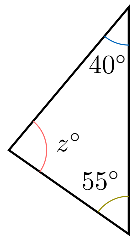


$y^\circ = \boxed{40}^\circ$

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} y^\circ + 60^\circ + 80^\circ &= 180^\circ \\ y^\circ + 140^\circ &= 180^\circ \\ y^\circ &= 180^\circ - 140^\circ \\ y^\circ &= 40^\circ \end{aligned}$$

Ex 15: Find the unknown angle in the triangle below:

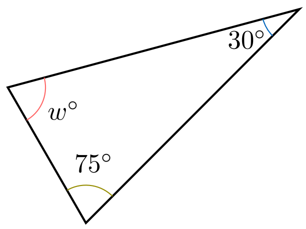


$z^\circ = \boxed{85}^\circ$

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} z^\circ + 40^\circ + 55^\circ &= 180^\circ \\ z^\circ + 95^\circ &= 180^\circ \\ z^\circ &= 180^\circ - 95^\circ \\ z^\circ &= 85^\circ \end{aligned}$$

Ex 16: Find the unknown angle in the triangle below:

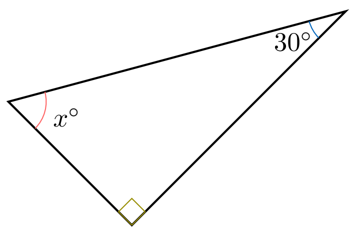


$w^\circ = \boxed{75}^\circ$

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} w^\circ + 30^\circ + 75^\circ &= 180^\circ \\ w^\circ + 105^\circ &= 180^\circ \\ w^\circ &= 180^\circ - 105^\circ \\ w^\circ &= 75^\circ \end{aligned}$$

Ex 17: Find the unknown angle in the triangle below:

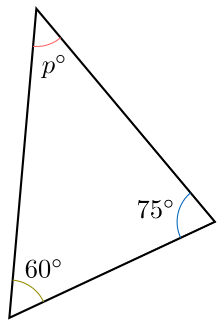


$x^\circ = \boxed{60}^\circ$

Answer: The sum of the angles in a triangle is 180°, and a right angle is 90°. Therefore:

$$\begin{aligned} x^\circ + 30^\circ + 90^\circ &= 180^\circ \\ x^\circ + 120^\circ &= 180^\circ \\ x^\circ &= 180^\circ - 120^\circ \\ x^\circ &= 60^\circ \end{aligned}$$

Ex 18: Find the unknown angle in the triangle below:

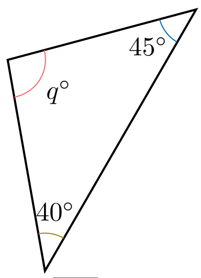


$p^\circ = \boxed{45}^\circ$

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} p^\circ + 75^\circ + 60^\circ &= 180^\circ \\ p^\circ + 135^\circ &= 180^\circ \\ p^\circ &= 180^\circ - 135^\circ \\ p^\circ &= 45^\circ \end{aligned}$$

Ex 19: Find the unknown angle in the triangle below:

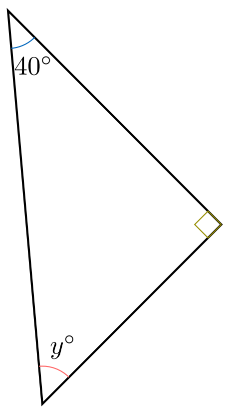


$q^\circ = \boxed{95}^\circ$

Answer: The sum of the angles in a triangle is 180°. Therefore:

$$\begin{aligned} q^\circ + 45^\circ + 40^\circ &= 180^\circ \\ q^\circ + 85^\circ &= 180^\circ \\ q^\circ &= 180^\circ - 85^\circ \\ q^\circ &= 95^\circ \end{aligned}$$

Ex 20: Find the unknown angle in the triangle below:

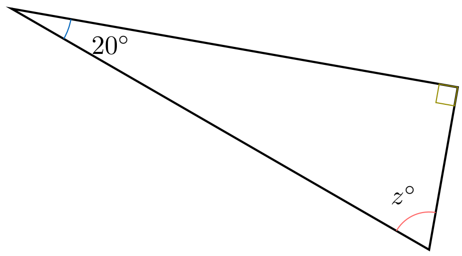


$y^\circ = \boxed{50}^\circ$

Answer: The sum of the angles in a triangle is 180°, and a right angle is 90°. Therefore:

$$\begin{aligned} y^\circ + 40^\circ + 90^\circ &= 180^\circ \\ y^\circ + 130^\circ &= 180^\circ \\ y^\circ &= 180^\circ - 130^\circ \\ y^\circ &= 50^\circ \end{aligned}$$

Ex 21: Find the unknown angle in the triangle below:



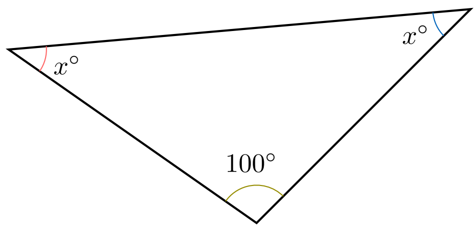
$z^\circ = \boxed{70}^\circ$

Answer: The sum of the angles in a triangle is 180°, and a right angle is 90°. Therefore:

$$\begin{aligned} z^\circ + 20^\circ + 90^\circ &= 180^\circ \\ z^\circ + 110^\circ &= 180^\circ \\ z^\circ &= 180^\circ - 110^\circ \\ z^\circ &= 70^\circ \end{aligned}$$

B.2 FINDING ANGLES IN ISOSCELES TRIANGLES

Ex 22: Find the unknown angle in the triangle below:

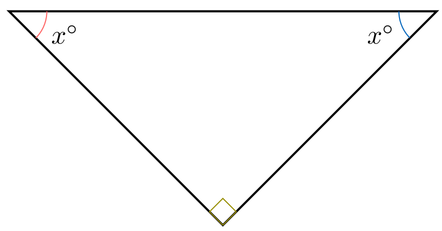


$x^\circ = \boxed{40}^\circ$

Answer: The sum of the angles in a triangle is 180°. Since two angles are equal (both x°), the triangle is isosceles. Therefore:

$$\begin{aligned} x^\circ + x^\circ + 100^\circ &= 180^\circ \\ 2x^\circ + 100^\circ &= 180^\circ \quad \text{(Collecting like terms)} \\ 2x^\circ &= 180^\circ - 100^\circ \quad \text{(Subtracting 100 from both sides)} \\ 2x^\circ &= 80^\circ \\ x^\circ &= 80^\circ \div 2 \quad \text{(Dividing both sides by 2)} \\ x^\circ &= 40^\circ \end{aligned}$$

Ex 23: Find the unknown angle in the triangle below:

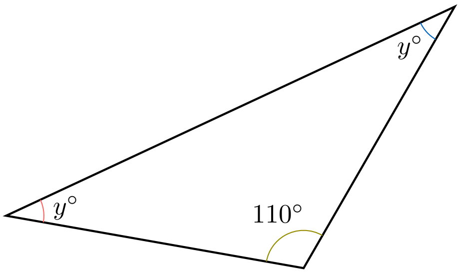


$x^\circ = \boxed{45}^\circ$

Answer: The sum of the angles in a triangle is 180°, and a right angle is 90°. Since two angles are equal (both $x°$), the triangle is isosceles. Therefore:

$$\begin{aligned} x° + x° + 90° &= 180° \\ 2x° + 90° &= 180° \quad \text{(Collecting like terms)} \\ 2x° &= 180° - 90° \quad \text{(Subtracting 90 from both sides)} \\ 2x° &= 90° \\ x° &= 90° \div 2 \quad \text{(Dividing both sides by 2)} \\ x° &= 45° \end{aligned}$$

Ex 24: Find the unknown angle in the triangle below:

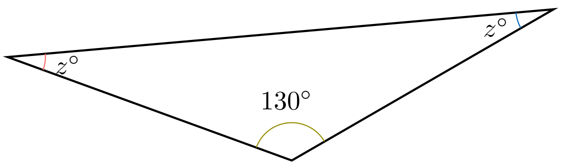


$y° = \boxed{35}°$

Answer: The sum of the angles in a triangle is 180°. Since two angles are equal (both $y°$), the triangle is isosceles. Therefore:

$$\begin{aligned} y° + y° + 110° &= 180° \\ 2y° + 110° &= 180° \quad \text{(Collecting like terms)} \\ 2y° &= 180° - 110° \quad \text{(Subtracting 110 from both sides)} \\ 2y° &= 70° \\ y° &= 70° \div 2 \quad \text{(Dividing both sides by 2)} \\ y° &= 35° \end{aligned}$$

Ex 25: Find the unknown angle in the triangle below:

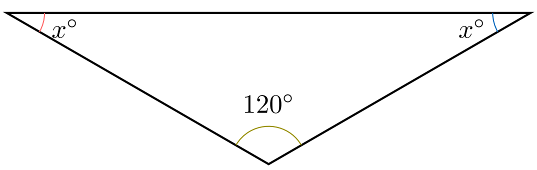


$z° = \boxed{25}°$

Answer: The sum of the angles in a triangle is 180°. Since two angles are equal (both $z°$), the triangle is isosceles. Therefore:

$$\begin{aligned} z° + z° + 130° &= 180° \\ 2z° + 130° &= 180° \quad \text{(Collecting like terms)} \\ 2z° &= 180° - 130° \quad \text{(Subtracting 130 from both sides)} \\ 2z° &= 50° \\ z° &= 50° \div 2 \quad \text{(Dividing both sides by 2)} \\ z° &= 25° \end{aligned}$$

Ex 26: Find the unknown angle in the triangle below:



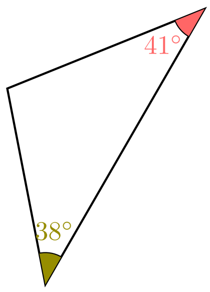
$x° = \boxed{30}°$

Answer: The sum of the angles in a triangle is 180°. Since two angles are equal (both $x°$), the triangle is isosceles. Therefore:

$$\begin{aligned} x° + x° + 120° &= 180° \\ 2x° + 120° &= 180° \quad \text{(Collecting like terms)} \\ 2x° &= 180° - 120° \quad \text{(Subtracting 120 from both sides)} \\ 2x° &= 60° \\ x° &= 60° \div 2 \quad \text{(Dividing both sides by 2)} \\ x° &= 30° \end{aligned}$$

B.3 CLASSIFYING ANGLES

MCQ 27: Classify the triangle:

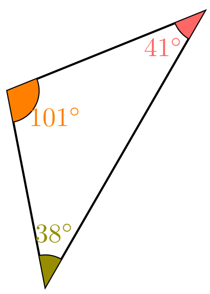


Choose one answer:

- ☐ Isosceles
- ☐ Equilateral
- ☐ Right-angle
- ☒ Scalene

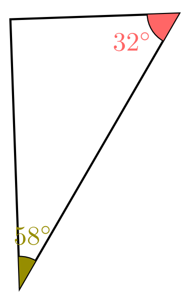
Answer: The third angle $x°$ is:

$$\begin{aligned} x° + 38° + 41° &= 180° \\ x° + 79° &= 180° \quad \text{(Adding known angles)} \\ x° &= 180° - 79° \quad \text{(Subtracting 79 from both sides)} \\ x° &= 101° \end{aligned}$$



Since the angles $(41°, 38°, 101°)$ are all different, the triangle is scalene.

MCQ 28: Classify the triangle:

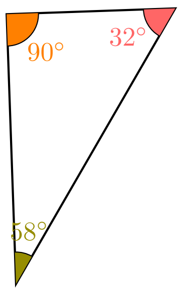


Choose one answer:

- ☐ Isosceles
- ☐ Equilateral
- ☒ Right-angle
- ☐ Scalene

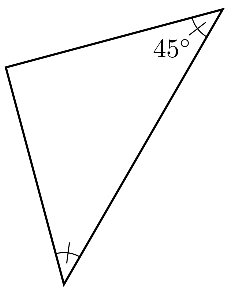
Answer: The third angle x° is:

$$\begin{aligned} x^\circ + 32^\circ + 58^\circ &= 180^\circ \\ x^\circ + 90^\circ &= 180^\circ \quad \text{(Adding known angles)} \\ x^\circ &= 180^\circ - 90^\circ \quad \text{(Subtracting 90 from both sides)} \\ x^\circ &= 90^\circ \end{aligned}$$



Since one angle is 90°, the triangle is right-angled.

MCQ 29: Classify the triangle:

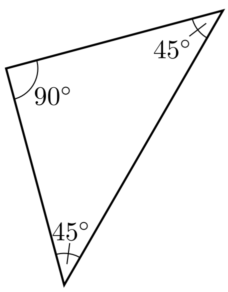


Choose two answers:

- ☒ Isosceles
- ☐ Equilateral
- ☒ Right-angle
- ☐ Scalene

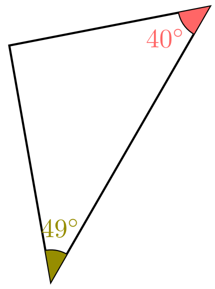
Answer: The triangle has two equal angles of 45°. The third angle x° is:

$$\begin{aligned} x^\circ + 45^\circ + 45^\circ &= 180^\circ \\ x^\circ + 90^\circ &= 180^\circ \quad \text{(Adding known angles)} \\ x^\circ &= 180^\circ - 90^\circ \quad \text{(Subtracting 90 from both sides)} \\ x^\circ &= 90^\circ \end{aligned}$$



Since two angles are equal (45°) and one angle is 90°, the triangle is isosceles and right-angled.

MCQ 30: Classify the triangle:

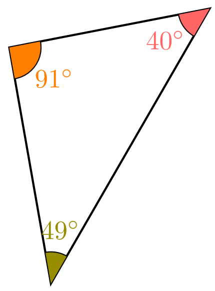


Choose one answer:

- ☐ Isosceles
- ☐ Equilateral
- ☐ Right-angle
- ☒ Scalene

Answer: The third angle x° is:

$$\begin{aligned} x^\circ + 40^\circ + 49^\circ &= 180^\circ \\ x^\circ + 89^\circ &= 180^\circ \quad \text{(Adding known angles)} \\ x^\circ &= 180^\circ - 89^\circ \quad \text{(Subtracting 89 from both sides)} \\ x^\circ &= 91^\circ \end{aligned}$$



Since the angles $(40^\circ, 49^\circ, 91^\circ)$ are all different, the triangle is scalene.

B.4 EVALUATING ANGLE PROPERTIES

MCQ 31: An equilateral triangle can be a right-angled triangle.

Choose one answer:

- ☐ True
- ☒ False

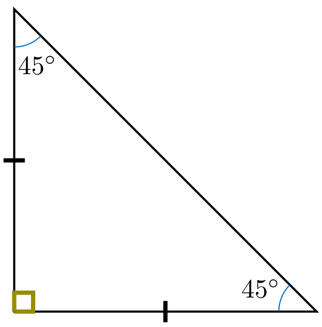
Answer: False. An equilateral triangle has three equal angles of 60°, summing to 180°. A right-angled triangle requires one angle of 90°, which would exceed 180° with two 60° angles, making it impossible.

MCQ 32: An isosceles triangle can be a right-angled triangle.

Choose one answer:

- ☒ True
- ☐ False

Answer: True. An isosceles right-angled triangle has two equal angles of 45° and one right angle of 90°, satisfying the angle sum of 180°. The two equal sides are the legs, and the third side is the hypotenuse.

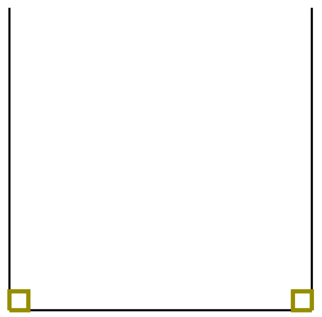


MCQ 33: A triangle can have two right angles.
Choose one answer:

☐ True

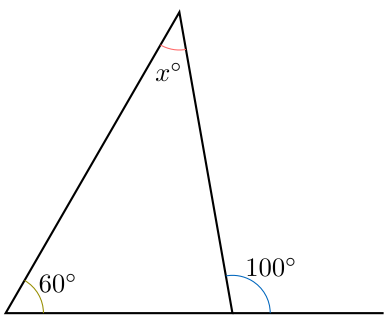
☒ False

Answer: False. The sum of angles in a triangle is 180°. If two angles were 90°, their sum would be 180°, leaving no room for a third angle, which is impossible for a triangle.



B.5 DEDUCTING ANGLES IN TRIANGLE CONFIGURATIONS

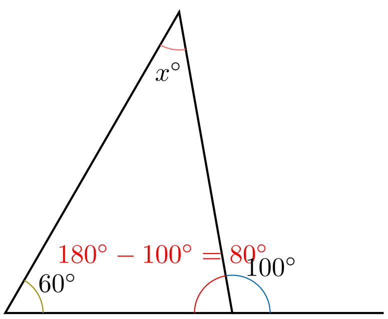
Ex 34: Find the unknown angle in the triangle below:



$x^\circ = \boxed{40}^\circ$

Answer:

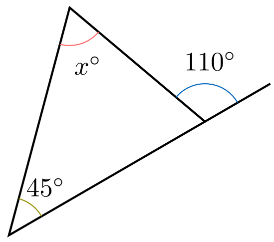
- The angles on a straight line sum to 180°. The exterior angle is 100°, so the interior angle is:



- The sum of the angles in a triangle is 180°. In the triangle, the angles are 60°, 80°, and x°. Therefore:

$$\begin{aligned} x^\circ + 60^\circ + 80^\circ &= 180^\circ \\ x^\circ + 140^\circ &= 180^\circ \quad \text{(Adding known angles)} \\ x^\circ &= 180^\circ - 140^\circ \quad \text{(Subtracting 140 from both sid} \\ x^\circ &= 40^\circ \end{aligned}$$

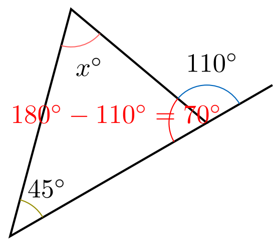
Ex 35: Find the unknown angle in the triangle below:



$x^\circ = \boxed{65}^\circ$

Answer:

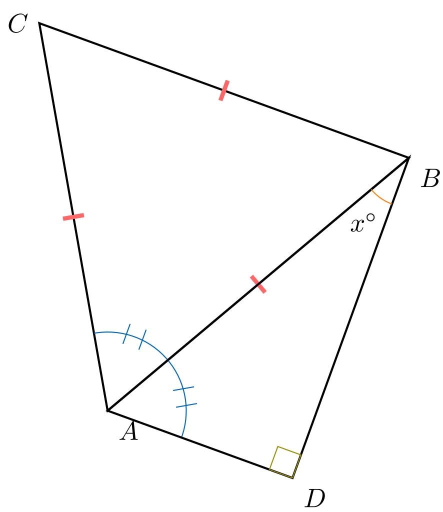
- The angles on a straight line sum to 180°. At point H, the exterior angle is 110°, so the interior angle is:



- The sum of the angles in a triangle is 180°. In the triangle, the angles are 45°, 70°, and x°. Therefore:

$$\begin{aligned} x^\circ + 45^\circ + 70^\circ &= 180^\circ \\ x^\circ + 115^\circ &= 180^\circ \quad \text{(Adding known angles)} \\ x^\circ &= 180^\circ - 115^\circ \quad \text{(Subtracting 115 from both sid} \\ x^\circ &= 65^\circ \end{aligned}$$

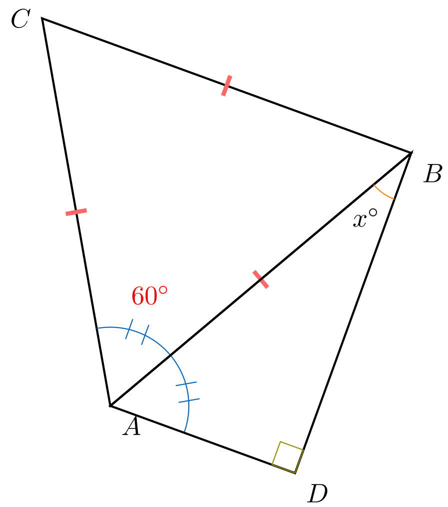
Ex 36: Find the unknown angle in the triangle below:

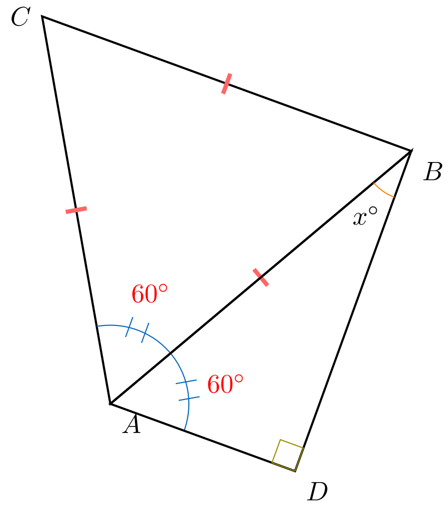


$x^\circ = \boxed{30}^\circ$

Answer: Consider the diagram:

- Triangle ABC is equilateral, as all sides are equal (marked with $|$). Thus, each angle in $\triangle ABC$ is $60°$. Specifically, $\angle BAC = 60°$.



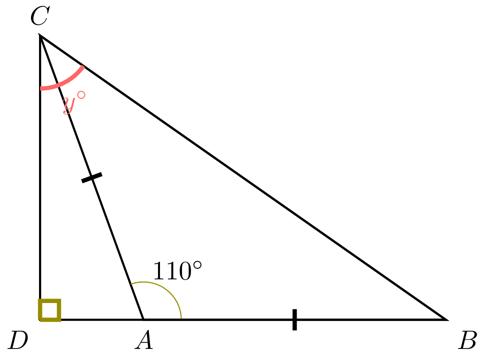


-

- In triangle ABD, $\angle ADB = 90°$ (right angle), and $\angle DAB = 60°$ (since $\angle DAB = \angle BAC$). The sum of angles in $\triangle ABD$ is $180°$. Therefore:

$$\begin{aligned} x° + 60° + 90° &= 180° \\ x° + 150° &= 180° \quad \text{(Adding known angles)} \\ x° &= 180° - 150° \quad \text{(Subtracting 150 from both sides)} \\ x° &= 30° \end{aligned}$$

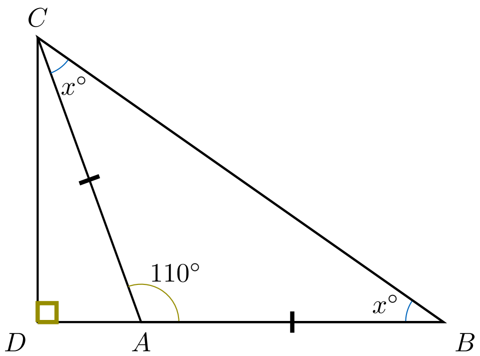
Ex 37: Find the unknown angle in the triangle below:



$y° = \boxed{55}°$

Answer:

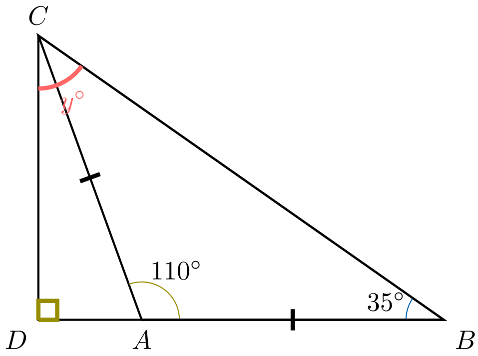
- Let $x°$ be the unknown angle in the triangle ABC



The sum of the angles of a triangle is $180°$. For the triangle ABC,

$$\begin{aligned} x° + x° + 110° &= 180° \\ 2x° + 110° &= 180° && \text{(Collecting like terms)} \\ \therefore 2x° &= 180° - 110° && \text{(Subtract 110 from both si} \\ \therefore 2x° &= 70° \\ \therefore x° &= 70° \div 2 && \text{(Divide by 2 from both side} \\ \therefore x° &= 35° \end{aligned}$$

- For the triangle DCB,

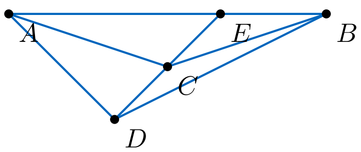


$$\begin{aligned} y^\circ + 90^\circ + 35^\circ &= 180^\circ \\ y^\circ + 125^\circ &= 180^\circ \\ \therefore y^\circ &= 180^\circ - 125^\circ \quad \text{(Subtract 125 from both sides)} \\ \therefore y^\circ &= 55^\circ \end{aligned}$$

C TRIANGLE INEQUALITY THEOREM

C.1 WRITING INEQUALITIES

Ex 38:

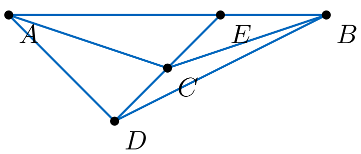


$AD \boxed{<} AC + CD$

Answer: In triangle ACD, the triangle inequality theorem states that the sum of any two sides is greater than the third side. Thus, $AC + CD > AD$. Therefore:

$$AD < AC + CD$$

Ex 39:

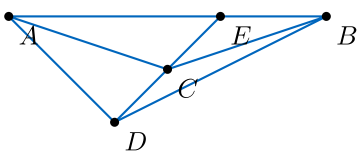


$BC + CA \boxed{>} AB$

Answer: In triangle ABC, the triangle inequality theorem states that the sum of any two sides is greater than the third side. Thus, $BC + CA > AB$. Therefore:

$$BC + CA > AB$$

Ex 40:

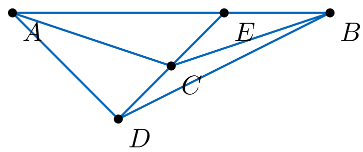


$BE + EA \boxed{=} BA$

Answer: Points B, E, and A are collinear, with E between B and A on segment AB. The length of the path $BE + EA$ equals the total length BA. Therefore:

$$BE + EA = BA$$

Ex 41:



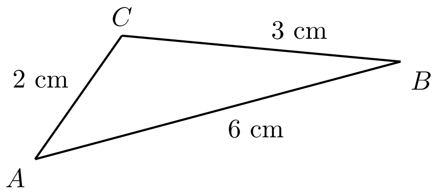
$AE + ED \boxed{>} AD$

Answer: In triangle AED, the triangle inequality theorem states that the sum of any two sides is greater than the third side. Thus, $AE + ED > AD$. Therefore:

$$AE + ED > AD$$

C.2 DETERMINING TRIANGLE EXISTENCE

MCQ 42:



Could these be the side lengths of a triangle?

☐ Yes

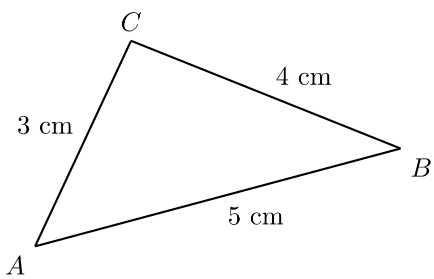
☒ No

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 6$ cm, $BC = 3$ cm, $AC = 2$ cm:

- $2 < 6 + 3 = 9$ (True) (Checking AC)
- $3 < 6 + 2 = 8$ (True) (Checking BC)
- $6 < 3 + 2 = 5$ (False) (Checking AB)

Since 6 is not less than $3 + 2$, the side lengths cannot form a triangle. Therefore, the answer is No.

MCQ 43:



Could these be the side lengths of a triangle?

☒ Yes

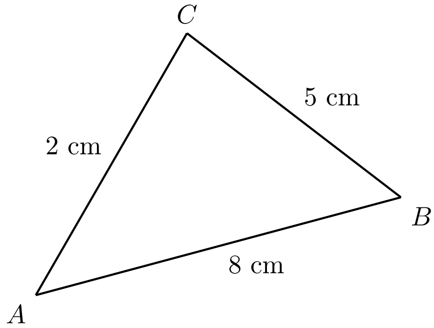
☐ No

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 5$ cm, $BC = 4$ cm, $AC = 3$ cm:

- $3 < 5 + 4 = 9$ (True) (Checking AC)
- $4 < 5 + 3 = 8$ (True) (Checking BC)
- $5 < 4 + 3 = 7$ (True) (Checking AB)

Since all inequalities are true, the side lengths can form a triangle. Therefore, the answer is Yes.

MCQ 44:



Could these be the side lengths of a triangle?

☐ Yes

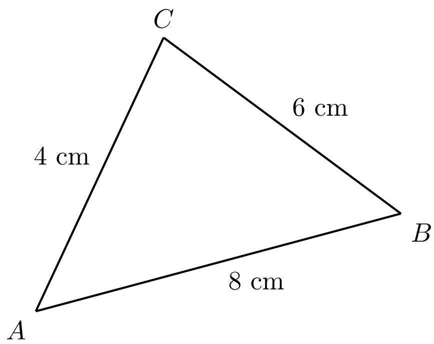
☒ No

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 8$ cm, $BC = 5$ cm, $AC = 2$ cm:

- $2 < 8 + 5 = 13$ (True) (Checking AC)
- $5 < 8 + 2 = 10$ (True) (Checking BC)
- $8 < 5 + 2 = 7$ (False) (Checking AB)

Since 8 is not less than $5 + 2$, the side lengths cannot form a triangle. Therefore, the answer is No.

MCQ 45:



Could these be the side lengths of a triangle?

☒ Yes

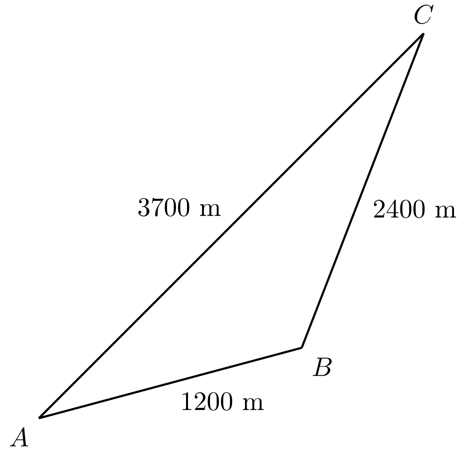
☐ No

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 8$ cm, $BC = 6$ cm, $AC = 4$ cm:

- $4 < 8 + 6 = 14$ (True) (Checking AC)
- $6 < 8 + 4 = 12$ (True) (Checking BC)
- $8 < 6 + 4 = 10$ (True) (Checking AB)

Since all inequalities are true, the side lengths can form a triangle. Therefore, the answer is Yes.

MCQ 46:



Could these be the side lengths of a triangle?

☐ Yes

☒ No

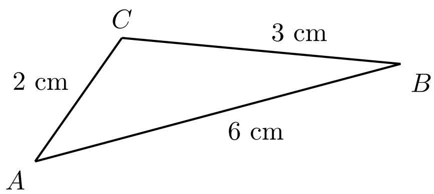
Answer: To form a triangle, each side must be less than the sum of the other two sides. Convert $AC = 3.7$ km to 3700 m. For sides $AB = 1200$ m, $BC = 2400$ m, $AC = 3700$ m:

- $1200 < 2400 + 3700 = 6100$ (True) (Checking AB)
- $2400 < 1200 + 3700 = 4900$ (True) (Checking BC)
- $3700 < 1200 + 2400 = 3600$ (False) (Checking AC)

Since 3700 is not less than $1200 + 2400$, the side lengths cannot form a triangle. Therefore, the answer is No.

C.3 DETERMINING TRIANGLE EXISTENCE

Ex 47:



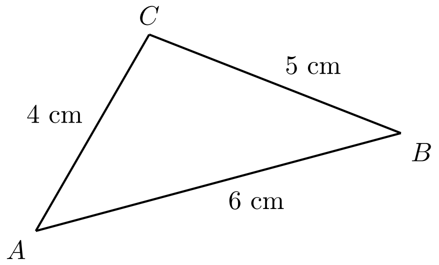
Could these side lengths form a triangle? Justify your answer.

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 6$ cm, $BC = 3$ cm, $AC = 2$ cm:

- $2 < 6 + 3 = 9$ (True) (Checking AC)
- $3 < 6 + 2 = 8$ (True) (Checking BC)
- $6 < 3 + 2 = 5$ (False) (Checking AB)

Since 6 is not less than $3 + 2$, the side lengths cannot form a triangle.

Ex 48:



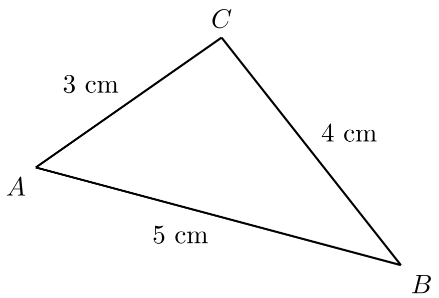
Could these side lengths form a triangle? Justify your answer.

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 6$ cm, $BC = 5$ cm, $AC = 4$ cm:

- $4 < 6 + 5 = 11$ (True) (Checking AC)
- $5 < 6 + 4 = 10$ (True) (Checking BC)
- $6 < 5 + 4 = 9$ (True) (Checking AB)

Since all inequalities are true, the side lengths can form a triangle.

Ex 49:



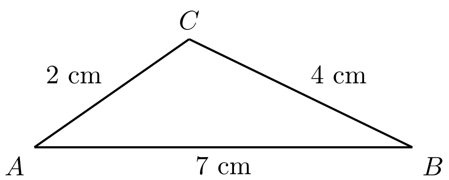
Could these side lengths form a triangle? Justify your answer.

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 5$ cm, $BC = 4$ cm, $AC = 3$ cm:

- $3 < 5 + 4 = 9$ (True) (Checking AC)
- $4 < 5 + 3 = 8$ (True) (Checking BC)
- $5 < 4 + 3 = 7$ (True) (Checking AB)

Since all inequalities are true, the side lengths can form a triangle.

Ex 50:



Could these side lengths form a triangle? Justify your answer.

Answer: To form a triangle, each side must be less than the sum of the other two sides. For sides $AB = 7$ cm, $BC = 4$ cm, $AC = 2$ cm:

- $2 < 7 + 4 = 11$ (True) (Checking AC)
- $4 < 7 + 2 = 9$ (True) (Checking BC)
- $7 < 4 + 2 = 6$ (False) (Checking AB)

Since 7 is not less than $4 + 2$, the side lengths cannot form a triangle.

C.4 EXPLORING TRIANGLE EXISTENCE

Ex 51: ABC is an isosceles triangle with C as the vertex of the equal sides. The perimeter is 10 cm, and $AB = 3$ cm. Can this triangle be constructed? Justify your answer.

Answer:

- **Given Data**
  Triangle ABC is isosceles at C, so $AC = BC$. The perimeter is 10 cm, and $AB = 3$ cm.
- **Calculating the Equal Sides**
  The perimeter is the sum of all sides:
  $$\begin{aligned} \text{Perimeter} &= AB + AC + BC \\ 10 &= 3 + AC + AC \quad (\text{Since } AC = BC) \\ 10 &= 3 + 2AC \\ 2AC &= 10 - 3 \quad (\text{Subtracting 3 from both sides}) \\ 2AC &= 7 \\ AC &= \frac{7}{2} = 3.5 \text{ cm} \quad (\text{Dividing both sides by 2}) \end{aligned}$$
  So, $AC = BC = 3.5$ cm.
- **Checking the Triangle Inequality**
  For sides $AB = 3$ cm, $AC = 3.5$ cm, $BC = 3.5$ cm, each side must be less than the sum of the other two:
  $$\begin{aligned} AB + AC > BC &: \quad 3 + 3.5 = 6.5 > 3.5 \quad (\text{True}) \\ AB + BC > AC &: \quad 3 + 3.5 = 6.5 > 3.5 \quad (\text{True}) \\ AC + BC > AB &: \quad 3.5 + 3.5 = 7 > 3 \quad (\text{True}) \end{aligned}$$
  Since all inequalities are true, the triangle can be constructed.

Ex 52: ABC is an isosceles triangle with C as the vertex of the equal sides. The perimeter is 10 cm, and $AC = 2$ cm. Can this triangle be constructed? Justify your answer.

Answer:

- **Given Data**
  Triangle ABC is isosceles at C, so $AC = BC$. The perimeter is 10 cm, and $AC = 2$ cm, so $BC = 2$ cm.
- **Calculating the Remaining Side**
  The perimeter is the sum of all sides:
  $$\begin{aligned} \text{Perimeter} &= AB + AC + BC \\ 10 &= AB + 2 + 2 \quad (\text{Since } AC = BC = 2 \text{ cm}) \\ 10 &= AB + 4 \\ AB &= 10 - 4 \quad (\text{Subtracting 4 from both sides}) \\ AB &= 6 \text{ cm} \end{aligned}$$
  So, $AB = 6$ cm.
- **Checking the Triangle Inequality**
  For sides $AB = 6$ cm, $AC = 2$ cm, $BC = 2$ cm, each side must be less than the sum of the other two:
  $$\begin{aligned} AB + AC > BC &: \quad 6 + 2 = 8 > 2 \quad (\text{True}) \\ AB + BC > AC &: \quad 6 + 2 = 8 > 2 \quad (\text{True}) \\ AC + BC > AB &: \quad 2 + 2 = 4 > 6 \quad (\text{False}) \end{aligned}$$
  Since $2 + 2 = 4$ is not greater than 6, the side lengths cannot form a triangle.

Ex 53: In triangle ABC, $AB = 5$ cm and $AC = 3$ cm. What are the possible integer lengths for segment $\overline{BC}$? Justify your answer.

Answer:

- **Given Data**
  In triangle ABC, $AB = 5$ cm, $AC = 3$ cm, and BC is an integer length to be determined.

- **Finding the Range for BC**
  To form a triangle, the side lengths must satisfy the triangle inequality theorem: the sum of any two sides must be greater than the third side. For sides $AB = 5$ cm, $AC = 3$ cm, BC, the inequalities are:

$$\begin{aligned} AB + AC > BC: \quad 5 + 3 &> BC \quad \text{(Sum of } AB \text{ and } AC) \\ 8 &> BC \quad \text{(Simplifying)} \\ BC &< 8 \text{ cm} \end{aligned}$$

$$\begin{aligned} AB + BC > AC: \quad 5 + BC &> 3 \quad \text{(Sum of } AB \text{ and } BC) \\ BC &> 3 - 5 \quad \text{(Subtracting 5 from both sides)} \\ BC &> -2 \text{ cm} \quad \text{(Always true since } BC > 0) \end{aligned}$$

$$\begin{aligned} AC + BC > AB: \quad 3 + BC &> 5 \quad \text{(Sum of } AC \text{ and } BC) \\ BC &> 5 - 3 \quad \text{(Subtracting 3 from both sides)} \\ BC &> 2 \text{ cm} \end{aligned}$$

  Combining the constraints, BC must satisfy:

$$2 < BC < 8 \text{ cm}$$

  Since BC must be an integer, the possible values are 3, 4, 5, 6, 7 cm.

- **Verifying the Solutions**
  Check the inequalities for each integer value:
  - $BC = 3$ cm: $5 + 3 > 3$ $(8 > 3)$, $5 + 3 > 3$ $(8 > 3)$, $3 + 3 > 5$ $(6 > 5)$. All true.
  - $BC = 4$ cm: $5 + 3 > 4$ $(8 > 4)$, $5 + 4 > 3$ $(9 > 3)$, $3 + 4 > 5$ $(7 > 5)$. All true.
  - $BC = 5$ cm: $5 + 3 > 5$ $(8 > 5)$, $5 + 5 > 3$ $(10 > 3)$, $3 + 5 > 5$ $(8 > 5)$. All true.
  - $BC = 6$ cm: $5 + 3 > 6$ $(8 > 6)$, $5 + 6 > 3$ $(11 > 3)$, $3 + 6 > 5$ $(9 > 5)$. All true.
  - $BC = 7$ cm: $5 + 3 > 7$ $(8 > 7)$, $5 + 7 > 3$ $(12 > 3)$, $3 + 7 > 5$ $(10 > 5)$. All true.

  Therefore, the possible integer lengths for BC are 3 cm, 4 cm, 5 cm, 6 cm, and 7 cm.