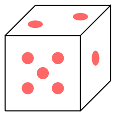
# PROBABILITY

## A ALGEBRA OF EVENTS

### A.1 SAMPLE SPACES

#### A.1.1 FINDING THE SAMPLE SPACES

**MCQ 1:** A fair six-sided die is rolled once.



Find the sample space.

☐ $\{1, 2, 3, 4, 5\}$

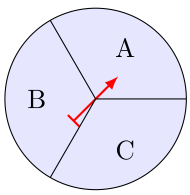
☐ $\{1, 2, 3, 4, 5, 6, 7\}$

☒ $\{1, 2, 3, 4, 5, 6\}$

*Answer:*

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

**MCQ 2:** You spin the arrow on the spinner below.




Find the sample space.

☒ $\{A, B, C\}$

☐ $\{A, B\}$

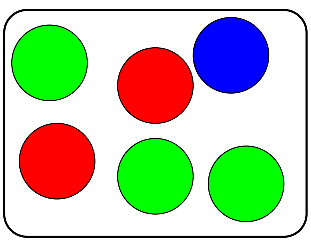
☐ $\{A, C\}$

*Answer:*

- The sample space is all possible outcomes of spinning the arrow.
- Here, the possible outcomes are $A$, $B$, and $C$.
- So, the sample space is $\{A, B, C\}$.

**MCQ 3:** A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls. To calculate probabilities, we treat each ball as a unique outcome.

Bag



Find the sample space.

☐ {Red, Blue, Green}

☐ {2 Red, 1 Blue, 3 Green}

☒ $\{R_1, R_2, B_1, G_1, G_2, G_3\}$

*Answer:*

- To make sure every outcome is equally likely, we must list every single ball in the sample space.
- We can give each ball a unique label:
  - Red balls: $R_1, R_2$
  - Blue ball: $B_1$
  - Green balls: $G_1, G_2, G_3$
- So, the complete sample space is: $\{R_1, R_2, B_1, G_1, G_2, G_3\}$.

**MCQ 4:** A letter is chosen randomly from the word BANANA. To calculate probabilities, we treat each letter's position as a unique outcome.
Find the sample space for the chosen letter.

☐ {B, N, A}

☐ {B, A, N, A, N, A}

☒ $\{B_1, A_2, N_3, A_4, N_5, A_6\}$

*Answer:*

- To ensure every outcome is equally likely, we must treat each of the 6 letters in the word as a distinct outcome. We can use subscripts to show their positions.
- The letters are: $B_1$ (first letter), $A_2$ (second), $N_3$ (third), $A_4$ (fourth), $N_5$ (fifth), and $A_6$ (sixth).
- So, the complete sample space is: $\{B_1, A_2, N_3, A_4, N_5, A_6\}$.

**MCQ 5:** A couple is expecting a baby. What is the sample space for this random experiment?

☒ {boy, girl}

☐ {boy}

☐ {girl}

*Answer:*

- The sample space is all possible outcomes for the sex of the baby.
- The possible outcomes are "boy" or "girl".
- So, the sample space is {boy, girl}.

## A.2 EVENTS

### A.2.1 FINDING EVENTS FOR DIE-ROLLING EVENTS

**MCQ 6:** If you roll a die, what is the set of outcomes for the event "getting a 3"?

☐ $\{1, 3, 5\}$

☐ $\{2, 3, 4\}$

☐ $\{1, 2, 3\}$

☒ $\{3\}$

*Answer:* The set of outcomes for the event "getting a 3" is $\{3\}$.

**MCQ 7:** If you roll a die, what is the set of outcomes for the event "getting a 5 or 6"?

☒ $\{5, 6\}$

☐ $\{4, 5, 6\}$

☐ $\{1, 2, 3\}$

☐ $\{3, 4, 5\}$

*Answer:* The set of outcomes for the event "getting a 5 or 6" is $\{5, 6\}$.

**MCQ 8:** If you roll a die, what is the set of outcomes for the event "getting a number greater than or equal to 4"?

☐ $\{1, 2, 3\}$

☒ $\{4, 5, 6\}$

☐ $\{3, 4, 5\}$

☐ $\{2, 3, 4\}$

*Answer:* The set of outcomes for the event "getting a number greater than or equal to 4" is $\{4, 5, 6\}$.

**MCQ 9:** If you roll a die, what is the set of outcomes for the event "even number"?

☐ $\{1, 3, 5\}$

☒ $\{2, 4, 6\}$

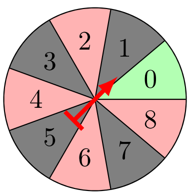
☐ $\{1, 2, 3, 4, 5, 6\}$

☐ $\{2, 3, 4, 5\}$

*Answer:* The set of outcomes for the event "even number" is $\{2, 4, 6\}$.

### A.2.2 FINDING EVENTS IN A CASINO SPINNER

**MCQ 10:** If you spin the spinner below, what is the set of outcomes for the event "getting a 2"?




☒ $\{2\}$

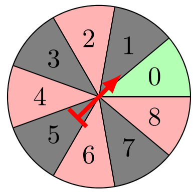
☐ $\{1, 2, 3\}$

☐ $\{2, 4, 6\}$

☐ $\{0, 1, 2\}$

*Answer:* The set of outcomes for the event "getting a 2" is $\{2\}$.

**MCQ 11:** If you spin the spinner below, what is the set of outcomes for the event "red"?




☐ $\{1, 3, 5, 7\}$

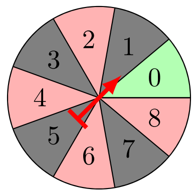
☐ $\{0\}$

☒ $\{2, 4, 6, 8\}$

☐ $\{1, 2, 3, 4\}$

*Answer:* The set of outcomes for the event "red" is $\{2, 4, 6, 8\}$.

**MCQ 12:** If you spin the spinner below, what is the set of outcomes for the event "getting an odd number"?




☐ $\{0, 1, 3\}$

☐ $\{2, 4, 6, 8\}$

☐ $\{1, 2, 3, 4\}$

☒ $\{1, 3, 5, 7\}$

*Answer:* The set of outcomes for the event "getting an odd number" is $\{1, 3, 5, 7\}$.

## A.3 COMPLEMENTARY EVENTS

### A.3.1 FINDING THE COMPLEMENTARY EVENTS

**MCQ 13:** If you roll a die, what is the set of outcomes for the event "not getting a 6"?

☐ $\{2, 3, 4\}$

☐ $\{1, 2, 3, 4, 5, 6\}$

☒ $\{1, 2, 3, 4, 5\}$

☐ $\{1, 3, 5\}$

*Answer:* The set of outcomes for the event "not getting a 6" is $\{1, 2, 3, 4, 5\}$.

**MCQ 14:** If you roll a die, what is the set of outcomes for the event "not getting an odd number"?

☒ $\{2,4,6\}$

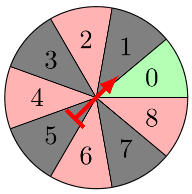
☐ $\{1,2,3,4,5,6\}$

☐ $\{1,2,3\}$

☐ $\{1,3,5\}$

*Answer:* The set of outcomes for the event "not getting an odd number" is $\{2,4,6\}$.

**MCQ 15:** If you spin the spinner below, what is the set of outcomes for the event "not getting a 4"?




☐ $\{1,2,3,4\}$

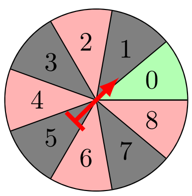
☒ $\{0,1,2,3,5,6,7,8\}$

☐ $\{2,4,6,8\}$

☐ $\{4,5,6\}$

*Answer:* The set of outcomes for the event "not getting a 4" is $\{0,1,2,3,5,6,7,8\}$.

**MCQ 16:** If you spin the spinner below, what is the set of outcomes for the event "not getting red"?




☒ $\{0,1,3,5,7\}$

☐ $\{2,4,6,8\}$

☐ $\{1,2,3,4,5,6,7,8\}$

☐ $\{0\}$

*Answer:* The set of outcomes for the event "not getting red" is $\{0,1,3,5,7\}$.

## A.4 MULTI-STEP RANDOM EXPERIMENTS

### A.4.1 FINDING OUTCOME IN A TABLE

**MCQ 17:** The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

| second child<br>first child | $B$ | $G$ |
|---|---|---|
| $B$ | $BB$ | ? |
| $G$ | $GB$ | $GG$ |

Find the missing outcome.

☐ $BB$

☒ $BG$

☐ $GB$

*Answer:*

- The first child is represented by the row ("first child"), and the second child by the column ("second child").
- The missing outcome is $BG$.

**MCQ 18:** The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

| letter 2<br>letter 1 | $M$ | $A$ | $T$ |
|---|---|---|---|
| $M$ | $MM$ | $MA$ | $MT$ |
| $A$ | $AM$ | $AA$ | $AT$ |
| $T$ | $TM$ | ? | $TT$ |

Find the missing outcome.

☐ $TT$

☒ $TA$

☐ $AT$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $TA$.

**MCQ 19:** The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

| letter 2<br>letter 1 | $C$ | $O$ | $D$ | $E$ |
|---|---|---|---|---|
| $C$ | $CC$ | $CO$ | $CD$ | $CE$ |
| $O$ | $OC$ | $OO$ | $OD$ | $OE$ |
| $D$ | $DC$ | ? | $DD$ | $DE$ |
| $E$ | $EC$ | $EO$ | $ED$ | $EE$ |

Find the missing outcome.

☒ $DO$

☐ $OD$

☐ $DC$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $DO$.

**MCQ 20:** The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

| letter 2 / letter 1 | $N$ | $O$ | $D$ | $E$ |
|---|---|---|---|---|
| $N$ | **X** | ? | $ND$ | $NE$ |
| $O$ | $ON$ | **X** | $OD$ | $OE$ |
| $D$ | $DN$ | $DO$ | **X** | $DE$ |
| $E$ | $EN$ | $EO$ | $ED$ | **X** |

Find the missing outcome.

☐ $NN$

☒ $NO$

☐ $ON$

*Answer:*

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is $NO$.

**MCQ 21:** The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

| Player 2 / Player 1 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | ? | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Find the missing outcome.

☒ $AB$

☐ $BA$

☐ $CA$

*Answer:*

- The first player is "Player 1" (row), and the second player is "Player 2" (column).
- The missing outcome is $AB$.

## A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

**Ex 22:** The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

| second child / first child | $B$ | $G$ |
|---|---|---|
| $B$ | $BB$ | $BG$ |
| $G$ | $GB$ | $GG$ |

Count the number of possible outcomes.

$\boxed{4}$ possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{BB, BG, GB, GG\}$.
- The number of possible outcomes is 4.

**Ex 23:** There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

| position 2 / position 1 | $C$ | $D$ |
|---|---|---|
| $A$ | $AC$ | $AD$ |
| $B$ | $BC$ | $BD$ |

Count the number of possible outcomes.

$\boxed{4}$ possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{AC, AD, BC, BD\}$.
- The number of possible outcomes is 4.

**Ex 24:** There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

| Player 2 / Player 1 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Count the number of possible outcomes.

$\boxed{12}$ possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC\}$
- The number of possible outcomes is 12.

**Ex 25:** There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

| Tuesday / Monday | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $X$ | **X** | $XY$ | $XZ$ |
| $Y$ | $YX$ | **X** | $YZ$ |
| $Z$ | $ZX$ | $ZY$ | **X** |

Count the number of possible outcomes.

$\boxed{6}$ possible outcomes.

*Answer:*

- The sample space (the possible outcomes) is $\{XY, XZ, YX, YZ, ZX, ZY\}$.
- The number of possible outcomes is 6.

### A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

**Ex 26:** There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

| Player 2 / Player 1 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Count the number of outcomes for the event that player A is selected.

$\boxed{6}$ outcomes.

*Answer:*

- The event "player A is selected" includes all outcomes where A is either Player 1 or Player 2.
- From the table:
  - When A is Player 1 (row A): $AB$, $AC$, $AD$ (3 outcomes).
  - When A is Player 2 (column A): $BA$, $CA$, $DA$ (3 outcomes).
- Total outcomes: $\{AB, AC, AD, BA, CA, DA\}$, which is 6 outcomes.

**Ex 27:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "double" (both dice show the same number).

$\boxed{6}$ outcomes.

*Answer:*

- The event "double" includes all outcomes where the red die and blue die show the same number.
- From the table:
  - Doubles: 11, 22, 33, 44, 55, 66 (6 outcomes).
- Total outcomes: 6.

**Ex 28:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

$\boxed{11}$ outcomes.

*Answer:*

- The event "at least one 6" includes all outcomes where at least one of the dice shows a 6.
- From the table:
  - Red die = 6 (row 6): 6 outcomes.
  - Blue die = 6 (column 6), excluding (6,6) already counted: 5 more outcomes.
- Total outcomes: 11.

**Ex 29:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is equal to 11."

$\boxed{2}$ outcomes.

*Answer:*

- The event "the sum of the dice is 11" includes all outcomes where the red die and blue die sum to 11.
- From the table:
  - Possible pairs: 56 (5+6=11), 65 (6+5=11).
- Total outcomes: 2.

**Ex 30:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is equal to 7."

$\boxed{6}$ outcomes.

*Answer:*

- The event "the sum of the dice is 7" includes all outcomes where the red die and blue die sum to 7.
- From the table:
  - Possible pairs: 16 (1+6), 25 (2+5), 34 (3+4), 43 (4+3), 52 (5+2), 61 (6+1).
- Total outcomes: 6.

**Ex 31:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Count the number of outcomes for the event "the sum of the dice is less than or equal to 3."

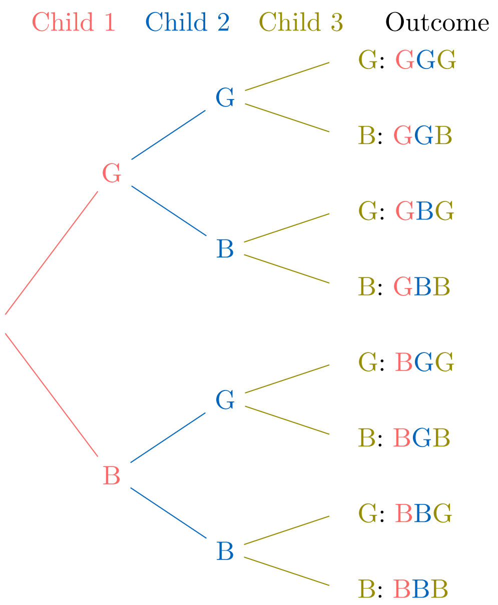
$\boxed{3}$ outcomes.

*Answer:*

- The event "the sum of the dice is less than or equal to 3" includes all outcomes where the sum is 2 or 3.
- From the table:
  - Possible pairs: 11 (1+1=2), 12 (1+2=3), 21 (2+1=3).
- Total outcomes: 3.

## A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TREE DIAGRAM

**Ex 32:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



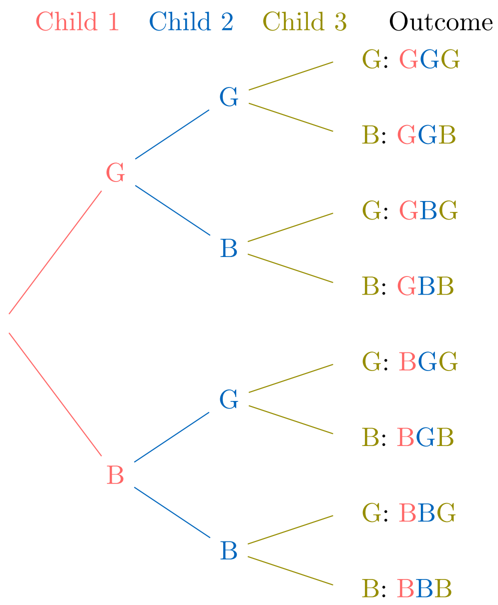

Count the number of possible outcomes for the event where the first child is a boy.

$\boxed{4}$

*Answer:*

- The event where the first child is a boy includes all outcomes starting with B, represented as $B**$.
- These outcomes are: BBB, BBG, BGB, BGG.
- The number of possible outcomes is 4.

**Ex 33:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



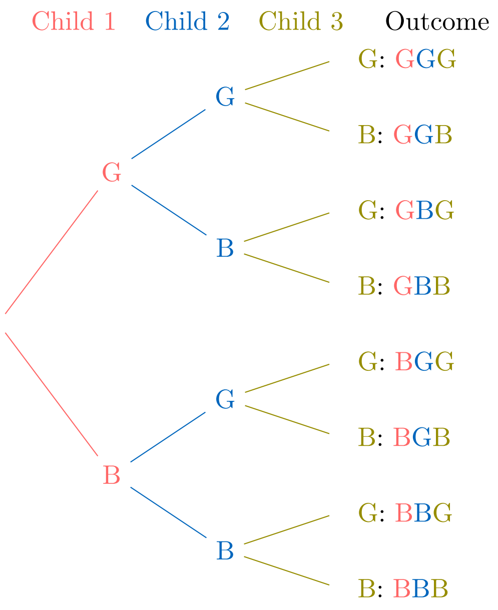

Count the number of possible outcomes for the event where there are exactly two girls.

$\boxed{3}$

*Answer:*

- The event where there are exactly two girls includes all outcomes with exactly two G's and one B.
- These outcomes are: BGG, GBG, GGB.
- The number of possible outcomes is 3.

**Ex 34:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible sex outcomes for the three children.



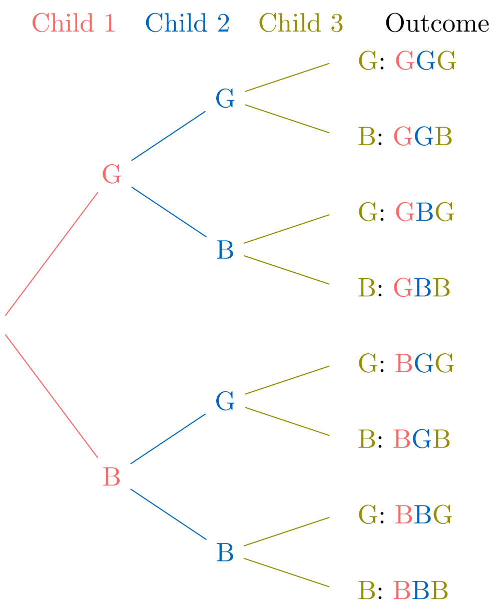

Count the number of possible outcomes for the event where there are at least two girls.

$\boxed{4}$

*Answer:*

- The event where there are at least two girls includes all outcomes with two or three girls.
- These outcomes are: BGG, GBG, GGB, GGG.
- The number of possible outcomes is 4.

**Ex 35:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.




Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

$\boxed{6}$

*Answer:*

- The event where the family has mixed-sex children includes all outcomes with at least one boy (B) and one girl (G), excluding all-boys (BBB) and all-girls (GGG).
- These outcomes are: BBG, BGB, BGG, GGB, GBG, GBB.
- The number of possible outcomes is 6.

## A.5 $E$ OR $F$

### A.5.1 FINDING THE UNION OF TWO EVENTS IN DICE EXPERIMENT

**MCQ 36:** Consider the roll of a standard six-sided die. Let event $E$ be the event of rolling an even number, and let event $F$ be the event of rolling a number less than 4. Find $E$ or $F$.

- ☐ $E$ or $F = \{2\}$
- ☒ $E$ or $F = \{1, 2, 3, 4, 6\}$
- ☐ $E$ or $F = \{1, 2, 3, 4, 5, 6\}$
- ☐ $E$ or $F = \{1, 2, 3\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $E$, rolling an even number, is $\{2, 4, 6\}$.
- Event $F$, rolling a number less than 4, is $\{1, 2, 3\}$.
- The event $E$ or $F$ (the union $E \cup F$) includes all outcomes that are in $E$, in $F$, or in both. Combining these, we get $\{1, 2, 3, 4, 6\}$.

**MCQ 37:** Consider the roll of a standard six-sided die. Let event $G$ be the event of rolling a number greater than 3, and let event $H$ be the event of rolling a prime number. Find $G$ or $H$.

- ☒ $G$ or $H = \{2, 3, 4, 5, 6\}$
- ☐ $G$ or $H = \{4, 5, 6\}$
- ☐ $G$ or $H = \{2, 3, 5\}$
- ☐ $G$ or $H = \{1, 2, 3, 4, 5, 6\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $G$, rolling a number greater than 3, is $\{4, 5, 6\}$.
- Event $H$, rolling a prime number, is $\{2, 3, 5\}$.
- The event $G$ or $H$ (the union $G \cup H$) includes all outcomes that are in $G$, in $H$, or in both. Combining these, we get $\{2, 3, 4, 5, 6\}$.

**MCQ 38:** Consider the roll of a standard six-sided die. Let event $I$ be the event of rolling a number divisible by 3, and let event $J$ be the event of rolling a number less than 5. Find $I$ or $J$.

- ☐ $I$ or $J = \{3, 6\}$
- ☐ $I$ or $J = \{1, 2, 3, 4\}$
- ☐ $I$ or $J = \{1, 2, 3, 4, 5, 6\}$
- ☒ $I$ or $J = \{1, 2, 3, 4, 6\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $I$, rolling a number divisible by 3, is $\{3, 6\}$.
- Event $J$, rolling a number less than 5, is $\{1, 2, 3, 4\}$.
- The event $I$ or $J$ (the union $I \cup J$) includes all outcomes that are in $I$, in $J$, or in both. Combining these, we get $\{1, 2, 3, 4, 6\}$.

### A.5.2 FINDING THE UNION OF TWO EVENTS IN FAMILY EXPERIMENT

**MCQ 39:** Consider a family with three children, assuming each child is equally likely to be a boy or a girl and the genders are independent. Let event $A$ be the event of having only boys, and let event $B$ be the event of having only girls. Find $A$ or $B$.

- ☐ $A$ or $B = \emptyset$
- ☒ $A$ or $B = \{\text{BBB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{BBB}\}$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, having only boys, is $\{\text{BBB}\}$.
- Event $B$, having only girls, is $\{\text{GGG}\}$.
- The event $A$ or $B$ (the union $A \cup B$) includes all outcomes that are in $A$, in $B$, or in both. Combining these, we get $\{\text{BBB}, \text{GGG}\}$.

**MCQ 40:** Consider a family with three children. Let event $A$ be the event that the first two children are girls, and let event $B$ be the event of having only girls. Find $A$ or $B$.

- ☐ $A$ or $B = \emptyset$
- ☒ $A$ or $B = \{\text{GGB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{GGG}\}$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, the first two children are girls, is $\{\text{GGB}, \text{GGG}\}$.
- Event $B$, having only girls, is $\{\text{GGG}\}$.
- The event $A$ or $B$ (the union $A \cup B$) includes all outcomes that are in $A$, in $B$, or in both. Combining these, we get $\{\text{GGB}, \text{GGG}\}$.

**MCQ 41:** Consider a family with three children. Let event $A$ be the event of having exactly two girls, and let event $B$ be the event that the first two children are girls. Find $A$ or $B$.

- ☐ $A$ or $B = \emptyset$
- ☒ $A$ or $B = \{\text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ $A$ or $B = \{\text{BGG}, \text{GBG}, \text{GGB}\}$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, having exactly two girls, is $\{\text{BGG}, \text{GBG}, \text{GGB}\}$.
- Event $B$, the first two children are girls, is $\{\text{GGB}, \text{GGG}\}$.
- The event $A$ or $B$ (the union $A \cup B$) includes all outcomes that are in $A$, in $B$, or in both. Combining these, we get $\{\text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$.

### A.5.3 FINDING THE UNION OF TWO EVENTS FROM A TABLE

**MCQ 42:** In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

**Table: Students**

| Name | Age | Gender |
|---|---|---|
| A | 15 | Female |
| B | 17 | Male |
| C | 16 | Female |
| D | 15 | Male |
| E | 14 | Female |
| F | 17 | Female |

Let event $A$ be the event of selecting a girl, and event $B$ be the event of selecting a student aged 17 or older.
Find $A$ or $B$.

- ☐ $A$ or $B = \{\text{A, B, C, D, E, F}\}$
- ☐ $A$ or $B = \{\text{C, F}\}$
- ☐ $A$ or $B = \{\text{A, B, C, D, F}\}$
- ☒ $A$ or $B = \{\text{A, B, C, E, F}\}$

*Answer:*

- The sample space is $\{\text{A, B, C, D, E, F}\}$.
- Event $A$, selecting a girl, is $\{\text{A, C, E, F}\}$.
- Event $B$, selecting a student older or equal than 17, is $\{\text{B, F}\}$.
- The event $A$ or $B$ (the union $A \cup B$) includes all outcomes that are in $A$, in $B$, or in both. Combining these, we get $\{\text{A, B, C, E, F}\}$.

**MCQ 43:** In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

**Table: Students**

| Name | Age | Instrument |
|---|---|---|
| G | 14 | Violin |
| H | 15 | Piano |
| I | 17 | Guitar |
| J | 16 | Drums |
| K | 15 | Flute |
| L | 18 | Violin |

Let event $X$ be the event of selecting a student who plays the violin, and event $Y$ be the event of selecting a student aged 16 or older. Find $X$ or $Y$.

☒ $X$ or $Y$ = {G, I, J, L}

☐ $X$ or $Y$ = {G, H, I, J, L}

☐ $X$ or $Y$ = {G, L}

☐ $X$ or $Y$ = {I, J, L}

*Answer:*

- The sample space is {G, H, I, J, K, L}.
- Event $X$, selecting a student who plays the violin, is {G, L}.
- Event $Y$, selecting a student aged 16 or older, is {I, J, L}.
- The event $X$ or $Y$ (the union $X \cup Y$) includes all outcomes that are in $X$, in $Y$, or in both. Combining these, we get {G, I, J, L}.

**MCQ 44:** In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

**Table: Players**

| Name | Height (cm) | Position |
|---|---|---|
| M | 180 | Forward |
| N | 170 | Goalkeeper |
| O | 185 | Defender |
| P | 175 | Midfielder |
| Q | 165 | Forward |
| R | 190 | Defender |

Let event $U$ be the event of selecting a defender, and event $V$ be the event of selecting a player taller than or equal to 180 cm. Find $U$ or $V$.

☒ $U$ or $V$ = {M, O, R}

☐ $U$ or $V$ = {O, R}

☐ $U$ or $V$ = {M, N, O, R}

☐ $U$ or $V$ = {M, O, P, R}

*Answer:*

- The sample space is {M, N, O, P, Q, R}.
- Event $U$, selecting a defender, is {O, R}.
- Event $V$, selecting a player taller than or equal to 180 cm, is {M, O, R}.
- The event $U$ or $V$ (the union $U \cup V$) includes all outcomes that are in $U$, in $V$, or in both. Combining these, we get {M, O, R}.

## A.6 $E$ AND $F$

### A.6.1 FINDING THE INTERSECTION OF TWO EVENTS IN DICE EXPERIMENT

**MCQ 45:** Consider the roll of a standard six-sided die. Let event $E$ be the event of rolling an even number, and let event $F$ be the event of rolling a number less than 4. Find $E$ and $F$.

☐ $E$ and $F = \{1, 2, 3, 4, 6\}$

☐ $E$ and $F = \{1, 2, 3\}$

☐ $E$ and $F = \{2, 4, 6\}$

☒ $E$ and $F = \{2\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $E$, rolling an even number, is $\{2, 4, 6\}$.
- Event $F$, rolling a number less than 4, is $\{1, 2, 3\}$.
- The event $E$ and $F$ (the intersection $E \cap F$) includes all outcomes that are in both $E$ and $F$. Combining these, we get $\{2\}$.

**MCQ 46:** Consider the roll of a standard six-sided die. Let event $G$ be the event of rolling a number greater than 3, and let event $H$ be the event of rolling a prime number. Find $G$ and $H$.

☐ $G$ and $H = \{2, 3, 4, 5, 6\}$

☐ $G$ and $H = \{2, 3, 5\}$

☒ $G$ and $H = \{5\}$

☐ $G$ and $H = \{4, 5, 6\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $G$, rolling a number greater than 3, is $\{4, 5, 6\}$.
- Event $H$, rolling a prime number, is $\{2, 3, 5\}$.
- The event $G$ and $H$ (the intersection $G \cap H$) includes all outcomes that are in both $G$ and $H$. Combining these, we get $\{5\}$.

**MCQ 47:** Consider the roll of a standard six-sided die. Let event $I$ be the event of rolling a number divisible by 3, and let event $J$ be the event of rolling a number less than 5. Find $I$ and $J$.

☐ $I$ and $J = \{1, 2, 3, 4, 6\}$

☐ $I$ and $J = \{1, 2, 3, 4\}$

☒ $I$ and $J = \{3\}$

☐ $I$ and $J = \{3, 6\}$

*Answer:*

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event $I$, rolling a number divisible by 3, is $\{3, 6\}$.
- Event $J$, rolling a number less than 5, is $\{1, 2, 3, 4\}$.
- The event $I$ and $J$ (the intersection $I \cap J$) includes all outcomes that are in both $I$ and $J$. Combining these, we get $\{3\}$.

## A.6.2 FINDING THE INTERSECTION OF TWO EVENTS IN FAMILY EXPERIMENT

**MCQ 48:** Consider a family with three children. Let event $A$ be the event of having only boys, and let event $B$ be the event of having only girls. Find $A$ and $B$.

- ☐ $A$ and $B = \{\text{BBB}, \text{GGG}\}$
- ☐ $A$ and $B = \{\text{BBB}\}$
- ☐ $A$ and $B = \{\text{GGG}\}$
- ☒ $A$ and $B = \emptyset$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, having only boys, is $\{\text{BBB}\}$.
- Event $B$, having only girls, is $\{\text{GGG}\}$.
- The event $A$ and $B$ (the intersection $A \cap B$) includes all outcomes that are in both $A$ and $B$. Combining these, we get $\emptyset$.

**MCQ 49:** Consider a family with three children. Let event $A$ be the event that the first two children are girls, and let event $B$ be the event of having only girls. Find $A$ and $B$.

- ☐ $A$ and $B = \{\text{GGB}, \text{GGG}\}$
- ☐ $A$ and $B = \emptyset$
- ☐ $A$ and $B = \{\text{GGB}\}$
- ☒ $A$ and $B = \{\text{GGG}\}$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, the first two children are girls, is $\{\text{GGB}, \text{GGG}\}$.
- Event $B$, having only girls, is $\{\text{GGG}\}$.
- The event $A$ and $B$ (the intersection $A \cap B$) includes all outcomes that are in both $A$ and $B$. Combining these, we get $\{\text{GGG}\}$.

**MCQ 50:** Consider a family with three children. Let event $A$ be the event of having exactly two girls, and let event $B$ be the event that the first two children are girls. Find $A$ and $B$.

- ☒ $A$ and $B = \{\text{GGB}\}$
- ☐ $A$ and $B = \{\text{BGG}, \text{GBG}, \text{GGB}\}$
- ☐ $A$ and $B = \{\text{GGG}\}$
- ☐ $A$ and $B = \{\text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$

*Answer:*

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event $A$, having exactly two girls, is $\{\text{BGG}, \text{GBG}, \text{GGB}\}$.
- Event $B$, the first two children are girls, is $\{\text{GGB}, \text{GGG}\}$.
- The event $A$ and $B$ (the intersection $A \cap B$) includes all outcomes that are in both $A$ and $B$. Combining these, we get $\{\text{GGB}\}$.

## A.6.3 FINDING THE INTERSECTION OF TWO EVENTS FROM A TABLE

**MCQ 51:** In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

**Table: Students**

| Name | Age | Gender |
|---|---|---|
| A | 15 | Female |
| B | 17 | Male |
| C | 16 | Female |
| D | 15 | Male |
| E | 14 | Female |
| F | 17 | Female |

Let event $A$ be the event of selecting a girl, and event $B$ be the event of selecting a student older or equal than 17 years. Find $A$ and $B$.

- ☐ $A$ and $B = \{\text{A, B, C, E, F}\}$
- ☐ $A$ and $B = \{\text{A, C, E, F}\}$
- ☐ $A$ and $B = \{\text{B, F}\}$
- ☒ $A$ and $B = \{\text{F}\}$

*Answer:*

- The sample space is $\{\text{A, B, C, D, E, F}\}$.
- Event $A$, selecting a girl, is $\{\text{A, C, E, F}\}$.
- Event $B$, selecting a student older or equal than 17, is $\{\text{B, F}\}$.
- The event $A$ and $B$ (the intersection $A \cap B$) includes all outcomes that are in both $A$ and $B$. Combining these, we get $\{\text{F}\}$.

**MCQ 52:** In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

**Table: Students**

| Name | Age | Instrument |
|---|---|---|
| G | 14 | Violin |
| H | 15 | Piano |
| I | 17 | Guitar |
| J | 16 | Drums |
| K | 15 | Flute |
| L | 18 | Violin |

Let event $X$ be the event of selecting a student who plays the violin, and event $Y$ be the event of selecting a student aged 16 or older. Find $X$ and $Y$.

- ☐ $X$ and $Y = \{\text{G, I, J, L}\}$
- ☐ $X$ and $Y = \{\text{G, L}\}$
- ☐ $X$ and $Y = \{\text{I, J, L}\}$
- ☒ $X$ and $Y = \{\text{L}\}$

*Answer:*

- The sample space is $\{\text{G, H, I, J, K, L}\}$.

- Event $X$, selecting a student who plays the violin, is {G, L}.
- Event $Y$, selecting a student aged 16 or older, is {I, J, L}.
- The event $X$ and $Y$ (the intersection $X \cap Y$) includes all outcomes that are in both $X$ and $Y$. Combining these, we get {L}.

**MCQ 53:** In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

**Table: Players**

| Name | Height (cm) | Position |
|---|---|---|
| M | 180 | Forward |
| N | 170 | Goalkeeper |
| O | 185 | Defender |
| P | 175 | Midfielder |
| Q | 165 | Forward |
| R | 190 | Defender |

Let event $U$ be the event of selecting a defender, and event $V$ be the event of selecting a player taller than or equal to 180 cm. Find $U$ and $V$.

☐ $U$ and $V = \{\text{M, O, R}\}$

☒ $U$ and $V = \{\text{O, R}\}$

☐ $U$ and $V = \emptyset$

☐ $U$ and $V = \{\text{O}\}$

*Answer:*

- The sample space is {M, N, O, P, Q, R}.
- Event $U$, selecting a defender, is {O, R}.
- Event $V$, selecting a player taller than or equal to 180 cm, is {M, O, R}.
- The event $U$ and $V$ (the intersection $U \cap V$) includes all outcomes that are in both $U$ and $V$. Combining these, we get {O, R}.

## A.7 MUTUALLY EXCLUSIVE

### A.7.1 DETERMINING MUTUAL EXCLUSIVITY

**MCQ 54:** Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event $E$: Rolling an even number.
- Event $F$: Rolling an odd number.

Are the events $E$ and $F$ mutually exclusive?

☒ Yes

☐ No

*Answer:*

- Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).
- Event $E$, rolling an even number, includes: 2, 4, 6 (3 outcomes).
- Event $F$, rolling an odd number, includes: 1, 3, 5 (3 outcomes).
- There is no number that is both even and odd, so the intersection $E \cap F = \emptyset$ (empty set).
- Since there are no common outcomes, events $E$ and $F$ are mutually exclusive.
- Therefore, the correct answer is: Yes.

**MCQ 55:** Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event $E$: Rolling a prime number.
- Event $F$: Rolling an even number.

Are the events $E$ and $F$ mutually exclusive?

☐ Yes

☒ No

*Answer:*

- Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).
- Event $E$, rolling a prime number, includes: 2, 3, 5 (3 outcomes). Note: 1 is not a prime number.
- Event $F$, rolling an even number, includes: 2, 4, 6 (3 outcomes).
- The number 2 is both a prime number (in $E$) and an even number (in $F$), so it is a common outcome.
- Since $E \cap F = \{2\}$ is not empty, events $E$ and $F$ are not mutually exclusive.
- Therefore, the correct answer is: No.

**MCQ 56:** Consider a standard deck of 52 playing cards (no jokers). Two events are defined as follows:

- Event $E$: Drawing a Queen.
- Event $F$: Drawing a Heart.

Are the events $E$ and $F$ mutually exclusive?

☐ Yes

☒ No

*Answer:*

- Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($E \cap F$) is empty—there are no outcomes common to both.
- Event $E$, drawing a Queen, includes the cards: Queen of Hearts, Queen of Spades, Queen of Clubs, and Queen of Diamonds (4 outcomes).
- Event $F$, drawing a Heart, includes all 13 Hearts: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King of Hearts.

- The Queen of Hearts is both a Queen (in $E$) and a Heart (in $F$), so it is a common outcome.
- Since $E \cap F = \{\text{Queen of Hearts}\}$ is not empty, events $E$ and $F$ are not mutually exclusive.
- Therefore, the correct answer is: No.

**MCQ 57:** Consider a family with exactly two children, where each child is a boy (B) or a girl (G). Two events are defined as follows:

- Event $E$: The family has only boys.
- Event $F$: The family has only girls.

Are the events $E$ and $F$ mutually exclusive?

☒ Yes

☐ No

*Answer:*

- Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).
- The sample space for a family with two children is: $\{BB, BG, GB, GG\}$, where B = boy and G = girl.
- Event $E$, having only boys, is: $\{BB\}$ (1 outcome).
- Event $F$, having only girls, is: $\{GG\}$ (1 outcome).
- There is no family outcome that has only boys and only girls at the same time, so the intersection $E \cap F = \emptyset$ (empty set).
- Since there are no common outcomes, events $E$ and $F$ are mutually exclusive.
- Therefore, the correct answer is: Yes.

## A.7.2 DETERMINING MUTUAL EXCLUSIVITY FROM TABLES

**MCQ 58:** Consider a sports club where members are listed with their preferred sport and age group. A member is selected at random from the following table:

**Table: Sports Club Members**

| Name | Preferred Sport | Age Group |
|---|---|---|
| S | Football | Under 16 |
| T | Basketball | 16-18 |
| U | Tennis | Over 18 |
| V | Swimming | Under 16 |
| W | Football | 16-18 |
| X | Basketball | Over 18 |

Let event $C$ be selecting a member whose preferred sport is Football, and event $D$ be selecting a member from the Over 18 age group. Are events $C$ and $D$ mutually exclusive?

☒ Yes

☐ No

*Answer:*

- Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($C \cap D$) is empty—there are no common outcomes.
- Event $C$, selecting a member whose preferred sport is Football, includes: S (Under 16), W (16-18) — 2 outcomes.
- Event $D$, selecting a member from the Over 18 age group, includes: U (Tennis), X (Basketball) — 2 outcomes.
- From the table, no member prefers Football and is in the Over 18 age group. The outcomes for $C$ (S, W) and $D$ (U, X) are distinct.
- Since $C \cap D = \emptyset$ (empty set), events $C$ and $D$ are mutually exclusive.
- Therefore, the correct answer is: Yes.

**MCQ 59:** Consider a bakery where items are listed with their type and topping. An item is selected at random from the following table:

**Table: Bakery Items**

| Item | Type | Topping |
|---|---|---|
| Muffin | Pastry | Chocolate |
| Cookie | Cookie | Sprinkles |
| Cake | Cake | Chocolate |
| Donut | Pastry | Glaze |
| Brownie | Cake | None |
| Croissant | Pastry | None |

Let event $T$ be selecting a Pastry, and event $U$ be selecting an item with Chocolate topping. Are events $T$ and $U$ mutually exclusive?

☐ Yes

☒ No

*Answer:*

- Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($T \cap U$) is empty—no common outcomes.
- Event $T$, selecting a Pastry, includes: Muffin (Chocolate), Donut (Glaze), Croissant (None) — 3 outcomes.
- Event $U$, selecting an item with Chocolate topping, includes: Muffin (Pastry), Cake (Cake) — 2 outcomes.
- The Muffin is both a Pastry (in $T$) and has Chocolate topping (in $U$), so there is a common outcome.
- Since $T \cap U = \{\text{Muffin}\}$ is not empty, events $T$ and $U$ are not mutually exclusive.
- Therefore, the correct answer is: No.

**MCQ 60:** Consider a library where books are listed with their genre and checkout status. A book is selected at random from the following table:

**Table: Library Books**

| Title | Genre | Status |
|---|---|---|
| A | Mystery | Checked Out |
| B | Fantasy | Available |
| C | Mystery | Available |
| D | Romance | Checked Out |
| E | Fantasy | Checked Out |
| F | Romance | Available |

Let event $P$ be selecting a Mystery book, and event $Q$ be selecting a Checked Out book. Are events $P$ and $Q$ mutually exclusive?

☐ Yes

☒ No

*Answer:*

- Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($P \cap Q$) is empty—no common outcomes.
- Event $P$, selecting a Mystery book, includes: A (Checked Out), C (Available) — 2 outcomes.
- Event $Q$, selecting a Checked Out book, includes: A (Mystery), D (Romance), E (Fantasy) — 3 outcomes.
- The book A is both a Mystery (in $P$) and Checked Out (in $Q$), so there is a common outcome.
- Since $P \cap Q = \{\text{A}\}$ is not empty, events $P$ and $Q$ are not mutually exclusive.
- Therefore, the correct answer is: No.

**MCQ 61:** Consider a zoo where animals are listed with their type and habitat. An animal is selected at random from the following table:

**Table: Zoo Animals**

| Name | Type | Habitat |
|---|---|---|
| Lion | Mammal | Savanna |
| Penguin | Bird | Arctic |
| Crocodile | Reptile | Swamp |
| Elephant | Mammal | Savanna |
| Parrot | Bird | Jungle |
| Snake | Reptile | Jungle |

Let event $R$ be selecting a Mammal, and event $S$ be selecting an animal from the Arctic habitat. Are events $R$ and $S$ mutually exclusive?

☒ Yes

☐ No

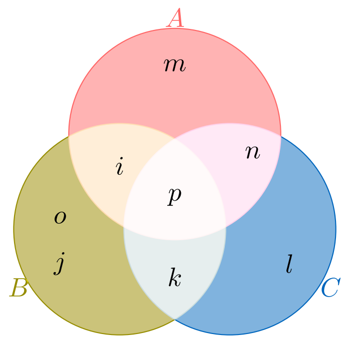
*Answer:*

- Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($R \cap S$) is empty—no common outcomes.
- Event $R$, selecting a Mammal, includes: Lion (Savanna), Elephant (Savanna) — 2 outcomes.
- Event $S$, selecting an animal from the Arctic habitat, includes: Penguin (Bird) — 1 outcome.
- From the table, no animal is both a Mammal and from the Arctic habitat. The outcomes for $R$ (Lion, Elephant) and $S$ (Penguin) are distinct.
- Since $R \cap S = \emptyset$ (empty set), events $R$ and $S$ are mutually exclusive.
- Therefore, the correct answer is: Yes.

## A.8 VENN DIAGRAM

### A.8.1 FINDING THE UNION OF TWO EVENTS IN A VENN DIAGRAM

**MCQ 62:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find $A$ or $B$.

☐ $A$ or $B = \{i, j, k, l, m, n\}$

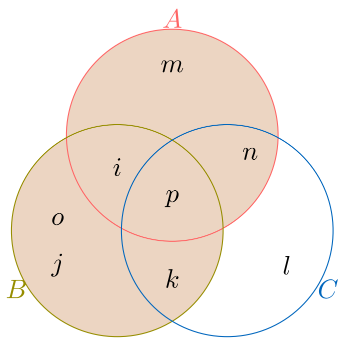
☒ $A$ or $B = \{i, j, k, m, n, o, p\}$

☐ $A$ or $B = \{i, j, k, l, m\}$

☐ $A$ or $B = \{i, j, k, l, m, o, p\}$

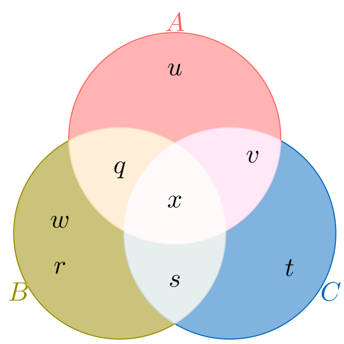
*Answer:*

- The union of events $A$ and $B$ ($A \cup B$) includes all elements that are in $A$, in $B$, or in both.
- From the Venn diagram:
  - Event $A = \{i, m, n, p\}$ (elements in the top circle).
  - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
  - Combining these, $A$ or $B = \{i, j, k, m, n, o, p\}$.
- The shaded area represents




$A$ or $B$:

**MCQ 63:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:



Find $A$ or $B$.

☐ $A$ or $B = \{q, r, s, t, u, v\}$

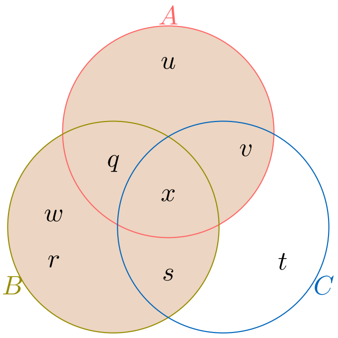
☒ $A$ or $B = \{q, r, s, u, w, x\}$

☐ $A$ or $B = \{q, r, s, t, u\}$

☐ $A$ or $B = \{q, r, s, t, u, w, x\}$

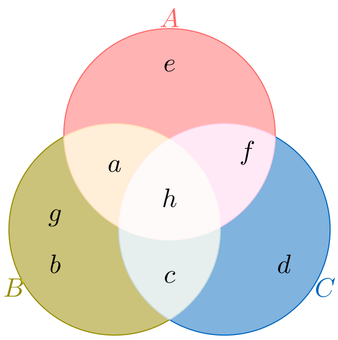
*Answer:*

- The union of events $A$ and $B$ ($A \cup B$) includes all elements that are in $A$, in $B$, or in both.
- From the Venn diagram:
  - Event $A = \{q, u, v, x\}$ (elements in the top circle).
  - Event $B = \{q, r, s, w, x\}$ (elements in the left circle).
  - Combining these, $A$ or $B = \{q, r, s, u, w, x\}$.
- The shaded area represents




$A$ or $B$:

**MCQ 64:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find $A$ or $C$.

☒ $A$ or $C = \{a, c, d, e, f, h\}$

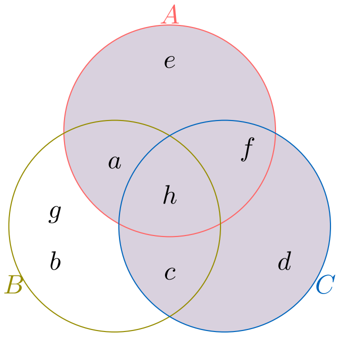
☐ $A$ or $C = \{a, b, e, g, h\}$

☐ $A$ or $C = \{a, b, c, d, e\}$

☐ $A$ or $C = \{a, b, c, d, e, g, h\}$

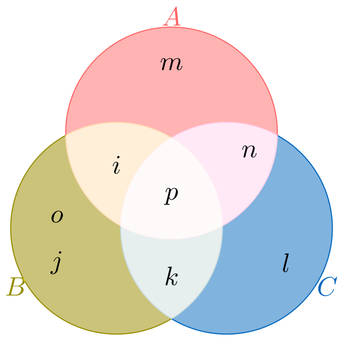
*Answer:*

- The union of events $A$ and $C$ ($A \cup C$) includes all elements that are in $A$, in $C$, or in both.
- From the Venn diagram:
  - Event $A = \{a, e, f, h\}$ (elements in the top circle).
  - Event $C = \{c, d, f, h\}$ (elements in the right circle).
  - Combining these, $A$ or $C = \{a, c, d, e, f, h\}$.
- The shaded area represents




$A$ or $C$:

**MCQ 65:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find $B$ or $C$.

☐ $B$ or $C = \{i, j, k, l, m, n\}$

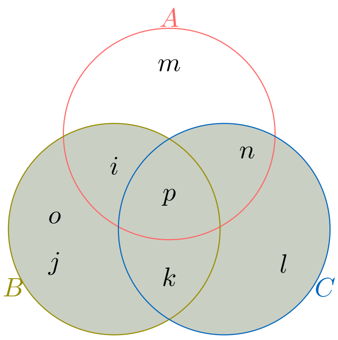
☐ $B$ or $C = \{i, j, k, l, o\}$

☒ $B$ or $C = \{i, j, k, l, n, o, p\}$

☐ $B$ or $C = \{i, j, k, o, p\}$

*Answer:*

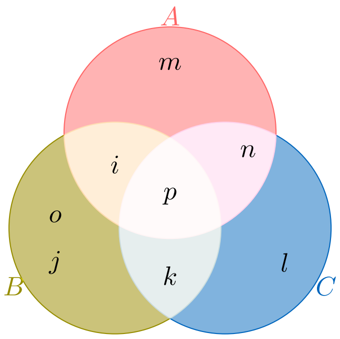
- The union of events $B$ and $C$ ($B \cup C$) includes all elements that are in $B$, in $C$, or in both.
- From the Venn diagram:
  - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
  - Event $C = \{k, l, n, p\}$ (elements in the right circle).
  - Combining these, $B$ or $C = \{i, j, k, l, n, o, p\}$.
- The shaded area represents




$B$ or $C$:

### A.8.2 FINDING THE INTERSECTION OF TWO EVENTS IN A VENN DIAGRAM

**MCQ 66:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find the intersection of $A$ and $B$.

☒ $A$ and $B = \{i, p\}$

☐ $A$ and $B = \{i, j, k, m, o, p\}$

☐ $A$ and $B = \{i, j, k, l, m\}$

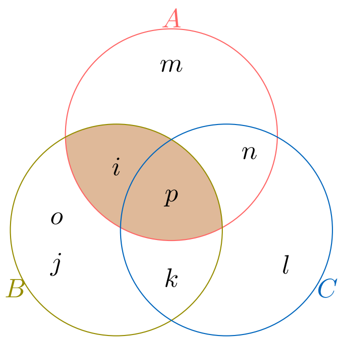
☐ $A$ and $B = \{j, k, o\}$

*Answer:*

1) The intersection of events $A$ and $B$ $(A \cap B)$ includes all elements that are in both $A$ and $B$.

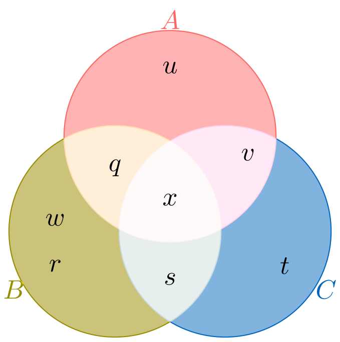
2) From the Venn diagram:
   - Event $A = \{i, m, n, p\}$ (elements in the top circle).
   - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
   - The common elements are $\{i, p\}$. Thus, $A$ and $B = \{i, p\}$.

3) The shaded area represents




$A$ and $B$:

**MCQ 67:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find the intersection of $A$ and $B$.

☒ $A$ and $B = \{q, x\}$

☐ $A$ and $B = \{q, r, s, u, w, x\}$

☐ $A$ and $B = \{r, s, w\}$

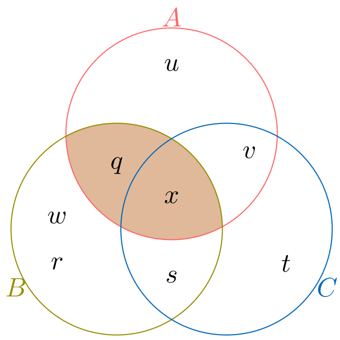
☐ $A$ and $B = \{q, r, s, t, u\}$

*Answer:*

1) The intersection of events $A$ and $B$ $(A \cap B)$ includes all elements that are in both $A$ and $B$.

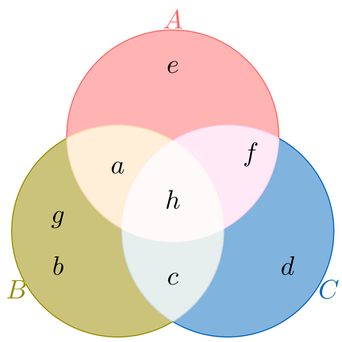
2) From the Venn diagram:
   - Event $A = \{q, u, v, x\}$ (elements in the top circle).
   - Event $B = \{q, r, s, w, x\}$ (elements in the left circle).
   - The common elements are $\{q, x\}$. Thus, $A$ and $B = \{q, x\}$.

3) The shaded area represents




$A$ and $B$:

**MCQ 68:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find the intersection of $A$ and $C$.

☐ $A$ and $C = \{a, c, d, e, f, h\}$

☐ $A$ and $C = \{a, e\}$

☒ $A$ and $C = \{f, h\}$

☐ $A$ and $C = \{c, d, f\}$

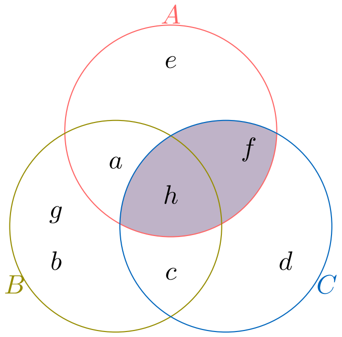
*Answer:*

1) The intersection of events $A$ and $C$ $(A \cap C)$ includes all elements that are in both $A$ and $C$.

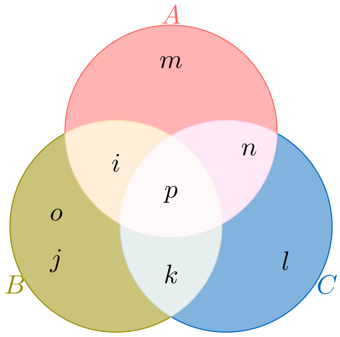
2) From the Venn diagram:
   - Event $A = \{a, e, f, h\}$ (elements in the top circle).
   - Event $C = \{c, d, f, h\}$ (elements in the right circle).

- The common elements are $\{f, h\}$. Thus, $A$ and $C = \{f, h\}$.

3) The shaded area represents $A$ and $C$:




**MCQ 69:** You are given this populated Venn diagram representing events $A$, $B$, and $C$:




Find the intersection of $B$ and $C$.

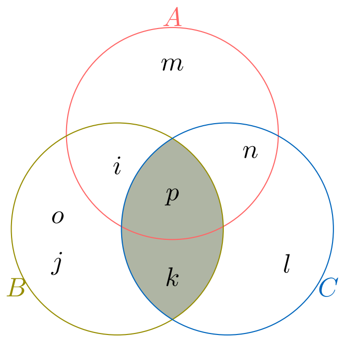
- ☐ $B$ and $C = \{i, j, k, l, o, p\}$
- ☒ $B$ and $C = \{k, p\}$
- ☐ $B$ and $C = \{j, k, o\}$
- ☐ $B$ and $C = \{l, n, p\}$

*Answer:*

1) The intersection of events $B$ and $C$ $(B \cap C)$ includes all elements that are in both $B$ and $C$.

2) From the Venn diagram:
   - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
   - Event $C = \{k, l, n, p\}$ (elements in the right circle).
   - The common elements are $\{k, p\}$. Thus, $B$ and $C = \{k, p\}$.

3) The shaded area represents $B$ and $C$:




# B AXIOMS AND RULES OF PROBABILITY

## B.1 AXIOMS OF PROBABILITY

### B.1.1 DESCRIBING PROBABILITIES WITH WORDS

**MCQ 70:** The probability of winning a game is $\frac{1}{10}$. Find the word to describe this probability.

- ☐ Impossible
- ☒ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

*Answer:* The correct answer is "Less Likely." The probability of winning is $\frac{1}{10}$, which means you have the chance to win 1 game out of 10 games played. So, it's Less Likely.

**MCQ 71:** The probability of winning a game is $\frac{4}{5}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☒ More Likely
- ☐ Certain

*Answer:* The correct answer is "More Likely." The probability of winning is $\frac{4}{5}$, which means you have the chance to win 4 games out of 5 games played. So, it's More Likely.

**MCQ 72:** The probability of winning a game is $\frac{1}{2}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☒ Even Chance
- ☐ More Likely
- ☐ Certain

*Answer:* The correct answer is "Even Chance." The probability of winning is $\frac{1}{2}$, which means you have the chance to win 1 game out of 2 games played. So, it's an Even Chance.

**MCQ 73:** The probability of winning a game is 0. Find the word to describe this probability.

- ☒ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

*Answer:* The correct answer is "Impossible." The probability of winning is 0, which means you have no chance to win the game. So, it's Impossible.

**MCQ 74:** The probability of winning a game is 1. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☒ Certain

*Answer:* The correct answer is "Certain." The probability of winning is 1, which means you will definitely win the game. So, it's Certain.

### B.1.2 MAKING DECISIONS USING PROBABILITIES

**MCQ 75:** Louis advises you to play because the probability of winning this game is $\frac{3}{4}$. Do you follow his advice?

- ☒ Yes
- ☐ No

*Answer:* The correct answer is "Yes." The probability of winning is $\frac{3}{4}$, which means you have the chance to win 3 games out of 4 games played. So it is more likely. Therefore, it's a good idea to follow Louis's advice and play.

**MCQ 76:** Louis advises you to play because the probability of winning this game is $\frac{1}{4}$. Do you follow his advice?

- ☐ Yes
- ☒ No

*Answer:* The correct answer is "No." The probability of winning is $\frac{1}{4}$, which means you have the chance to win 1 game out of 4 games played. So it is less likely. Therefore, it's not a good idea to follow Louis's advice and play.

**MCQ 77:** The probability of scoring a penalty is $\frac{1}{2}$ for Louis and $\frac{3}{4}$ for Hugo. Which player do you choose to take the penalty?

- ☐ Louis
- ☒ Hugo

*Answer:* The correct answer is "Hugo." The probability of succeeding for Louis is $\frac{1}{2}$, which means he has an even chance to succeed. For Hugo, it's $\frac{3}{4}$, which means he is more likely to succeed because he has the chance to succeed in 3 out of 4 penalties. So, Hugo is the better choice to take the penalty.

**MCQ 78:** The probability of scoring a penalty is $\frac{1}{4}$ for Louis and $\frac{3}{5}$ for Hugo. Which player do you choose to take the penalty?

- ☐ Louis
- ☒ Hugo

*Answer:* The correct answer is "Hugo." The probability of succeeding for Louis is $\frac{1}{4}$, which means he is less likely to succeed because he has the chance to succeed in 1 out of 4 penalties. For Hugo, it's $\frac{3}{5}$, which means he is more likely to succeed because he has the chance to succeed in 3 out of 5 penalties. So, Hugo is the better choice to take the penalty.

### B.1.3 FINDING PROBABILITY FOR MUTUALLY EXCLUSIVE EVENTS

**Ex 79:** Let $P(G) = 0.6$ and $P(H) = 0.2$. Assume that events $G$ and $H$ are mutually exclusive.
Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \boxed{0.8}$$

*Answer:*

- Since $G$ and $H$ are mutually exclusive events, they cannot occur at the same time.
- The probability of either event $G$ or event $H$ occurring is the sum of their individual probabilities.
- We are given $P(G) = 0.6$ and $P(H) = 0.2$.
- Therefore,
$$\begin{aligned} P(G \text{ or } H) &= P(G) + P(H) \\ &= 0.6 + 0.2 \\ &= 0.8. \end{aligned}$$
- So, the probability of either event $G$ or event $H$ occurring is 0.8.

**Ex 80:** Let $P(C) = 0.4$ and $P(D) = 0.5$. Assume that events $C$ and $D$ are mutually exclusive.
Calculate $P(C \text{ or } D)$.

$$P(C \text{ or } D) = \boxed{0.9}$$

*Answer:*

- Since $C$ and $D$ are mutually exclusive events, they cannot occur at the same time.
- The probability of either event $C$ or event $D$ occurring is the sum of their individual probabilities.
- We are given $P(C) = 0.4$ and $P(D) = 0.5$.
- Therefore,
$$\begin{aligned} P(C \text{ or } D) &= P(C) + P(D) \\ &= 0.4 + 0.5 \\ &= 0.9. \end{aligned}$$
- So, the probability of either event $C$ or event $D$ occurring is 0.9.

**Ex 81:** Let $P(A) = 0.5$ and $P(B) = 0.3$. Assume that events $A$ and $B$ are mutually exclusive.
Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \boxed{0.8}$$

*Answer:*

- Since $A$ and $B$ are mutually exclusive events, they cannot occur at the same time.
- The probability of either event $A$ or event $B$ occurring is the sum of their individual probabilities.
- We are given $P(A) = 0.5$ and $P(B) = 0.3$.

- Therefore,
$$\begin{aligned}P(A \text{ or } B) &= P(A) + P(B)\\ &= 0.5 + 0.3\\ &= 0.8.\end{aligned}$$
- So, the probability of either event $A$ or event $B$ occurring is 0.8.

## B.2 FUNDAMENTAL PROBABILITY RULES

### B.2.1 APPLYING THE COMPLEMENT RULE

**Ex 82:** I toss a fair coin. The probability of getting heads is $\frac{1}{2}$.
Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The probability of getting heads is $\frac{1}{2}$.
- The event "Getting tails" is the complement of "Getting heads."
- Using the complement rule:
$$\begin{aligned}P(\text{"Getting tails"}) &= 1 - P(\text{"Getting heads"})\\ &= 1 - \frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$
- So, the probability of getting tails is $\frac{1}{2} = 50\%$.

**Ex 83:** A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{30}\%$$

*Answer:*

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement of "Laughing."
- Using the complement rule:
$$\begin{aligned}P(\text{"Not laughing"}) &= 100\% - P(\text{"Laughing"})\\ &= 100\% - 70\%\\ &= 30\%\end{aligned}$$
- Therefore, the probability that a student does not laugh at the joke is 30%.

**Ex 84:** I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \boxed{\frac{1}{10}}$$

*Answer:*

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:
$$\begin{aligned}P(\text{"Selecting a boy"}) &= 1 - P(\text{"Selecting a girl"})\\ &= 1 - \frac{9}{10}\\ &= \frac{1}{10}\end{aligned}$$
- So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

**Ex 85:** The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \boxed{30}\%$$

*Answer:*

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of "Rain".
- Using the complement rule:
$$\begin{aligned}P(\text{"No rain"}) &= 1 - P(\text{"Rain"})\\ &= 100\% - 70\%\\ &= 30\%\end{aligned}$$
- Therefore, the probability that it will not rain tomorrow is 30%.

**Ex 86:** In a loto game, the probability of winning is $\frac{1}{100}$.
Find the probability of losing.

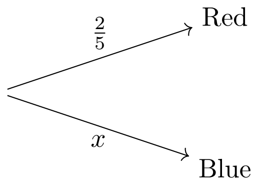
$$P(\text{"Losing"}) = \boxed{\frac{99}{100}}$$

*Answer:*

- The probability of winning is $\frac{1}{100}$.
- The event "Losing" is the complement of "Winning."
- Using the complement rule:
$$\begin{aligned}P(\text{"Losing"}) &= 1 - P(\text{"Winning"})\\ &= 1 - \frac{1}{100}\\ &= \frac{100}{100} - \frac{1}{100}\\ &= \frac{99}{100}\end{aligned}$$
- So, the probability of losing is $\frac{99}{100} = 99\%$.

## B.2.2 COMPLETING A PROBABILITY TREE DIAGRAM

**Ex 87:** From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability $x$ of choosing a blue ball.




$$x = \frac{\boxed{3}}{\boxed{5}}$$

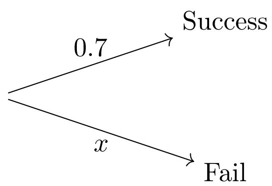
*Answer:*

- The probability of choosing a red ball from the bag is given as $\frac{2}{5}$.
- Since the only other option is choosing a blue ball, the probability of choosing a blue ball is calculated as follows:

$$\begin{aligned} P(\text{"Blue"}) &= 1 - P(\text{"Red"}) \\ &= 1 - \frac{2}{5} \\ &= \frac{5}{5} - \frac{2}{5} \\ &= \frac{3}{5} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{5}$.

**Ex 88:** Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability $x$ that he misses his first shot.




$$x = \boxed{0.3}$$

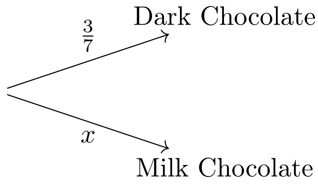
*Answer:*

- The probability that Jasper makes his first shot is 0.7.
- Since the complement event is missing the shot, the probability $x$ is calculated as follows:

$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.7 \\ &= 0.3 \end{aligned}$$

- So, the correct answer is $x = 0.3$.

**Ex 89:** In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability $x$ of picking a milk chocolate.




$$x = \frac{\boxed{4}}{\boxed{7}}$$

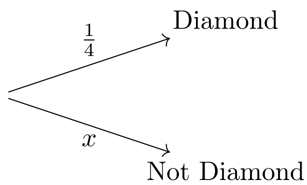
*Answer:*

- The probability of picking a dark chocolate from the box is $\frac{3}{7}$.
- Since the only other option is picking a milk chocolate, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Milk chocolate"}) &= 1 - P(\text{"Dark chocolate"}) \\ &= 1 - \frac{3}{7} \\ &= \frac{7}{7} - \frac{3}{7} \\ &= \frac{4}{7} \end{aligned}$$

- So, the correct answer is $x = \frac{4}{7}$.

**Ex 90:** In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability $x$ of drawing a card that is not a diamond.




$$x = \frac{\boxed{3}}{\boxed{4}}$$

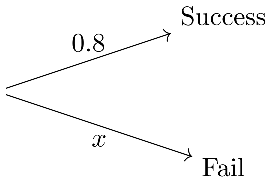
*Answer:*

- The probability of drawing a diamond from a deck of cards is $\frac{1}{4}$.
- Since the only other option is drawing a card that is not a diamond, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Not diamond"}) &= 1 - P(\text{"Diamond"}) \\ &= 1 - \frac{1}{4} \\ &= \frac{4}{4} - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{4}$.

**Ex 91:** Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability $x$ that she fails to complete the level.




$$x = \boxed{0.2}$$

*Answer:*

- The probability that Emma completes the level is 0.8.
- Since the complement event is failing to complete the level, the probability $x$ is calculated as follows:

$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

- So, the correct answer is $x = 0.2$.

## B.2.3 CALCULATING PROBABILITIES FOR UNION OF EVENTS

**Ex 92:** Let $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \boxed{0.7}$$

*Answer:*

- To calculate $P(A \text{ or } B)$, we use the general probability addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

- We are given:

$$P(A) = 0.5, \quad P(B) = 0.3, \quad \text{and} \quad P(A \text{ and } B) = 0.1.$$

- Substitute the given values into the formula:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.5 + 0.3 - 0.1 \\ &= 0.7. \end{aligned}$$

- So, the probability of either event $A$ or event $B$ occurring is 0.7.

**Ex 93:** Let $P(E) = 0.8$, $P(F) = 0.3$ and $P(E \text{ and } F) = 0.2$. Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \boxed{0.9}$$

*Answer:*

- To calculate $P(E \text{ or } F)$, we use the general probability addition rule:

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F).$$

- We are given:

$$P(E) = 0.8, \quad P(F) = 0.3, \quad \text{and} \quad P(E \text{ and } F) = 0.2.$$

- Substitute the given values into the formula:

$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= 0.8 + 0.3 - 0.2 \\ &= 0.9. \end{aligned}$$

- So, the probability of either event $E$ or event $F$ occurring is 0.9.

**Ex 94:** Let $P(G) = 0.6$, $P(H) = 0.2$ and $P(G \text{ and } H) = 0.1$. Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \boxed{0.7}$$

*Answer:*

- To calculate $P(G \text{ or } H)$, we use the general probability addition rule:

$$P(G \text{ or } H) = P(G) + P(H) - P(G \text{ and } H).$$

- We are given:

$$P(G) = 0.6, \quad P(H) = 0.2, \quad \text{and} \quad P(G \text{ and } H) = 0.1.$$

- Substitute the given values into the formula:

$$\begin{aligned} P(G \text{ or } H) &= P(G) + P(H) - P(G \text{ and } H) \\ &= 0.6 + 0.2 - 0.1 \\ &= 0.7. \end{aligned}$$

- So, the probability of either event $G$ or event $H$ occurring is 0.7.

**Ex 95:** Let $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \text{ and } Y) = 0.2$. Calculate $P(X \text{ or } Y)$.

$$P(X \text{ or } Y) = \boxed{0.9}$$

*Answer:*

- To calculate $P(X \text{ or } Y)$, we use the general probability addition rule:

$$P(X \text{ or } Y) = P(X) + P(Y) - P(X \text{ and } Y).$$

- We are given:

$$P(X) = 0.7, \quad P(Y) = 0.4, \quad \text{and} \quad P(X \text{ and } Y) = 0.2.$$

- Substitute the given values into the formula:

$$\begin{aligned} P(X \text{ or } Y) &= P(X) + P(Y) - P(X \text{ and } Y) \\ &= 0.7 + 0.4 - 0.2 \\ &= 0.9. \end{aligned}$$

- So, the probability of either event $X$ or event $Y$ occurring is 0.9.

### B.2.4 CALCULATING PROBABILITIES FOR UNION OF EVENTS IN REAL-WORLD PROBLEMS

**Ex 96:** In a school survey, the probability of a student liking math is 0.6, and the probability of liking science is 0.4. The probability of a student liking both math and science is 0.25.
What is the probability that a randomly selected student likes either math or science?

$$P(\text{Math or Science}) = \boxed{0.75}$$

*Answer:*

- Let $M$ be the event of liking math, and let $S$ be the event of liking science. We are given:
$$P(M) = 0.6, \quad P(S) = 0.4, \quad \text{and} \quad P(M \text{ and } S) = 0.25.$$
- The probability of a student liking either math or science is given by $P(M \text{ or } S)$.
- Using the addition rule of probability:
$$P(M \text{ or } S) = P(M) + P(S) - P(M \text{ and } S).$$
- Substituting the given values:
$$\begin{aligned} P(M \text{ or } S) &= 0.6 + 0.4 - 0.25 \\ &= 0.75. \end{aligned}$$
- Therefore, the probability that a student likes either math or science is 0.75.

**Ex 97:** In a city survey, the probability of a resident using public transportation is 0.7, and the probability of using a bicycle is 0.3. The probability of a resident using both public transportation and a bicycle is 0.15.
What is the probability that a randomly selected resident uses either public transportation or a bicycle?

$$P(\text{Public Transport or Bicycle}) = \boxed{0.85}$$

*Answer:*

- Let $T$ be the event of using public transportation, and let $B$ be the event of using a bicycle. We are given:
$$P(T) = 0.7, \quad P(B) = 0.3, \quad \text{and} \quad P(T \text{ and } B) = 0.15.$$
- The probability of a resident using either public transportation or a bicycle is given by $P(T \text{ or } B)$.
- Using the addition rule of probability:
$$P(T \text{ or } B) = P(T) + P(B) - P(T \text{ and } B).$$
- Substituting the given values:
$$\begin{aligned} P(T \text{ or } B) &= 0.7 + 0.3 - 0.15 \\ &= 0.85. \end{aligned}$$
- Therefore, the probability that a resident uses either public transportation or a bicycle is 0.85.

**Ex 98:** In a company survey, the probability of an employee enjoying team meetings is 0.5, and the probability of enjoying training sessions is 0.4. The probability of an employee enjoying both team meetings and training sessions is 0.2.
What is the probability that a randomly selected employee enjoys either team meetings or training sessions?

$$P(\text{Team Meetings or Training}) = \boxed{0.7}$$

*Answer:*

- Let $R$ be the event of enjoying team meetings, and let $F$ be the event of enjoying training sessions. We are given:
$$P(R) = 0.5, \quad P(F) = 0.4, \quad \text{and} \quad P(R \text{ and } F) = 0.2.$$
- The probability of an employee enjoying either team meetings or training sessions is given by $P(R \text{ or } F)$.
- Using the addition rule of probability:
$$P(R \text{ or } F) = P(R) + P(F) - P(R \text{ and } F).$$
- Substituting the given values:
$$\begin{aligned} P(R \text{ or } F) &= 0.5 + 0.4 - 0.2 \\ &= 0.7. \end{aligned}$$
- Therefore, the probability that an employee enjoys either team meetings or training sessions is 0.7.

**Ex 99:** In a neighborhood survey, the probability of a household owning a dog is 0.5, and the probability of owning a cat is 0.35. The probability of a household owning both a dog and a cat is 0.2.
What is the probability that a randomly selected household owns either a dog or a cat?

$$P(\text{Dog or Cat}) = \boxed{0.65}$$

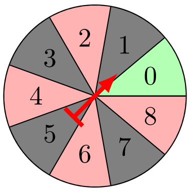
*Answer:*

- Let $D$ be the event of owning a dog, and let $C$ be the event of owning a cat. We are given:
$$P(D) = 0.5, \quad P(C) = 0.35, \quad \text{and} \quad P(D \text{ and } C) = 0.2.$$
- The probability of a household owning either a dog or a cat is given by $P(D \text{ or } C)$.
- Using the addition rule of probability:
$$P(D \text{ or } C) = P(D) + P(C) - P(D \text{ and } C).$$
- Substituting the given values:
$$\begin{aligned} P(D \text{ or } C) &= 0.5 + 0.35 - 0.2 \\ &= 0.65. \end{aligned}$$
- Therefore, the probability that a household owns either a dog or a cat is 0.65.

## B.3 EQUALLY LIKELY

### B.3.1 FINDING PROBABILITIES IN A CASINO SPINNER

**Ex 100:** You spin the casino spinner shown below. Calculate the probability of the event "getting a 2".



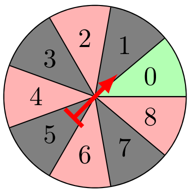

$$P(\text{"getting a 2"}) = \boxed{\frac{1}{9}}$$

*Answer:*

- The number of outcomes in the sample space when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "getting a 2" is 1, as there is one section labeled 2 on the spinner.
- Therefore, the probability of getting a 2 is given by:

$$\begin{aligned} P(\text{"getting a 2"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}} \\ &= \frac{1}{9} \end{aligned}$$

**Ex 101:** You spin the casino spinner shown below. Calculate the probability of the event "not getting a 4".



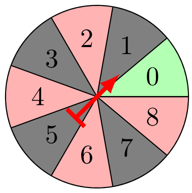

$$P(\text{"not getting a 4"}) = \boxed{\frac{8}{9}}$$

*Answer:*

- The number of outcomes in the sample space when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "not getting a 4" is 8, as there are eight sections that are not 4: 0, 1, 2, 3, 5, 6, 7, and 8.
- Therefore, the probability of not getting a 4 is given by:

$$\begin{aligned} P(\text{"not getting a 4"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}} \\ &= \frac{8}{9} \end{aligned}$$

**Ex 102:** You spin the casino spinner shown below. Calculate the probability of the event "red".



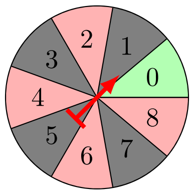

$$P(\text{"red"}) = \boxed{\frac{4}{9}}$$

*Answer:*

- The number of outcomes in the sample space when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "red" is 4, as there are four red sections on the spinner: 2, 4, 6, and 8.
- Therefore, the probability of landing on a red section is given by:

$$\begin{aligned} P(\text{"red"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}} \\ &= \frac{4}{9} \end{aligned}$$

**Ex 103:** You spin the casino spinner shown below. Calculate the probability of the event "getting an odd number".



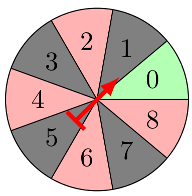

$$P(\text{"getting an odd number"}) = \boxed{\frac{4}{9}}$$

*Answer:*

- The number of outcomes in the sample space when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "getting an odd number" is 4, as there are four odd numbers on the spinner: 1, 3, 5, and 7.
- Therefore, the probability of getting an odd number is given by:

$$\begin{aligned} P(\text{"odd number"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}} \\ &= \frac{4}{9} \end{aligned}$$

**Ex 104:** You spin the casino spinner shown below. Calculate the probability of the event "not getting red".




$$P(\text{"not getting red"}) = \boxed{\frac{5}{9}}$$

*Answer:*

- The number of outcomes in the sample space when spinning the casino spinner is 9, since there are 9 sections (0 to 8).
- The number of outcomes for the event "not getting red" is 5, as there are five sections that are not red: 0, 1, 3, 5, and 7.
- Therefore, the probability of not getting red is given by:

$$\begin{aligned} P(\text{"not getting red"}) &= \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}} \\ &= \frac{5}{9} \end{aligned}$$

## B.3.2 FINDING PROBABILITIES IN A DICE EXPERIMENT

**Ex 105:** If you roll a die, what is the probability of the event "getting a 3"?

$$P(\text{"getting a 3"}) = \boxed{\frac{1}{6}}$$

*Answer:* There is 1 outcome (3) out of 6 possible outcomes, so the probability is $\frac{1}{6}$.

**Ex 106:** If you roll a die, what is the probability of the event "getting a 5 or 6"?

$$P(\text{"getting a 5 or 6"}) = \boxed{\frac{1}{3}}$$

*Answer:* There are 2 outcomes (5 or 6) out of 6 possible outcomes, so the probability is $\frac{2}{6} = \frac{1}{3}$.

**Ex 107:** If you roll a die, what is the probability of the event "getting a number greater than or equal to 4"?

$$P(\text{number} \geq 4) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (4, 5, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

**Ex 108:** If you roll a die, what is the probability of the event "even number"?

$$P(\text{"even number"}) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (2, 4, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

**Ex 109:** If you roll a die, what is the probability of the event "not getting a 6"?

$$P(\text{"not getting a 6"}) = \boxed{\frac{5}{6}}$$

*Answer:* There are 5 outcomes (1, 2, 3, 4, or 5) out of 6 possible outcomes, so the probability is $\frac{5}{6}$.

**Ex 110:** If you roll a die, what is the probability of the event "not getting an odd number"?

$$P(\text{"not getting an odd number"}) = \boxed{\frac{1}{2}}$$

*Answer:* There are 3 outcomes (2, 4, or 6) out of 6 possible outcomes, so the probability is $\frac{3}{6} = \frac{1}{2}$.

## B.3.3 CALCULATING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

**Ex 111:** A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

| Player 2 / Player 1 | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | **X** | $AB$ | $AC$ | $AD$ |
| $B$ | $BA$ | **X** | $BC$ | $BD$ |
| $C$ | $CA$ | $CB$ | **X** | $CD$ |
| $D$ | $DA$ | $DB$ | $DC$ | **X** |

Calculate the probability that player C is selected as either Player 1 or Player 2.

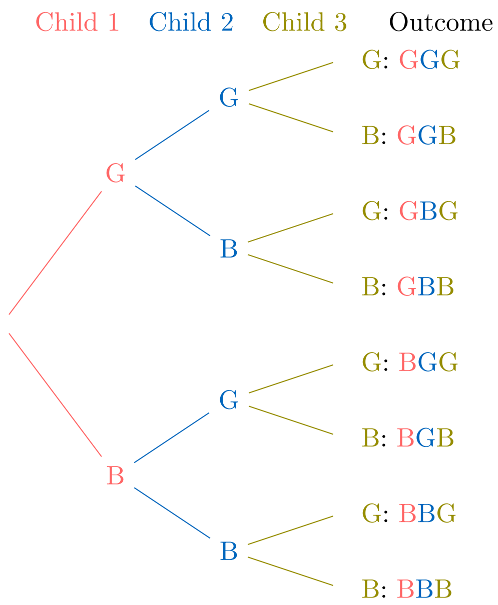
$$P(\text{"selecting player C"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The sample space consists of all possible pairs of players: AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. This totals 12 outcomes.
- The event that player C is selected includes pairs where C is either Player 1 or Player 2: CA, CB, CD, AC, BC, DC. This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"C is selected"}) &= \frac{\text{number of outcomes where C is selected}}{\text{number of possible outcomes}} \\ &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

**Ex 112:** Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.Calculate the probability that the family has at least two girls.



$$P(\text{"at least two girls"}) = \boxed{\frac{1}{2}}$$

*Answer:*

- The sample space consists of all possible gender outcomes for three children: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG. This totals 8 outcomes.
- The event that the family has at least two girls includes outcomes with two or three girls: BGG, GBG, GGB, GGG. This totals 4 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"at least 2 girls"}) &= \frac{\text{Number of outcomes with at least 2 girls}}{\text{Total number of outcomes}} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

**Ex 113:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is exactly 7.

$$P(\text{"sum is 7"}) = \boxed{\frac{1}{6}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for Die 1 times 6 outcomes for Die 2).
- The event that the sum is exactly 7 includes pairs: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 7"}) &= \frac{\text{Number of outcomes with sum 7}}{\text{Total number of outcomes}} \\ &= \frac{6}{36} \\ &= \frac{1}{6} \end{aligned}$$

**Ex 114:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is greater than or equal to 11.

$$P(\text{"sum } \geq 11\text{"}) = \boxed{\frac{1}{12}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is greater than or equal to 11 includes pairs: (5,6), (6,5), (6,6). This totals 3 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum } \geq 11\text{"}) &= \frac{\text{Number of outcomes with sum } \geq 11}{\text{Total number of outcomes}} \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

**Ex 115:** A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

| blue die / red die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 |
| 4 | 41 | 42 | 43 | 44 | 45 | 46 |
| 5 | 51 | 52 | 53 | 54 | 55 | 56 |
| 6 | 61 | 62 | 63 | 64 | 65 | 66 |

Calculate the probability that the sum of the two dice is exactly 6 or 8.

$$P(\text{"sum is 6 or 8"}) = \boxed{\frac{5}{18}}$$

*Answer:*

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is exactly 6 or 8 includes pairs: (1,5), (2,4), (3,3), (4,2), (5,1) for sum 6, and (2,6), (3,5), (4,4), (5,3), (6,2) for sum 8. This totals 10 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 6 or 8"}) &= \frac{\text{Number of outcomes with sum 6 or 8}}{\text{Total number of outcomes}} \\ &= \frac{10}{36} \\ &= \frac{5}{18} \end{aligned}$$

## B.4 PROBABILITY OF INDEPENDENT EVENTS

### B.4.1 DRAW A PROBABILITY TREE FOR TWO INDEPENDENT EVENTS

**Ex 116:** Let $E$ be the event "drawing a red marble" from a bag of 10 marbles with 6 red marbles. Let $F$ be the event "spinning a 1" on a spinner numbered 1 to 10.
Suppose the two events are independent.
**Draw the probability tree diagram.**

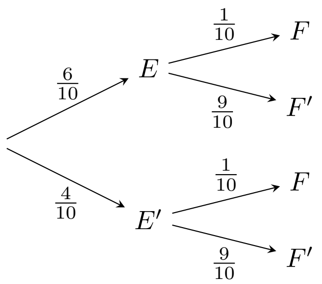
*Answer:* First, calculate the probabilities of the events:

- $P(E) = \frac{6}{10}$
- $P(F) = \frac{1}{10}$

Then, calculate the probabilities of the complementary events:

- $P(E') = 1 - \frac{6}{10} = \frac{10}{10} - \frac{6}{10} = \frac{4}{10}$
- $P(F') = 1 - \frac{1}{10} = \frac{10}{10} - \frac{1}{10} = \frac{9}{10}$

The probability tree diagram is:




**Ex 117:** Let $A$ be the event "drawing an ace" from a standard deck of 52 cards. Let $B$ be the event "rolling a 6" on a regular six-sided die.
Suppose the two events are independent.
**Draw the probability tree diagram.**

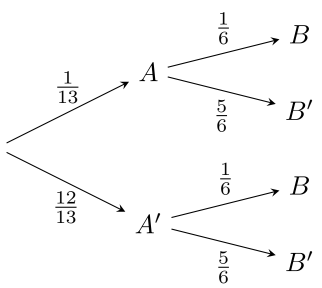
*Answer:* First, calculate the probabilities of the events:

- $P(A) = \frac{4}{52} = \frac{1}{13}$
- $P(B) = \frac{1}{6}$

Then, calculate the probabilities of the complementary events:

- $P(A') = 1 - P(A) = \frac{12}{13}$
- $P(B') = 1 - P(B) = \frac{5}{6}$

The probability tree diagram is:




**Ex 118:** Let $E$ be the event "player $A$ succeeds his basketball shot" ($P(E) = 60\%$).
Let $F$ be the event "player $B$ succeeds his basketball shot" ($P(F) = 70\%$).
Assume the two shots are independent.
**Draw the probability tree diagram.**

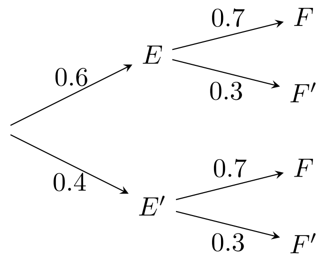
*Answer:* First, calculate the probabilities of the events:

- $P(E) = 60\% = 0.6$
- $P(F) = 70\% = 0.7$

Then, calculate the probabilities of the complementary events:

- $P(E') = 1 - 0.6 = 0.4$ (so, 40% chance $A$ misses)
- $P(F') = 1 - 0.7 = 0.3$ (so, 30% chance $B$ misses)

Now, draw the probability tree diagram:




**Ex 119:** Let $E$ be the event "Julia passes her computer test" ($P(E) = 80\%$).
Let $F$ be the event "Julia passes her English test" ($P(F) = 90\%$).
Assume the two tests are independent.
**Draw the probability tree diagram.**

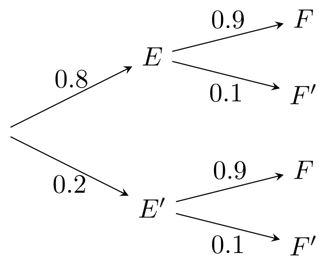
*Answer:* First, calculate the probabilities of the events:

- $P(E) = 80\% = 0.8$
- $P(F) = 90\% = 0.9$

Then, calculate the probabilities of the complementary events:

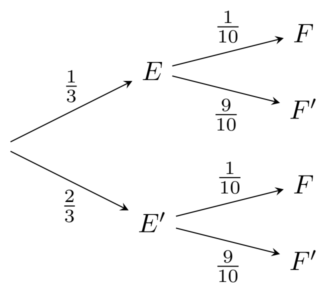
- $P(E') = 1 - 0.8 = 0.2$
- $P(F') = 1 - 0.9 = 0.1$

Now, draw the probability tree diagram:




### B.4.2 CALCULATING PROBABILITIES FROM A TREE DIAGRAM

**Ex 120:** Consider the following probability tree diagram. The two events $E$ and $F$ are independent.



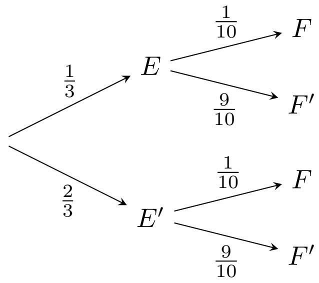

Calculate the probability that both $E$ and $F$ occur

$$P(E \text{ and } F) = \boxed{\frac{1}{30}}$$

*Answer:* To find $P(E \text{ and } F)$, multiply the probabilities along the path $E \to F$:

$$\begin{aligned}P(E \text{ and } F) &= P(E) \times P(F)\\ &= \frac{1}{3} \times \frac{1}{10}\\ &= \frac{1}{30}\end{aligned}$$

**Ex 121:** Consider the following probability tree diagram. The two events $E$ and $F$ are independent.



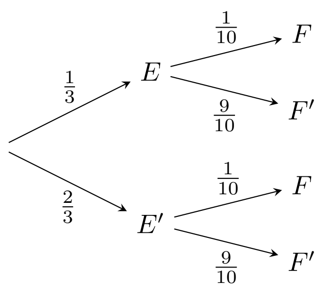

Calculate the probability that neither $E$ nor $F$ occur (i.e., both the complement events $E'$ and $F'$ happen):

$$P(E' \text{ and } F') = \boxed{\frac{3}{5}}$$

*Answer:* To find $P(E' \text{ and } F')$, multiply the probabilities along the path $E' \to F'$:

$$\begin{aligned}P(E' \text{ and } F') &= P(E') \times P(F')\\ &= \frac{2}{3} \times \frac{9}{10}\\ &= \frac{18}{30}\\ &= \frac{3}{5}\end{aligned}$$

**Ex 122:** Consider the following probability tree diagram. The two events $E$ and $F$ are independent.



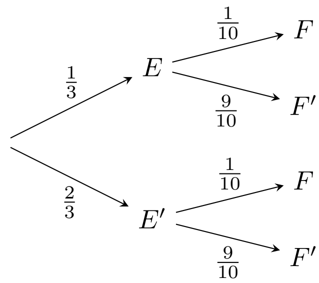

Calculate the probability that $E$ occurs and $F$ does not occur:

$$P(E \text{ and } F') = \boxed{\frac{3}{10}}$$

*Answer:* To find $P(E \text{ and } F')$, multiply the probabilities along the path $E \to F'$:

$$\begin{aligned}P(E \text{ and } F') &= P(E) \times P(F')\\ &= \frac{1}{3} \times \frac{9}{10}\\ &= \frac{9}{30}\\ &= \frac{3}{10}\end{aligned}$$

**Ex 123:** Consider the following probability tree diagram. The two events $E$ and $F$ are independent.




Calculate the probability that $E'$ occurs and $F$ occurs:

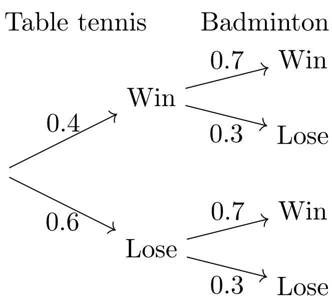
$$P(E' \text{ and } F) = \boxed{\frac{1}{15}}$$

*Answer:* To find $P(E' \text{ and } F)$, multiply the probabilities along the path $E' \to F$:

$$\begin{aligned}P(E' \text{ and } F) &= P(E') \times P(F)\\ &= \frac{2}{3} \times \frac{1}{10}\\ &= \frac{2}{30}\\ &= \frac{1}{15}\end{aligned}$$

### B.4.3 CALCULATING PROBABILITIES FROM A TREE DIAGRAM

**Ex 124:** Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability of winning table tennis is 0.4 and the probability of winning badminton is 0.7. The two events are independent. The probability tree is shown below:




Calculate the probability that Niamh wins both games.

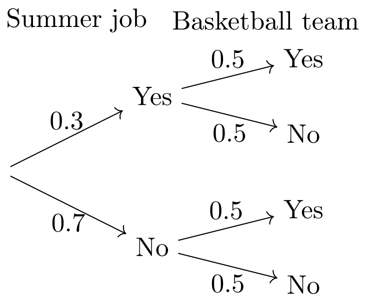
$$P(\text{"Win both"}) = \boxed{0.28}$$

*Answer:* To find the probability that Niamh wins both games, follow the path on the tree where she wins table tennis **and** wins badminton, then multiply the probabilities along that path:

- Probability of winning table tennis: 0.4
- Probability of winning badminton (after winning table tennis): 0.7
- Multiply: $0.4 \times 0.7 = 0.28$

$$
\begin{aligned}
P(\text{"Win both"}) &= P(\text{"Win Tennis" and "Win Badminton"}) \\
&= P(\text{"Win Tennis"}) \times P(\text{"Win Badminton"}) \\
&= 0.4 \times 0.7 \\
&= 0.28
\end{aligned}
$$

**Ex 125:** Noah is applying for a summer job and also trying out for a basketball team. The probability that Noah gets the job is 0.3, and the probability that Noah is selected for the basketball team is 0.5. The two events are independent. The probability tree is shown below:




Calculate the probability that Noah does **not** get the job **and** is **not** selected for the basketball team.

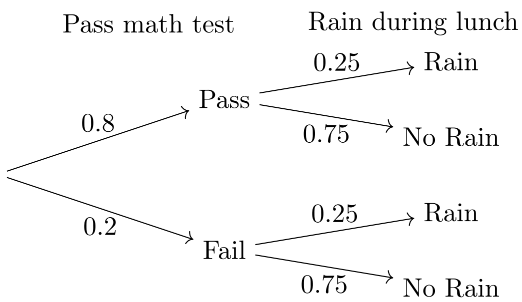
$$P(\text{"No job and not selected for team"}) = \boxed{0.35}$$

*Answer:* To find the probability that Noah does **not** get the job **and** is **not** selected for the basketball team, follow the path on the tree where both are "No", and multiply the probabilities along that path:

- Probability that Noah does not get the job: 0.7
- Probability that Noah is not selected for the basketball team: 0.5
- Multiply: $0.7 \times 0.5 = 0.35$

$$
\begin{aligned}
P(\text{"No job and not selected for team"}) &= P(\text{"No job" and "No team"}) \\
&= P(\text{"No job"}) \times P(\text{"No team"}) \\
&= 0.7 \times 0.5 \\
&= 0.35
\end{aligned}
$$

**Ex 126:** Maria has a math test and there is a chance of rain during lunch. The probability that Maria passes her math test is 0.8, and the probability that it rains during lunch is 0.25. The two events are independent. The probability tree is shown below:




Calculate the probability that Maria passes her math test **and** it rains during lunch.

$$P(\text{"Pass and Rain"}) = \boxed{0.20}$$

*Answer:* To find the probability that Maria passes her math test **and** it rains during lunch, follow the path on the tree where both happen, and multiply the probabilities along that path:

- Probability that Maria passes her math test: 0.8
- Probability that it rains during lunch (given she passed): 0.25
- Multiply: $0.8 \times 0.25 = 0.20$

$$
\begin{aligned}
P(\text{"Pass and Rain"}) &= P(\text{"Pass test" and "Rain during lunch"}) \\
&= P(\text{"Pass test"}) \times P(\text{"Rain during lunch"}) \\
&= 0.8 \times 0.25 \\
&= 0.20
\end{aligned}
$$

### B.4.4 CALCULATING THE PROBABILITY OF TWO INDEPENDENT EVENTS

**Ex 127:** Let $A$ be the event "drawing an ace" from a standard deck of 52 cards. Let $B$ be the event "rolling a 6" on a regular six-sided die.
Suppose the two events are independent.

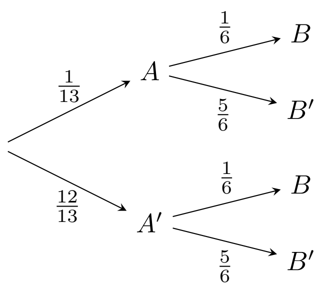
1. Draw the probability tree diagram.
2. Calculate the probability of "drawing an ace" and "rolling a 6".

*Answer:*

1. First, calculate the probabilities of the events:
   - $P(A) = \frac{4}{52} = \frac{1}{13}$
   - $P(B) = \frac{1}{6}$

   Then, calculate the probabilities of the complementary events:
   - $P(A') = 1 - \frac{1}{13} = \frac{12}{13}$
   - $P(B') = 1 - \frac{1}{6} = \frac{5}{6}$

   The probability tree diagram is:




2. To find $P(A \text{ and } B)$, multiply the probabilities along the path $A \to B$:

$$
\begin{aligned}
P(A \text{ and } B) &= P(A) \times P(B) \\
&= \frac{1}{13} \times \frac{1}{6} \\
&= \frac{1}{78}
\end{aligned}
$$

**Ex 128:** Let $E$ be the event "player A succeeds in their basketball shot" ($P(E) = 40\%$). Let $F$ be the event "player B succeeds in their basketball shot" ($P(F) = 40\%$).
Suppose the two events are independent.

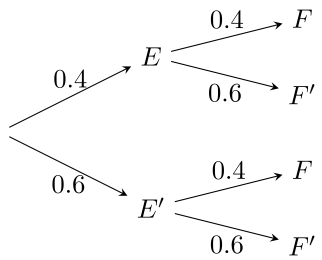
1. Draw the probability tree diagram.
2. Calculate the probability that both players fail their shot.

*Answer:*

1. First, calculate the probabilities of the events:
   - $P(E) = 0.4$
   - $P(F) = 0.4$

   Then, calculate the probabilities of the complementary events (failure):
   - $P(E') = 1 - 0.4 = 0.6$
   - $P(F') = 1 - 0.4 = 0.6$

   The probability tree diagram is:




2. To find $P(E' \text{ and } F')$, multiply the probabilities along the path $E' \to F'$:

$$\begin{aligned} P(E' \text{ and } F') &= P(E') \times P(F') \\ &= 0.6 \times 0.6 \\ &= 0.36 \end{aligned}$$

**Ex 129:** Let $E$ be the event "drawing a red marble" from a bag of 10 marbles with 6 red marbles. Let $F$ be the event "spinning a 1" on a spinner numbered 1 to 10.
Suppose the two events are independent.

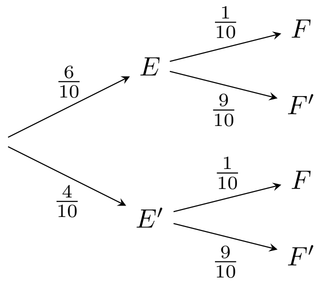
1. Draw the probability tree diagram.
2. Calculate the probability of "drawing a red marble" and "spinning a 1".

*Answer:*

1. First, calculate the probabilities of the events:
   - $P(E) = \frac{6}{10}$
   - $P(F) = \frac{1}{10}$

   Then, calculate the probabilities of the complementary events:
   - $P(E') = 1 - \frac{6}{10} = \frac{4}{10}$
   - $P(F') = 1 - \frac{1}{10} = \frac{9}{10}$

   The probability tree diagram is:




2. To find $P(E \text{ and } F)$, multiply the probabilities along the path $E \to F$:

$$\begin{aligned} P(E \text{ and } F) &= P(E) \times P(F) \\ &= \frac{6}{10} \times \frac{1}{10} \\ &= \frac{6}{100} \\ &= \frac{3}{50} \end{aligned}$$

**Ex 130:** Let $E$ be the event "the machine produces a defective item" ($P(E) = 15\%$). Let $F$ be the event "the item is selected for quality control" ($P(F) = 20\%$).
Suppose the two events are independent.

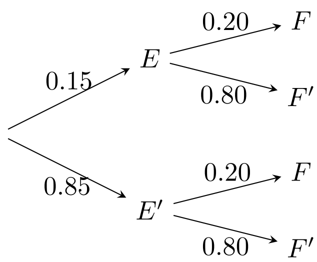
1. Draw the probability tree diagram.
2. Calculate the probability that the machine produces a non-defective item and the item is not selected for quality control.

*Answer:*

1. First, calculate the probabilities of the events:
   - $P(E) = 0.15$
   - $P(F) = 0.20$

   Then, calculate the probabilities of the complementary events:
   - $P(E') = 1 - 0.15 = 0.85$
   - $P(F') = 1 - 0.20 = 0.80$

   The probability tree diagram is:




2. To find $P(E' \text{ and } F')$, multiply the probabilities along the path $E' \to F'$:

$$\begin{aligned} P(E' \text{ and } F') &= P(E') \times P(F') \\ &= 0.85 \times 0.80 \\ &= 0.68 \end{aligned}$$

## B.5 EXPERIMENTAL PROBABILITY

### B.5.1 CALCULATING EXPERIMENTAL PROBABILITIES IN PERCENTAGE FORM

**Ex 131:** During a classroom experiment, Ethan flips a coin 50 times and records that it lands on heads 30 times. Calculate the experimental probability that the coin lands on heads, and express the result in percentage form.

$$P(\text{"landing on heads"}) \approx \boxed{60}\ \%$$

*Answer:*

- The total number of trials in the experiment is 50, since Ethan flipped the coin 50 times.
- The number of successful outcomes for the event "landing on heads" is 30, as the coin landed on heads 30 times.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"landing on heads"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{30}{50} \\ &\approx 30 \div 50 \\ &\approx 0.6 \\ &\approx 0.6 \times 100\% \\ &\approx 60\% \end{aligned}$$

**Ex 132:** During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt, and express the result in percentage form.

$$P(\text{"making the next attempt"}) \approx \boxed{75}\%$$

*Answer:*

- The total number of trials in the experiment is 60, since Mia made 60 free-throw attempts.
- The number of successful outcomes for the event "making the next attempt" is 45, as Mia successfully made 45 free-throws.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"making the next attempt"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{45}{60} \\ &\approx 45 \div 60 \\ &\approx 0.75 \\ &\approx 0.75 \times 100\% \\ &\approx 75\% \end{aligned}$$

**Ex 133:** During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the experimental probability that the next student will choose a vegetarian meal, and express the result in percentage form.

$$P(\text{choosing a vegetarian meal}) \approx \boxed{80}\%$$

*Answer:*

- The total number of trials in the experiment is 150, since 150 students were recorded.
- The number of successful outcomes for the event "choosing a vegetarian meal" is 120, as 120 students chose a vegetarian meal.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"vegetarian meal"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{120}{150} \\ &\approx 120 \div 150 \\ &\approx 0.8 \\ &\approx 0.8 \times 100\% \\ &\approx 80\% \end{aligned}$$

**Ex 134:** Over the course of a year, it rained on 146 days out of 365 recorded days. Estimate the experimental probability that it will rain, and express the result in percentage form.

$$P(\text{"raining"}) \approx \boxed{40}\%$$

*Answer:*

- The total number of trials in the experiment is 365, since 365 days were recorded.
- The number of successful outcomes for the event "raining" is 146, as it rained on 146 days.
- Calculate the experimental probability:

$$\begin{aligned} P(\text{"raining"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{146}{365} \\ &\approx 146 \div 365 \\ &\approx 0.4 \\ &\approx 0.4 \times 100\% \\ &\approx 40\% \end{aligned}$$

### B.5.2 CONDUCTING EXPERIMENTS TO ESTIMATE PROBABILITIES

**Ex 135:** In an experiment, you are asked to toss a fair coin at least 30 times. Follow these steps:

1. Note the number of times the coin lands on heads.
2. Note the total number of trials (tosses).
3. Calculate the experimental probability that the coin lands on heads, and express the result in decimal form.

*Answer:* To demonstrate the process, let's assume a sample result from the experiment: I conducted this experiment and noted each result using tally marks.

1. Number of heads = 卌卌卌|||
   Number of heads = 18
2. Number of trials = 卌卌卌卌卌卌卌卌
   Number of trials = 40
3. Calculate the experimental probability that the coin lands on heads:

$$\begin{aligned} P(\text{"landing on heads"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{18}{40} \\ &\approx 18 \div 40 \\ &\approx 0.45 \end{aligned}$$

This is a sample result; your actual probability will depend on your experiment's outcomes.

**Ex 136:** In a classroom experiment, you are asked to ask at least 10 friends to randomly choose a single number from 1 to 5. Follow these steps:

1. Note the number of times the answer is 5.
2. Note the total number of trials (friends asked).
3. Calculate the experimental probability that a friend chooses the number 5, and express the result in decimal form.

*Answer:* To demonstrate the process, let's assume a sample result from the experiment: I conducted this survey by asking 40 friends, and I noted each result using tally marks.

1. Number of times the answer is 5 = 卌卌||
   Number of times the answer is 5 = 12
2. Number of trials = 卌卌卌卌卌卌卌卌
   Number of trials = 40
3. Calculate the experimental probability that a friend chooses the number 5:

$$\begin{aligned} P(\text{"choosing the number 5"}) &\approx \frac{\text{number of successful outcomes}}{\text{total number of trials}} \\ &\approx \frac{12}{40} \\ &\approx 12 \div 40 \\ &\approx 0.3 \end{aligned}$$

This is a sample result; your actual probability will depend on your experiment's outcomes.

# C CONDITIONAL PROBABILITY

## C.1 DEFINITION

### C.1.1 EXPLORING PROBABILITIES WITH TWO-WAY TABLES

**Ex 137:** Consider a two-way table showing students' preferences about math (love / don't love), categorized by gender:

| | Loves Math | Don't Love Math | Total |
|---|---|---|---|
| Girls | 35 | 16 | 51 |
| Boys | 30 | 19 | 49 |
| Total | 65 | 35 | 100 |

A student is randomly selected from the class. Find the probability that the selected student is a girl.

$$P(\text{"Girl"}) = \boxed{\frac{51}{100}}$$

*Answer:*

- Total number of girls: 51.
- Total number of students: 100.
- Since all students are equally likely to be selected, the probability of picking a girl is:

$$\begin{aligned} P(\text{"Girl"}) &= \frac{\text{Number of girls}}{\text{Total number of students}} \\ &= \frac{51}{100}. \end{aligned}$$

**Ex 138:** Consider a two-way table showing students' preferences for participating in a school drama club, categorized by gender:

| | Likes Drama Club | Dislikes Drama Club | Total |
|---|---|---|---|
| Girls | 28 | 12 | 40 |
| Boys | 32 | 18 | 50 |
| Total | 60 | 30 | 90 |

A student is randomly selected from the group. Find the probability that the selected student likes the drama club.

$$P(\text{"Likes Drama Club"}) = \boxed{\frac{2}{3}}$$

*Answer:*

- Total number of students who like the drama club: 60.
- Total number of students: 90.
- Since all students are equally likely to be selected, the probability that a randomly selected student likes the drama club is:

$$\begin{aligned} P(\text{"Likes Drama Club"}) &= \frac{\text{Number of students who like drama club}}{\text{Total number of students}} \\ &= \frac{60}{90} \\ &= \frac{2}{3} \text{ (simplify).} \end{aligned}$$

**Ex 139:** Consider a two-way table showing students' preferences for a short school trip, categorized by grade level:

| | Likes Trip | Dislikes Trip | Total |
|---|---|---|---|
| Grade 9 | 15 | 5 | 20 |
| Grade 10 | 25 | 15 | 40 |
| Total | 40 | 20 | 60 |

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9 and likes the trip.

$$P(\text{"Grade 9 and Likes Trip"}) = \boxed{\frac{15}{60}}$$

*Answer:*

- Number of Grade 9 students who like the trip: 15.
- Total number of students: 60.
- Since all students are equally likely to be selected, the probability that a randomly selected student is in Grade 9 and likes the trip is:

$$\begin{aligned} P(\text{"Grade 9 and Likes Trip"}) &= \frac{\text{Number of Grade 9 students who like the trip}}{\text{Total number of students}} \\ &= \frac{15}{60} \\ &= \frac{1}{4} \text{ (simplify).} \end{aligned}$$

**Ex 140:** Consider a two-way table showing students' preferences for science, categorized by gender:

| | Likes Science | Dislikes Science | Total |
|---|---|---|---|
| Girls | 18 | 12 | 30 |
| Boys | 24 | 6 | 30 |
| Total | 42 | 18 | 60 |

A student is randomly selected from the group. Find the probability that the selected student is a boy and likes science.

$$P(\text{"Boy and Likes Science"}) = \boxed{\frac{24}{60}}$$

*Answer:*

- Number of boys who like science: 24.
- Total number of students: 60.
- Since all students are equally likely to be selected, the probability that a randomly selected student is a boy and likes science is:

$$\begin{aligned} P(\text{"Boy and Likes Science"}) &= \frac{\text{Number of boys who like science}}{\text{Total number of students}} \\ &= \frac{24}{60} \\ &= \frac{2}{5} \text{ (simplify).} \end{aligned}$$

**Ex 141:** Consider a two-way table showing students' preferences for science, categorized by gender:

| | Likes Science | Dislikes Science | Total |
|---|---|---|---|
| Girls | 18 | 12 | 30 |
| Boys | 24 | 6 | 30 |
| Total | 42 | 18 | 60 |

A student is randomly selected from the group. Find the probability that the selected student likes science, given that the student is a boy.

$$P(\text{"Likes Science"} \mid \text{"Boy"}) = \boxed{\frac{24}{30}}$$

*Answer:*

- Number of boys who like science: 24.
- Total number of boys: 30.
- Since all boys are equally likely to be selected, the conditional probability that a student likes science, given that the student is a boy, is:

$$\begin{aligned} P(\text{"Likes Science"} \mid \text{"Boy"}) &= \frac{\text{Number of boys who like science}}{\text{Total number of boys}} \\ &= \frac{24}{30} \\ &= \frac{4}{5} \text{ (simplify).} \end{aligned}$$

**Ex 142:** Consider a two-way table showing students' preferences for music, categorized by grade level:

| | Likes Music | Dislikes Music | Total |
|---|---|---|---|
| Grade 9 | 20 | 10 | 30 |
| Grade 10 | 15 | 15 | 30 |
| Total | 35 | 25 | 60 |

A student is randomly selected from the group. Find the probability that the selected student likes music, given that the student is in Grade 9.

$$P(\text{"Likes Music"} \mid \text{"Grade 9"}) = \boxed{\frac{20}{30}}$$

*Answer:*

- Number of Grade 9 students who like music: 20.
- Total number of Grade 9 students: 30.
- Since all Grade 9 students are equally likely to be selected, the conditional probability that a student likes music, given that the student is in Grade 9, is:

$$\begin{aligned} P(\text{"Likes Music"} \mid \text{"Grade 9"}) &= \frac{\text{Number of Grade 9 students who like music}}{\text{Total number of Grade 9 students}} \\ &= \frac{20}{30} \\ &= \frac{2}{3} \text{ (simplify).} \end{aligned}$$

**Ex 143:** Consider a two-way table showing students' preferences for art, categorized by grade level:

| | Likes Art | Dislikes Art | Total |
|---|---|---|---|
| Grade 9 | 12 | 8 | 20 |
| Grade 10 | 18 | 12 | 30 |
| Total | 30 | 20 | 50 |

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes art.

$$P(\text{"Grade 9"} \mid \text{"Likes Art"}) = \boxed{\frac{12}{30}}$$

*Answer:*

- Number of Grade 9 students who like art: 12.
- Total number of students who like art: 30.
- Since all students who like art are equally likely to be selected, the conditional probability that a student is in Grade 9, given that the student likes art, is:

$$\begin{aligned}P(\text{"Grade 9"} \mid \text{"Likes Art"}) &= \frac{\text{Number of Grade 9 students who like art}}{\text{Total number of students who like art}}\\&= \frac{12}{30}\\&= \frac{2}{5} \text{ (simplify).}\end{aligned}$$

**Ex 144:** Consider a two-way table showing students' preferences for art, categorized by grade level:

| | Likes Art | Dislikes Art | Total |
|---|---|---|---|
| Grade 9 | 12 | 8 | 20 |
| Grade 10 | 18 | 12 | 30 |
| Total | 30 | 20 | 50 |

A student is randomly selected from the group. Find the probability that the selected student likes art, given that the student is in Grade 9.

$$P(\text{"Likes Art"} \mid \text{"Grade 9"}) = \boxed{\frac{12}{20}}$$

*Answer:*

- Number of Grade 9 students who like art: 12.
- Total number of Grade 9 students: 20.
- Since all Grade 9 students are equally likely to be selected, the conditional probability that a student likes art, given that the student is in Grade 9, is:

$$\begin{aligned}P(\text{"Likes Art"} \mid \text{"Grade 9"}) &= \frac{\text{Number of Grade 9 students who like art}}{\text{Total number of Grade 9 students}}\\&= \frac{12}{20}\\&= \frac{3}{5} \text{ (simplify).}\end{aligned}$$

**Ex 145:** Consider a two-way table showing students' preferences for music, categorized by grade level:

| | Likes Music | Dislikes Music | Total |
|---|---|---|---|
| Grade 9 | 20 | 10 | 30 |
| Grade 10 | 15 | 15 | 30 |
| Total | 35 | 25 | 60 |

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes music.

$$P(\text{"Grade 9"} \mid \text{"Likes Music"}) = \boxed{\frac{20}{35}}$$

*Answer:*

- Number of Grade 9 students who like music: 20.
- Total number of students who like music: 35.
- Since all students who like music are equally likely to be selected, the conditional probability that a student is in Grade 9, given that the student likes music, is:

$$\begin{aligned}P(\text{"Grade 9"} \mid \text{"Likes Music"}) &= \frac{\text{Number of Grade 9 students who like music}}{\text{Total number of students who like music}}\\&= \frac{20}{35}\\&= \frac{4}{7} \text{ (simplify).}\end{aligned}$$

### C.1.2 CALCULATING CONDITIONAL PROBABILITIES

**Ex 146:** Given that $P(E \text{ and } F) = 0.1$ and $P(F) = 0.4$, find :

$$P(E \mid F) = \boxed{0.25}$$

*Answer:* Applying the conditional probability formula,

$$\begin{aligned}P(E \mid F) &= \frac{P(E \text{ and } F)}{P(F)}\\&= \frac{0.1}{0.4}\\&= 0.25\end{aligned}$$

**Ex 147:** Given that $P(A \text{ and } B) = 0.15$ and $P(B) = 0.5$, find :

$$P(A \mid B) = \boxed{0.3}$$

*Answer:* Applying the conditional probability formula,

$$\begin{aligned}P(A \mid B) &= \frac{P(A \text{ and } B)}{P(B)}\\&= \frac{0.15}{0.5}\\&= 0.3\end{aligned}$$

**Ex 148:** Given that $P(X \text{ and } Y) = 0.12$ and $P(Y) = 0.3$, find :

$$P(X \mid Y) = \boxed{0.4}$$

*Answer:* Applying the conditional probability formula,

$$\begin{aligned}P(X \mid Y) &= \frac{P(X \text{ and } Y)}{P(Y)}\\&= \frac{0.12}{0.3}\\&= 0.4\end{aligned}$$

**Ex 149:** Given that $P(M \text{ and } N) = 0.25$ and $P(N) = 0.8$, find :

$$P(M \mid N) = \boxed{0.3125}$$

*Answer:* Applying the conditional probability formula,

$$\begin{aligned}P(M \mid N) &= \frac{P(M \text{ and } N)}{P(N)}\\&= \frac{0.25}{0.8}\\&= 0.3125\end{aligned}$$

### C.1.3 CALCULATING CONDITIONAL PROBABILITIES IN REAL-WORLD PROBLEMS

**Ex 150:** In a certain town, it is found that 30% of the families own a pet dog, and 15% of the families own both a pet dog and a cat. If a randomly selected family owns a dog, what is the probability that they also own a cat?

$$P(\text{"Own a cat"} \mid \text{"Own a dog"}) = \boxed{0.5}$$

*Answer:* To find the conditional probability, use the formula:

$$\begin{aligned} P(\text{"Own a cat"} \mid \text{"Own a dog"}) &= \frac{P(\text{"Own a dog and a cat"})}{P(\text{"Own a dog"})} \\ &= \frac{0.15}{0.3} \\ &= 0.5 \end{aligned}$$

This means that if a family owns a dog, there is a 50% chance they also own a cat.

**Ex 151:** In a school, it is found that 40% of the students enjoy reading books, and 20% of the students enjoy both reading books and watching movies. If a randomly selected student enjoys reading books, what is the probability that they also enjoy watching movies?

$$P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) = \boxed{0.5}$$

*Answer:* To find the conditional probability, use the formula:

$$\begin{aligned} P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) &= \frac{P(\text{"Enjoy books and movies"})}{P(\text{"Enjoy books"})} \\ &= \frac{0.20}{0.4} \\ &= 0.5 \end{aligned}$$

This means that if a student enjoys reading books, there is a 50% chance they also enjoy watching movies.

**Ex 152:** In a neighborhood, it is found that 60% of the households own a car, and 24% of the households own both a car and a bicycle. If a randomly selected household owns a car, what is the probability that they also own a bicycle?

$$P(\text{"Own a bicycle"} \mid \text{"Own a car"}) = \boxed{0.4}$$

*Answer:* To find the conditional probability, use the formula:

$$\begin{aligned} P(\text{"Own a bicycle"} \mid \text{"Own a car"}) &= \frac{P(\text{"Own a car and a bicycle"})}{P(\text{"Own a car"})} \\ &= \frac{0.24}{0.6} \\ &= 0.4 \end{aligned}$$

This means that if a household owns a car, there is a 40% chance they also own a bicycle.

**Ex 153:** In a club, it is found that 25% of the members play soccer, and 10% of the members play both soccer and basketball. If a randomly selected member plays soccer, what is the probability that they also play basketball?

$$P(\text{"Play basketball"} \mid \text{"Play soccer"}) = \boxed{0.4}$$

*Answer:* To find the conditional probability, use the formula:

$$\begin{aligned} P(\text{"Play basketball"} \mid \text{"Play soccer"}) &= \frac{P(\text{"Play soccer and basketball"})}{P(\text{"Play soccer"})} \\ &= \frac{0.10}{0.25} \\ &= 0.4 \end{aligned}$$

This means that if a member plays soccer, there is a 40% chance they also play basketball.

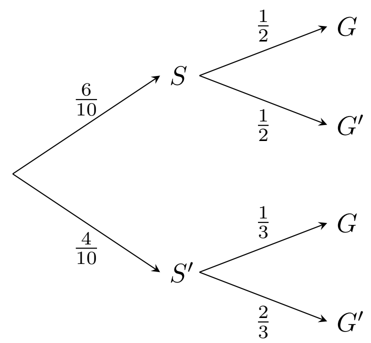
## C.2 CONDITIONAL PROBABILITY TREE DIAGRAMS

### C.2.1 IDENTIFYING CONDITIONAL PROBABILITY TREE DIAGRAMS

**MCQ 154:** The probability that Sam is coaching a game is $\frac{6}{10}$, and the probability that Alex is coaching is $\frac{4}{10}$.

- With Coach Sam, the probability of being Goalkeeper is $\frac{1}{2}$.
- With Coach Alex, the probability of being Goalkeeper is $\frac{2}{3}$.

Let $S$ be the event that Sam is the coach.
Let $G$ be the event of being the goalkeeper.
**Choose the correct probability tree diagram:**

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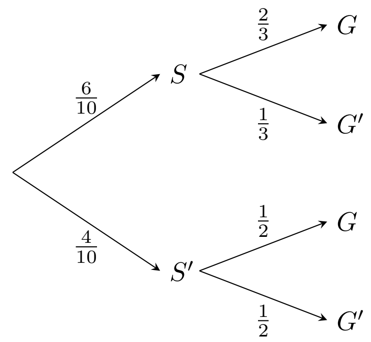


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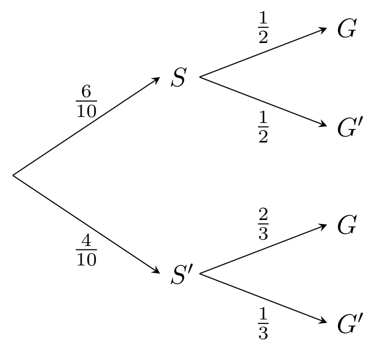


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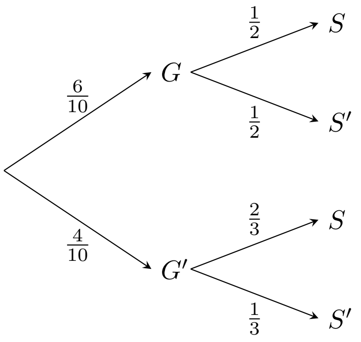
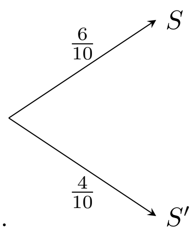
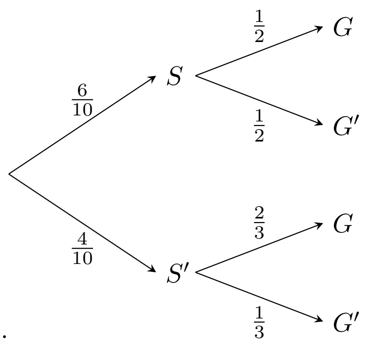



☐

*Answer:*

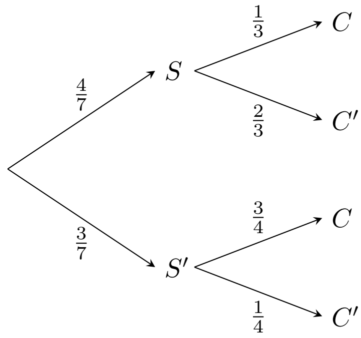
- Let $S'$ be the event that Alex is the coach and $G'$ be the event of not being the goalkeeper.
- The probability that Sam is coaching, denoted as $P(S)$, is $\frac{6}{10}$.
- The probability that Alex is coaching, denoted as $P(S')$, is $\frac{4}{10}$.
- So, the first step is: 

- From the Sam and Alex branches, we have two events: being the goalkeeper ($G$) and not being the goalkeeper ($G'$).
- The probability of being the goalkeeper when Sam is the coach is given as $P(G \mid S) = \frac{1}{2}$.
- The probability of not being the goalkeeper when Sam is the coach is $P(G' \mid S) = 1 - \frac{1}{2} = \frac{1}{2}$.
- The probability of being the goalkeeper when Alex is the coach is given as $P(G \mid S') = \frac{2}{3}$.
- The probability of not being the goalkeeper when Alex is the coach is $P(G' \mid S') = 1 - \frac{2}{3} = \frac{1}{3}$.
- So, the correct probability tree diagram is: 


**MCQ 155:** In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$, and the probability of it being rainy is $\frac{3}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.
- On a rainy day, the probability that the park is crowded is $\frac{1}{4}$.

Let $S$ be the event that it's a sunny day.
Let $C$ be the event of the park being crowded.
**Choose the correct probability tree diagram:**

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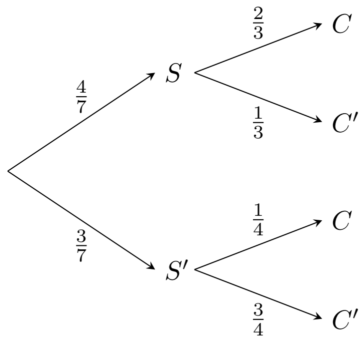


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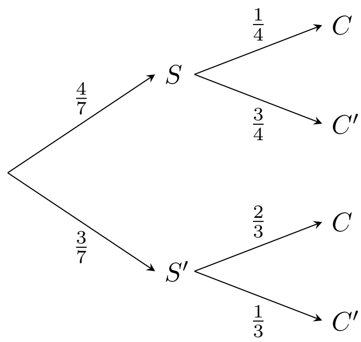


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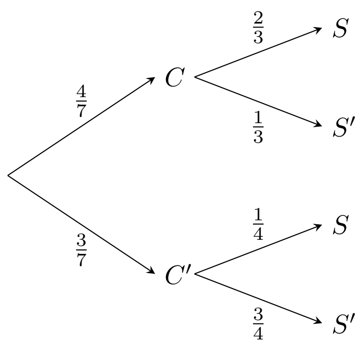
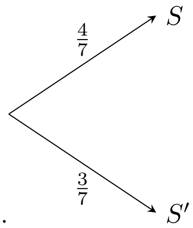



☐

*Answer:*

- Let $S'$ be the event that it's a rainy day and $C'$ be the event that the park is not crowded.
- The probability that it's a sunny day, denoted as $P(S)$, is $\frac{4}{7}$.
- The probability that it's a rainy day, denoted as $P(S')$, is $\frac{3}{7}$.

- So, the first step is:




- From the sunny and rainy branches, we have two events: the park being crowded ($C$) and the park not being crowded ($C'$).

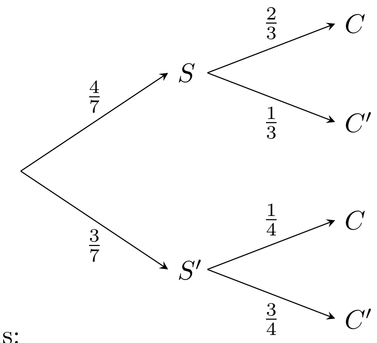
- The probability that the park is crowded on a sunny day is given as $P(C \mid S) = \frac{2}{3}$.

- The probability that the park is not crowded on a sunny day is $P(C' \mid S) = 1 - \frac{2}{3} = \frac{1}{3}$.

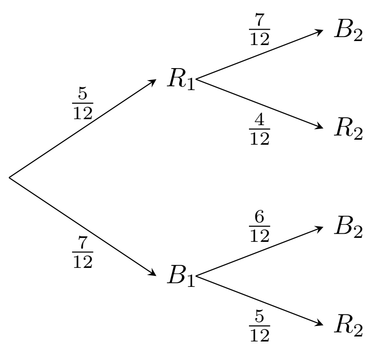
- The probability that the park is crowded on a rainy day is given as $P(C \mid S') = \frac{1}{4}$.

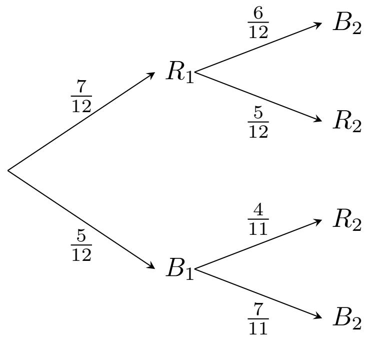
- The probability that the park is not crowded on a rainy day is $P(C' \mid S') = 1 - \frac{1}{4} = \frac{3}{4}$.

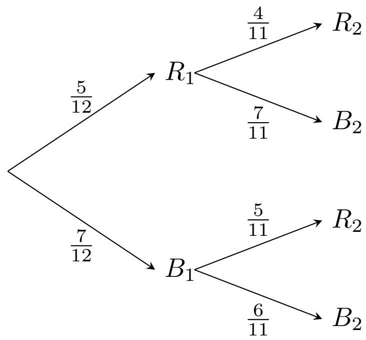
- So, the correct probability tree diagram is:




**MCQ 156:** A bag contains 5 red (R) and 7 blue (B) marbles. One marble is drawn and not replaced, and then a second marble is drawn.
Let $R_1$ be the event of drawing a red marble first, and let $B_1$ be the event of drawing a blue marble first.
Let $R_2$ be the event of drawing a red marble second, and let $B_2$ be the event of drawing a blue marble second.
**Choose the correct probability tree diagram:**

☐ 


☐ 


☒ 

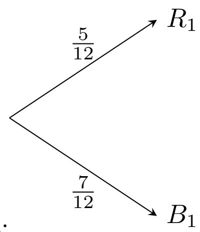

*Answer:*

- The total number of marbles is $5 + 7 = 12$.

- The probability of drawing a red marble first, denoted as $P(R_1)$, is $\frac{5}{12}$.

- The probability of drawing a blue marble first, denoted as $P(B_1)$, is $\frac{7}{12}$.

- So, the first step is:




- From the red and blue branches, we have two possibilities for the second draw: drawing a red marble ($R_2$) or a blue marble ($B_2$).

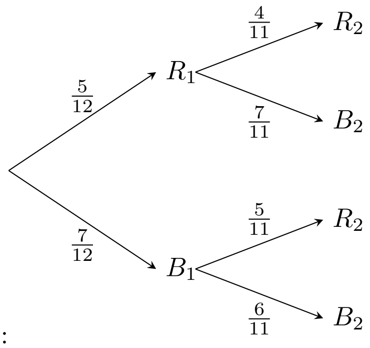
- After drawing a red marble first (leaving 4 red and 7 blue, total 11), the probability of drawing a red marble second is $P(R_2 \mid R_1) = \frac{4}{11}$.

- The probability of drawing a blue marble second after a red marble first is $P(B_2 \mid R_1) = \frac{7}{11}$.

- After drawing a blue marble first (leaving 5 red and 6 blue, total 11), the probability of drawing a red marble second is $P(R_2 \mid B_1) = \frac{5}{11}$.

- The probability of drawing a blue marble second after a blue marble first is $P(B_2 \mid B_1) = \frac{6}{11}$.

- So, the correct probability tree diagram

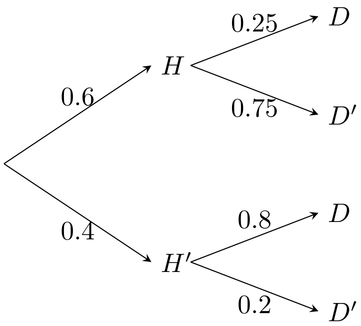



is:

**MCQ 157:** In a town, the probability that a randomly chosen morning has heavy traffic is 0.6, and the probability that it has light traffic is 0.4.

- On a morning with heavy traffic, the probability of a bus being delayed is 0.75.
- On a morning with light traffic, the probability of a bus being delayed is 0.2.

Let $H$ be the event that the morning has heavy traffic.
Let $D$ be the event that the bus is delayed.
**Choose the correct probability tree diagram:**

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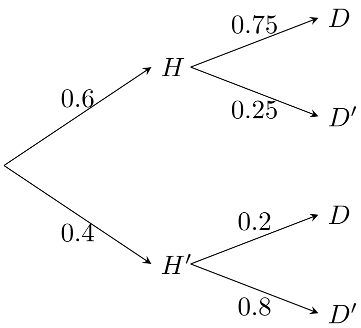


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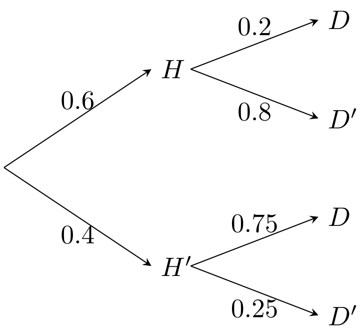


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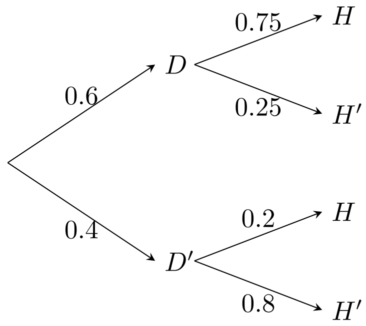
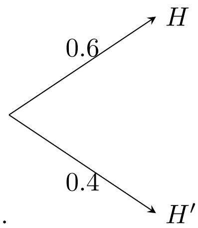


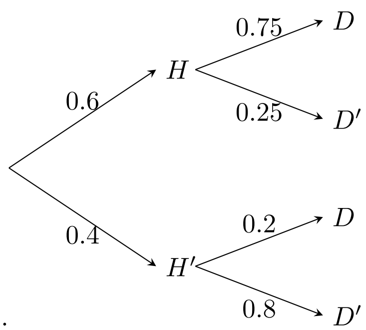

☐

*Answer:*

- Let $H'$ be the event that the morning has light traffic and $D'$ be the event that the bus is not delayed.
- The probability that the morning has heavy traffic, denoted as $P(H)$, is 0.6.
- The probability that the morning has light traffic, denoted as $P(H')$, is 0.4.
- So, the first step is:




- From the heavy and light traffic branches, we have two events: the bus being delayed ($D$) and the bus not being delayed ($D'$).
- The probability that the bus is delayed on a morning with heavy traffic is given as $P(D \mid H) = 0.75$.
- The probability that the bus is not delayed on a morning with heavy traffic is $P(D' \mid H) = 1 - 0.75 = 0.25$.
- The probability that the bus is delayed on a morning with light traffic is given as $P(D \mid H') = 0.2$.
- The probability that the bus is not delayed on a morning with light traffic is $P(D' \mid H') = 1 - 0.2 = 0.8$.
- So, the correct probability tree diagram




is:

## C.2.2 DRAWING CONDITIONAL PROBABILITY TREE DIAGRAMS

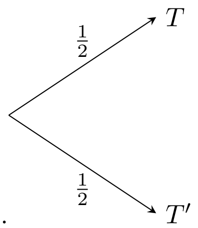
**Ex 158:** The probability that a team wins the coin toss in a cricket match is $\frac{1}{2}$.

- If the team wins the toss, the probability that it wins the match is $\frac{3}{5}$.
- If the team loses the toss, the probability that it wins the match is $\frac{2}{5}$.

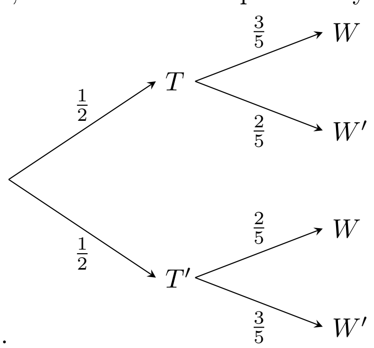
Let $T$ be the event that the team wins the toss.
Let $W$ be the event that the team wins the match.
Draw the probability tree diagram

*Answer:*

- Let $T'$ be the event that the team loses the toss and $W'$ be the event that the team does not win the match.
- The probability that the team wins the toss, denoted as $P(T)$, is $\frac{1}{2}$.
- The probability that the team loses the toss, denoted as $P(T')$, is $\frac{1}{2}$.
- So, the first step is:




- From the wins toss and loses toss branches, we have two events: winning the match $(W)$ and not winning the match $(W')$.
- The probability that the team wins the match when it wins the toss is given as $P(W \mid T) = \frac{3}{5}$.
- The probability that the team does not win the match when it wins the toss is $P(W' \mid T) = 1 - \frac{3}{5} = \frac{2}{5}$.
- The probability that the team wins the match when it loses the toss is given as $P(W \mid T') = \frac{2}{5}$.
- The probability that the team does not win the match when it loses the toss is $P(W' \mid T') = 1 - \frac{2}{5} = \frac{3}{5}$.
- So, the probability tree diagram is:



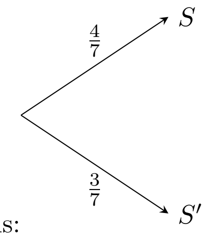

**Ex 159:** In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.
- On a non-sunny day, the probability that the park is crowded is $\frac{1}{4}$.

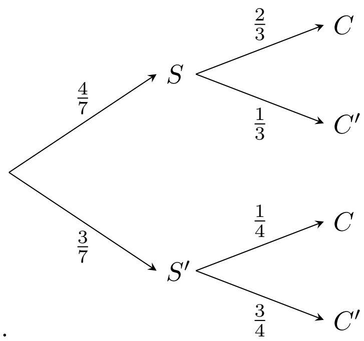
Let $S$ be the event that it's a sunny day.
Let $C$ be the event that the park is crowded.
Draw the probability tree diagram

*Answer:*

- Let $S'$ be the event that it is a non-sunny day and $C'$ be the event that the park is not crowded.
- The probability that it is sunny, denoted as $P(S)$, is $\frac{4}{7}$.
- The probability that it is not sunny, denoted as $P(S')$, is $1 - \frac{4}{7} = \frac{3}{7}$.
- So, the first step is:




- From the sunny and not sunny branches, we have two events: the park being crowded $(C)$ and the park not being crowded $(C')$.
- The probability that the park is crowded on a sunny day is given as $P(C \mid S) = \frac{2}{3}$.
- The probability that the park is not crowded on a sunny day is $P(C' \mid S) = 1 - \frac{2}{3} = \frac{1}{3}$.
- The probability that the park is crowded on a non-sunny day is given as $P(C \mid S') = \frac{1}{4}$.
- The probability that the park is not crowded on a non-sunny day is $P(C' \mid S') = 1 - \frac{1}{4} = \frac{3}{4}$.
- So, the probability tree diagram is:




**Ex 160:** The probability that a person has a rare disease is $\frac{1}{100}$, and the probability that the person does not have the disease is $\frac{99}{100}$.

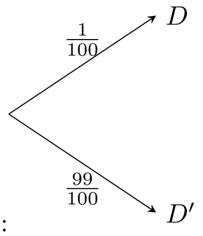
- If the person has the disease, the probability that the test detects it (positive result) is $\frac{99}{100}$.
- If the person does not have the disease, the probability that the test falsely detects it (positive result) is $\frac{1}{100}$.

Let $D$ be the event that the person has the disease.
Let $T^+$ be the event that the test is positive.
Draw the probability tree diagram

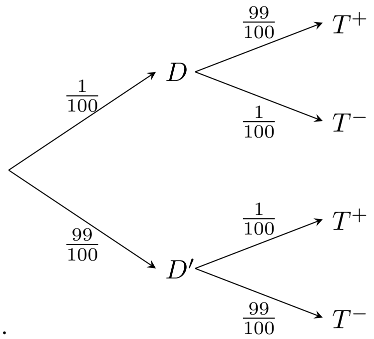
*Answer:*

- Let $D'$ be the event that the person does not have the disease and $T^-$ be the event that the test is negative.

- The probability that the person has the disease, denoted as $P(D)$, is $\frac{1}{100}$.
- The probability that the person does not have the disease, denoted as $P(D')$, is $\frac{99}{100}$.
- So, the first step is:



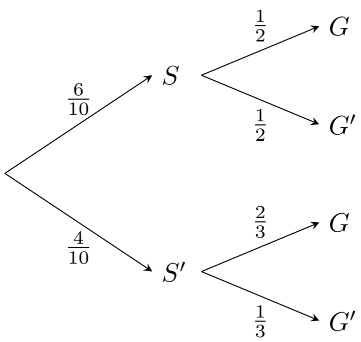

- From the disease and no disease branches, we have two events: positive test ($T^+$) and negative test ($T^-$).
- The probability of a positive test when the person has the disease is given as $P(T^+ \mid D) = \frac{99}{100}$.
- The probability of a negative test when the person has the disease is $P(T^- \mid D) = 1 - \frac{99}{100} = \frac{1}{100}$.
- The probability of a positive test when the person does not have the disease is given as $P(T^+ \mid D') = \frac{1}{100}$.
- The probability of a negative test when the person does not have the disease is $P(T^- \mid D') = 1 - \frac{1}{100} = \frac{99}{100}$.
- So, the probability tree diagram is:




## C.3 JOINT PROBABILITY: $P(E \cap F)$

### C.3.1 CALCULATING JOINT PROBABILITIES WITH TREES

**Ex 161:** For this probability tree diagram:

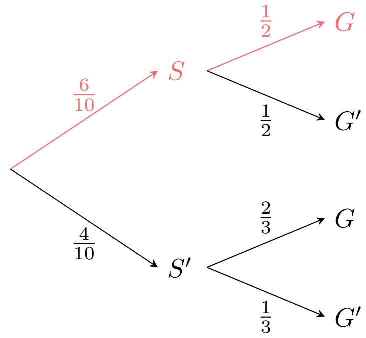



find the probability :

$$P(S \text{ and } G) = \boxed{\frac{3}{10}}$$

*Answer:*

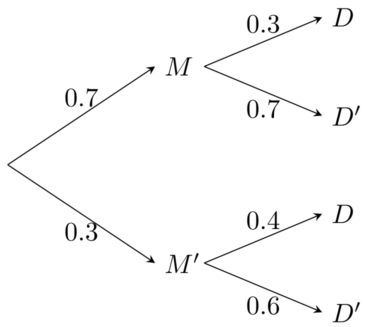
- **Path:** $S$ to $G$ (highlighted):




- **Calculate:**

$$\begin{aligned} P(S \text{ and } G) &= P(S) \times P(G \mid S) \\ &= \frac{6}{10} \times \frac{1}{2} \\ &= \frac{6}{20} \\ &= \frac{3}{10} \end{aligned}$$

**Ex 162:** For this probability tree diagram:

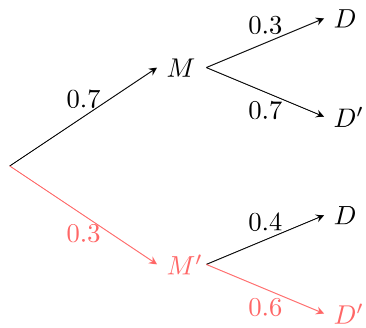



find the probability :

$$P(M' \text{ and } D') = \boxed{0.18}$$

*Answer:*

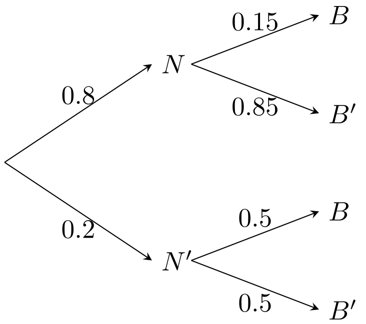
- **Path:** $M'$ to $D'$ (highlighted):




- **Calculate:**

$$\begin{aligned} P(M' \text{ and } D') &= P(M') \times P(D' \mid M') \\ &= 0.3 \times 0.6 \\ &= 0.18 \end{aligned}$$

**Ex 163:** For this probability tree diagram:

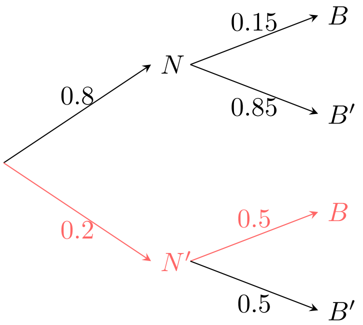



find the probability :

$$P(N' \text{ and } B) = \boxed{0.1}$$

*Answer:*

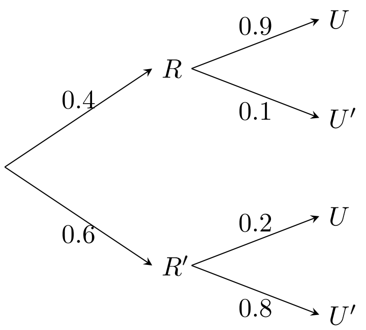
- **Path:** $N'$ to $B$ (highlighted):




- **Calculate:**

$$\begin{aligned} P(N' \text{ and } B) &= P(N') \times P(B \mid N') \\ &= 0.2 \times 0.5 \\ &= 0.1 \end{aligned}$$

**Ex 164:** For this probability tree diagram:

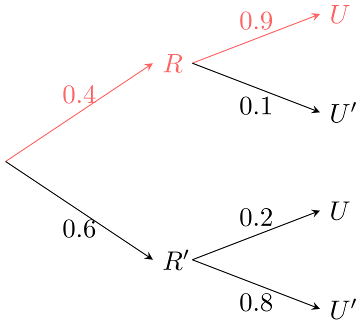



find the probability :

$$P(R \text{ and } U) = \boxed{0.36}$$

*Answer:*

- **Path:** $R$ to $U$ (highlighted):



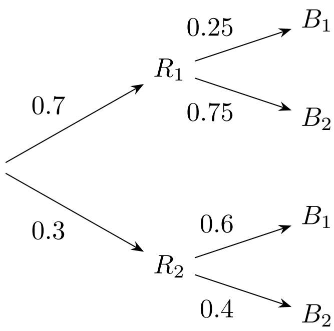

- **Calculate:**

$$\begin{aligned} P(R \text{ and } U) &= P(R) \times P(U \mid R) \\ &= 0.4 \times 0.9 \\ &= 0.36 \end{aligned}$$

## C.4 LAW OF TOTAL PROBABILITY

### C.4.1 CALCULATING PROBABILITIES WITH TREES

**Ex 165:** For this probability tree:

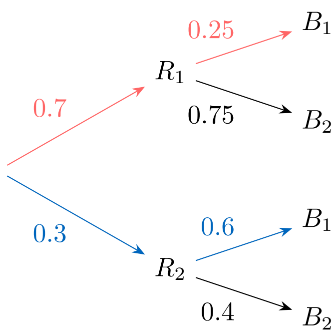



calculate the probability :

$$P(B_1) = \boxed{0.355}$$

*Answer:*

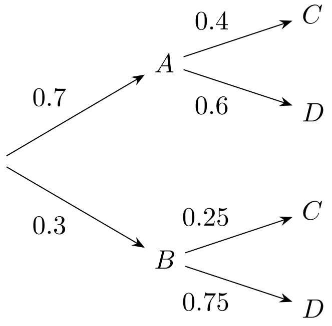
1. Paths to $B_1$:




2. Calculate:

$$\begin{aligned} P(B_1) &= P(R_1) \times P(B_1 \mid R_1) + P(R_2) \times P(B_1 \mid R_2) \\ &= 0.7 \times 0.25 + 0.3 \times 0.6 \\ &= 0.175 + 0.18 \\ &= 0.355 \end{aligned}$$

**Ex 166:** For this probability tree:

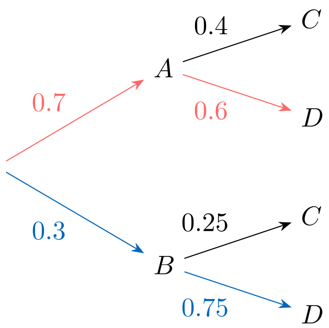



calculate the probability :

$$P(D) = \boxed{0.645}$$

*Answer:*

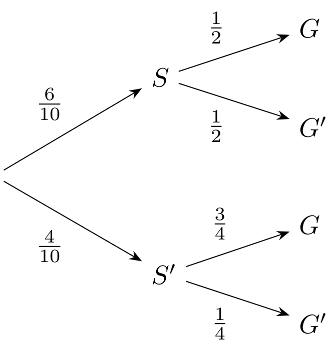
1. Paths to $D$:




2. Calculate:

$$\begin{aligned}
P(D) &= P(A) \times P(D \mid A) + P(B) \times P(D \mid B) \\
&= 0.7 \times 0.6 + 0.3 \times 0.75 \\
&= 0.42 + 0.225 \\
&= 0.645
\end{aligned}$$

**Ex 167:** For this probability tree,

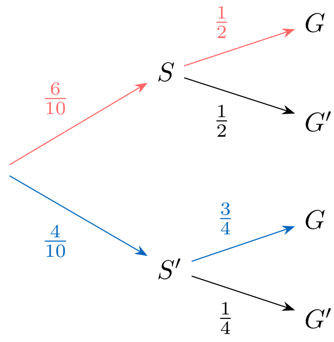



calculate the probability :

$$P(G) = \boxed{\frac{3}{5}}$$

*Answer:*

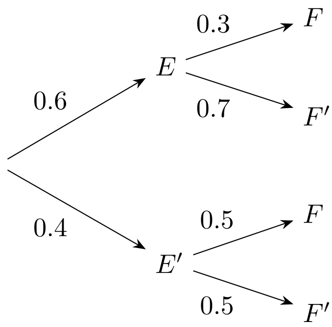
1. Paths to $G$:




2. Calculate:

$$\begin{aligned}
P(G) &= P(S) \times P(G \mid S) + P(S') \times P(G \mid S') \\
&= \frac{6}{10} \times \frac{1}{2} + \frac{4}{10} \times \frac{3}{4} \\
&= \frac{6}{20} + \frac{12}{40} \\
&= \frac{12}{40} + \frac{12}{40} \\
&= \frac{24}{40} \\
&= \frac{3}{5}.
\end{aligned}$$

**Ex 168:** For this probability tree:

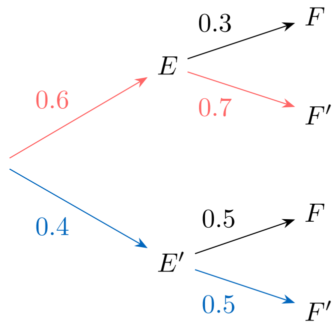



calculate the probability :

$$P(F') = \boxed{0.62}$$

*Answer:*

1. Paths to $F'$:




2. Calculate:

$$\begin{aligned}
P(F') &= P(E) \times P(F' \mid E) + P(E') \times P(F' \mid E') \\
&= 0.6 \times 0.7 + 0.4 \times 0.5 \\
&= 0.42 + 0.2 \\
&= 0.62
\end{aligned}$$

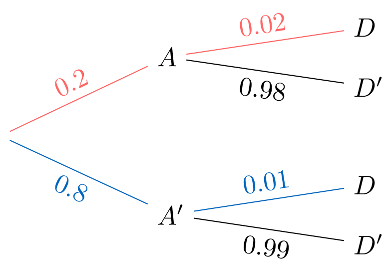
## C.4.2 CALCULATING PROBABILITIES IN REAL-WORLD PROBLEMS

**Ex 169:** A company produces two types of parts: A and B. 20% of parts are type A and 80% are type B. The probability that a part is defective given type A is 0.02, and the probability that a part is defective given type B is 0.01.
Find the probability that a part is defective :

$$P(\text{"Defective"}) = \boxed{0.012}$$

*Answer:*

- **Define the events:**
  - $A$: Part is type A.
  - $D$: Part is defective.
- **Define the probabilities:**
  - $P(A) = 20\% = \frac{20}{100} = 0.2$, and $P(A') = 80\% = \frac{80}{100} = 0.8$.
  - $P(D|A) = 0.02$, and $P(D|A') = 0.01$.
- **Paths to $D$:**




- **Law of total probability:**

$$\begin{aligned} P(D) &= P(A) \times P(D|A) + P(A') \times P(D|A') \\ &= 0.2 \times 0.02 + 0.8 \times 0.01 \\ &= 0.004 + 0.008 \\ &= 0.012 \end{aligned}$$

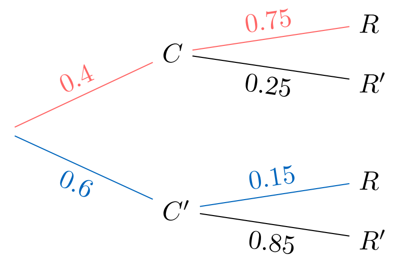
- The probability that a part is defective is 0.012.

**Ex 170:** A meteorologist observes cloud conditions to predict rain. On a given day, 40% of the time the sky is cloudy, and 60% of the time it is clear. The probability of rain given a cloudy sky is 0.75, and the probability of rain given a clear sky is 0.15.
Find the probability that it rains :

$$P(\text{Rain}) = \boxed{0.39}$$

*Answer:*

- **Define the events:**
  - $C$: Sky is cloudy.
  - $R$: It rains.
- **Define the probabilities:**
  - $P(C) = 40\% = \frac{40}{100} = 0.4$, and $P(C') = 60\% = \frac{60}{100} = 0.6$.
  - $P(R|C) = 0.75$, and $P(R|C') = 0.15$.
- **Paths to $R$:**




- **Law of total probability:**

$$\begin{aligned} P(R) &= P(C) \times P(R|C) + P(C') \times P(R|C') \\ &= 0.4 \times 0.75 + 0.6 \times 0.15 \\ &= 0.3 + 0.09 \\ &= 0.39 \end{aligned}$$

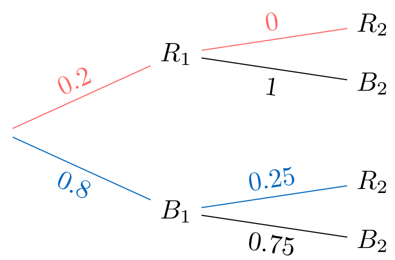
- The probability that it rains is 0.39.

**Ex 171:** An urn contains 1 red ball and 4 blue balls. A first ball is drawn without replacement. Then a second ball is drawn from the remaining balls.
Find the probability that the second ball drawn is red :

$$P(R_2) = \boxed{0.2}$$

*Answer:*

- **Define the events:**
  - $R_1$: First ball drawn is red.
  - $B_1$: First ball drawn is blue.
  - $R_2$: Second ball drawn is red.
- **Define the probabilities:**
  - Total balls $=$ 5 (1 red $+$ 4 blue).
  - $P(R_1) = \frac{1}{5} = 0.2$, and $P(B_1) = \frac{4}{5} = 0.8$.
  - $P(R_2|R_1) = \frac{0}{4} = 0$ (if first is red, 0 red remain out of 4).
  - $P(R_2|B_1) = \frac{1}{4} = 0.25$ (if first is blue, 1 red remains out of 4).
- **Paths to $R_2$:**




- **Law of total probability:**

$$\begin{aligned} P(R_2) &= P(R_1) \times P(R_2|R_1) + P(B_1) \times P(R_2|B_1) \\ &= 0.2 \times 0 + 0.8 \times 0.25 \\ &= 0 + 0.2 \\ &= 0.2 \end{aligned}$$

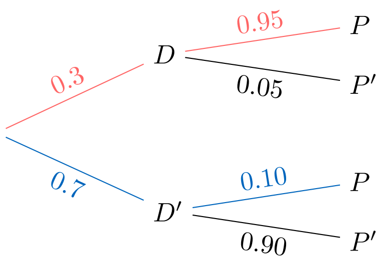
- The probability that the second ball drawn is red is 0.2.

**Ex 172:** A population is tested for a disease. 30% of the population has the disease. The probability that a test is positive given the person has the disease is 0.95, and the probability that a test is positive given the person does not have the disease is 0.10.
Find the probability that a test is positive :

$$P(\text{Positive}) = \boxed{0.355}$$

*Answer:*

- **Define the events:**
  - $D$: Person has the disease.
  - $P$: Test is positive.
- **Define the probabilities:**
  - $P(D) = 30\% = \frac{30}{100} = 0.3$, and $P(D') = 1 - 0.3 = 0.7$.
  - $P(P|D) = 0.95$, and $P(P|D') = 0.10$.
- **Paths to $P$:**




- **Law of total probability:**

$$\begin{aligned} P(P) &= P(D) \times P(P|D) + P(D') \times P(P|D') \\ &= 0.3 \times 0.95 + 0.7 \times 0.10 \\ &= 0.285 + 0.07 \\ &= 0.355 \end{aligned}$$

- The probability that a test is positive is 0.355.

## C.5 BAYES' THEOREM

### C.5.1 UNVEILING THE HIDDEN CAUSE: BAYES' THEOREM IN RARE EVENT DETECTION

**Ex 173:** Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

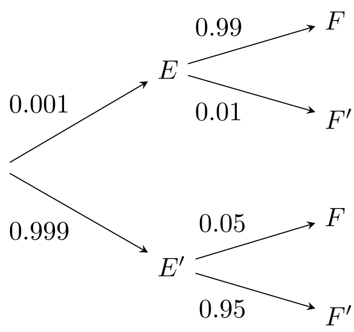
- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

Find the probability in percent that a person actually has the disease if their test result is positive (round to 1 decimal place):

$$P(\text{Disease} \mid \text{Test positive}) = \boxed{1.9}\%$$

*Answer:*

- **Define the events:**
  - $E$: The person has the disease.
  - $F$: The test result is positive.
- **Define the probabilities:**
  - $P(E) = \frac{1}{1000} = 0.001$, thus $P(E') = 1 - 0.001 = 0.999$.
  - $P(F|E) = 0.99$, hence $P(F'|E) = 1 - 0.99 = 0.01$.
  - $P(F'|E') = 0.95$, hence $P(F|E') = 1 - 0.95 = 0.05$.
- **Draw the probability tree:**




- **Calculate $P(F)$:**

$$\begin{aligned} P(F) &= P(E) \times P(F|E) + P(E') \times P(F|E') \\ &= (0.001 \times 0.99) + (0.999 \times 0.05) \\ &= 0.00099 + 0.04995 \\ &= 0.05094 \end{aligned}$$

- **Calculate $P(E \mid F)$:** Using Bayes' theorem:

$$\begin{aligned} P(E \mid F) &= \frac{P(E) \times P(F|E)}{P(F)} \\ &= \frac{0.001 \times 0.99}{0.05094} \\ &\approx 0.01943 \\ &\approx 1.943\% \quad \text{(convert to percent)} \\ &\approx 1.9\% \quad \text{(round to 1 decimal place)} \end{aligned}$$

- The probability that a person actually has the disease, given a positive test result, is approximately 1.9%. This highlights a key issue with screening tests for rare conditions: even highly accurate tests can yield a significant proportion of false positives.

**Ex 174:** Consider a rare alien signal that is present in approximately 1 out of every 10,000 radio scans conducted by a space observatory. A signal detector has the following characteristics:

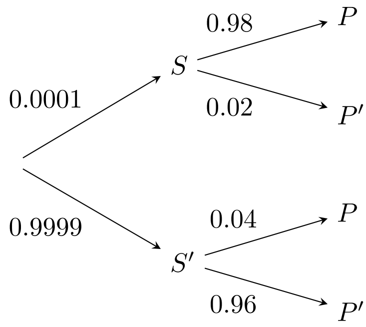
- Sensitivity: If an alien signal is present, the detector correctly identifies it as positive 98 % of the time.
- Specificity: If no alien signal is present, the detector correctly identifies it as negative 96 % of the time.

Find the probability in percent that an alien signal is actually present if the detector returns a positive result (round to 1 decimal place):

$$P(\text{Signal} \mid \text{Positive}) = \boxed{0.2}\%$$

*Answer:*

- **Define the events:**
  - $S$: An alien signal is present.
  - $P$: The detector returns a positive result.
- **Define the probabilities:**
  - $P(S) = \frac{1}{10000} = 0.0001$, thus $P(S') = 1 - 0.0001 = 0.9999$.
  - $P(P|S) = 0.98$, hence $P(P'|S) = 1 - 0.98 = 0.02$.
  - $P(P'|S') = 0.96$, hence $P(P|S') = 1 - 0.96 = 0.04$.
- **Draw the probability tree:**




- **Calculate $P(P)$:**

$$\begin{aligned} P(P) &= P(S) \times P(P|S) + P(S') \times P(P|S') \\ &= (0.0001 \times 0.98) + (0.9999 \times 0.04) \\ &= 0.000098 + 0.039996 \\ &= 0.040094 \end{aligned}$$

- **Calculate $P(S \mid P)$:**Using Bayes' theorem:

$$\begin{aligned} P(S \mid P) &= \frac{P(S) \times P(P|S)}{P(P)} \\ &= \frac{0.0001 \times 0.98}{0.040094} \\ &\approx 0.002445 \\ &\approx 0.2445\% \quad \text{(convert to percent)} \\ &\approx 0.2\% \quad \text{(round to 1 decimal place)} \end{aligned}$$

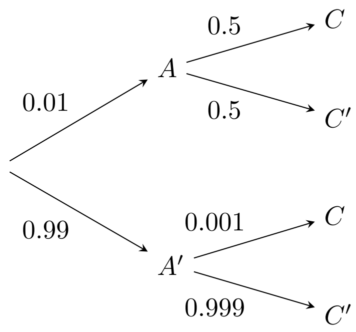
- The probability that an alien signal is actually present, given a positive detector result, is approximately 0.2%. This demonstrates how rare events, even with a highly accurate detector, can lead to many false positives.

**Ex 175:** In a city, 1 out of every 100 drivers drives with alcohol in their system. The probability of having an accident given that a driver has alcohol is $1/2$, and the probability of having an accident given that a driver has no alcohol is $1/1000$. Find the probability in percent that a driver has alcohol in their system if they have had an accident (round to 1 decimal place):

$$P(\text{Alcohol} \mid \text{Accident}) = \boxed{83.5}\%$$

*Answer:*

- **Define the events:**
  - $A$: The driver has alcohol in their system.
  - $C$: The driver has an accident.
- **Define the probabilities:**
  - $P(A) = \frac{1}{100} = 0.01$, thus $P(A') = 1 - 0.01 = 0.99$.
  - $P(C|A) = \frac{1}{2} = 0.5$, hence $P(C'|A) = 1 - 0.5 = 0.5$.
  - $P(C|A') = \frac{1}{1000} = 0.001$, hence $P(C'|A') = 1 - 0.001 = 0.999$.
- **Draw the probability tree:**




- **Calculate $P(C)$:**

$$\begin{aligned} P(C) &= P(A) \times P(C|A) + P(A') \times P(C|A') \\ &= (0.01 \times 0.5) + (0.99 \times 0.001) \\ &= 0.005 + 0.00099 \\ &= 0.00599 \end{aligned}$$

- **Calculate $P(A \mid C)$:**Using Bayes' theorem:

$$\begin{aligned} P(A \mid C) &= \frac{P(A) \times P(C|A)}{P(C)} \\ &= \frac{0.01 \times 0.5}{0.00599} \\ &\approx 0.8347 \\ &\approx 83.47\% \quad \text{(convert to percent)} \\ &\approx 83.5\% \quad \text{(round to 1 decimal place)} \end{aligned}$$

- This indicates that if an accident has occurred, there is a very high probability (83.5%) that the driver involved had been drinking.

**Ex 176:** In a futuristic society, 1 out of every 500 devices contains a rare quantum crystal as its power source. A crystal detector has been invented with the following properties:

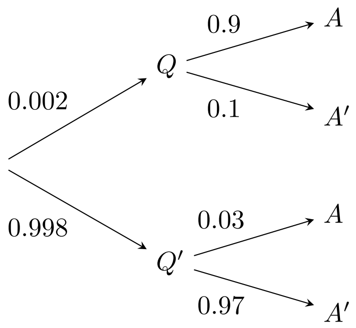
- Sensitivity: If a device has a quantum crystal, the detector correctly registers it as active 90% of the time.
- Specificity: If a device does not have a quantum crystal, the detector correctly registers it as inactive 97% of the time.

Find the probability in percent that a device actually has a quantum crystal if the detector registers it as active (round to 1 decimal place):

$$P(\text{Crystal} \mid \text{Active}) = \boxed{5.7}\%$$

*Answer:*

- **Define the events:**
  - $Q$: The device has a quantum crystal.
  - $A$: The detector registers the device as active.
- **Define the probabilities:**
  - $P(Q) = \frac{1}{500} = 0.002$, thus $P(Q') = 1 - 0.002 = 0.998$.
  - $P(A|Q) = 0.9$, hence $P(A'|Q) = 1 - 0.9 = 0.1$.
  - $P(A'|Q') = 0.97$, hence $P(A|Q') = 1 - 0.97 = 0.03$.
- **Draw the probability tree:**




- **Calculate $P(A)$:**

$$\begin{aligned} P(A) &= P(Q) \times P(A|Q) + P(Q') \times P(A|Q') \\ &= (0.002 \times 0.9) + (0.998 \times 0.03) \\ &= 0.0018 + 0.02994 \\ &= 0.03174 \end{aligned}$$

- **Calculate $P(Q \mid A)$:** Using Bayes' theorem:

$$\begin{aligned} P(Q \mid A) &= \frac{P(Q) \times P(A|Q)}{P(A)} \\ &= \frac{0.002 \times 0.9}{0.03174} \\ &\approx 0.056710 \\ &\approx 5.6710\% \quad \text{(convert to percent)} \\ &\approx 5.7\% \quad \text{(round to 1 decimal place)} \end{aligned}$$

- The probability that a device actually has a quantum crystal, given the detector registers it as active, is approximately 5.7%. This reflects how the rarity of quantum crystals leads to a low probability of a true positive despite a reliable detector.