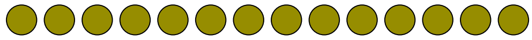
PROPERTIES OF INTEGERS

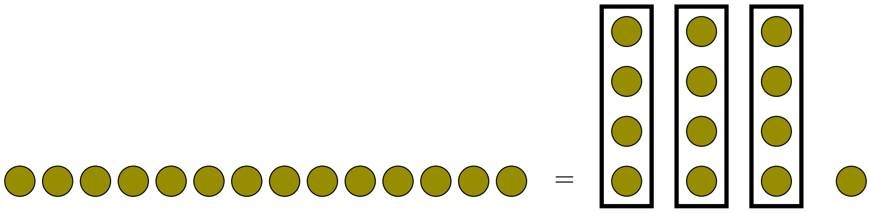
A DIVISION WITH REMAINDERS

Discover: Hugo has 13 marbles to divide evenly into 3 groups.



How many marbles will be in each group, and how many will be left over?

Answer:



There are 4 marbles in each group, with 1 marble left over. We call the leftover marble the remainder. We can write this division as:

$$13 \div 3 = 4\text{R}1$$

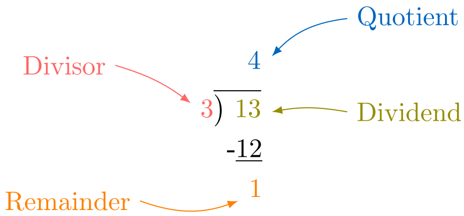
To check our answer, we can use multiplication and addition:

$$13 = 3 \times 4 + 1.$$

Theorem Division with Remainder

For any two whole numbers, a dividend and a non-zero divisor, there exists a unique quotient and a unique remainder that are also whole numbers, such that:

$$\begin{array}{ccccccccc} 13 & = & 3 & \times & 4 & + & 1 & \text{with} & 1 < 3 \\ \text{Dividend} & = & \text{Divisor} & \times & \text{Quotient} & + & \text{Remainder} & \text{with} & \text{Remainder} < \text{Divisor} \end{array}$$



B DIVISIBILITY

Definition Divisible

A whole number is said to be **divisible** by another non-zero whole number if the remainder is zero when you divide the first number by the second number. In this case, we say that the second number is a **divisor** of the first number.

Ex: Is 10 divisible by 5?

Answer: Yes, 10 is divisible by 5 because the remainder of the division is 0: $10 = 5 \times 2 + 0$.

Ex: Is 13 divisible by 5?

Answer: No, 13 is not divisible by 5 because the remainder of the division is 3: $13 = 5 \times 2 + 3$.

Definition Multiple

A whole number is said to be a **multiple** of another non-zero whole number if the first number can be obtained by multiplying the second number by some whole number. In other words, the first number appears in the multiplication table of the second number. In this case, we say that the second number is a **factor** of the first number.

Ex: Is 10 a multiple of 5?

Answer: Yes, 10 is a multiple of 5 because $5 \times 2 = 10$. So 5 is a factor of 10.

Theorem Divisible $\Leftrightarrow$ Multiple

For two whole numbers, the first number is **divisible** by the second number if and only if the first number is a **multiple** of the second number.

Proof

This is a direct consequence of the uniqueness of the quotient and remainder in the division with remainder theorem. If the remainder is zero, then the dividend is exactly the product of the divisor and the quotient, which is the definition of a multiple. Conversely, if a number is a multiple of another, then by definition it can be written as a product, implying a zero remainder upon division.

Ex: Is 1000 divisible by 10?

Answer: Yes, as $1000 = 100 \times 10$, 1000 is a multiple of 10. Therefore, 1000 is divisible by 10.

C DIVISIBILITY CRITERIA

Divisibility criteria are methods that allow us to quickly determine if a whole number is divisible by another whole number without performing long division. These rules are useful for simplifying calculations and understanding number properties. Here are some common divisibility criteria:

Proposition Divisibility Criteria for 2 and 5

- A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).
- A number is divisible by 5 if its last digit is 0 or 5.

Ex: Determine whether 946 is divisible by 2.

Answer: 946 is divisible by 2 because its last digit is 6, which is even.

Ex: Determine whether 947 is divisible by 5.

Answer: 947 is not divisible by 5 because its last digit is 7, which is not 0 or 5.

Proposition Divisibility Criteria for 3 and 9

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 9 if the sum of its digits is divisible by 9.

Ex: Determine whether 948 is divisible by 3.

Answer: 948 is divisible by 3 because the sum of its digits, $9 + 4 + 8 = 21$, is divisible by 3 ($21 = 3 \times 7$).

Ex: Determine whether 948 is divisible by 9.

Answer: 948 is not divisible by 9 because the sum of its digits, $9 + 4 + 8 = 21$, is not divisible by 9 ($21 = 9 \times 2 + 3$).

Proposition Divisibility Criteria for 4

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex: Determine whether 917 is divisible by 4.

Answer: 917 is not divisible by 4 because the number formed by its last two digits, 17, is not divisible by 4 ($17 = 4 \times 4 + 1$).