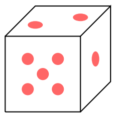
PROBABILITY

A ALGEBRA OF EVENTS

A.1 SAMPLE SPACES

A.1.1 FINDING THE SAMPLE SPACES

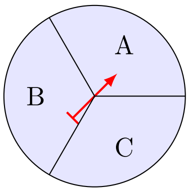
MCQ 1: A fair six-sided die is rolled once.



Find the sample space.

- ☐ $\{1,2,3,4,5\}$
- ☐ $\{1,2,3,4,5,6,7\}$
- ☐ $\{1,2,3,4,5,6\}$

MCQ 2: You spin the arrow on the spinner below.

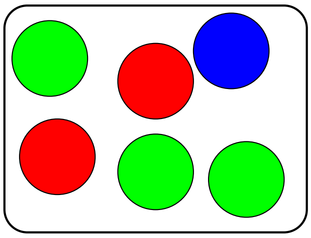


Find the sample space.

- ☐ $\{A,B,C\}$
- ☐ $\{A,B\}$
- ☐ $\{A,C\}$

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls. To calculate probabilities, we treat each ball as a unique outcome.

Bag



Find the sample space.

- ☐ {Red, Blue, Green}
- ☐ {2 Red, 1 Blue, 3 Green}
- ☐ $\{R_1,R_2,B_1,G_1,G_2,G_3\}$

MCQ 4: A letter is chosen randomly from the word BANANA. To calculate probabilities, we treat each letter's position as a unique outcome.
Find the sample space for the chosen letter.

- ☐ {B, N, A}
- ☐ {B, A, N, A, N, A}
- ☐ $\{B_1,A_2,N_3,A_4,N_5,A_6\}$

MCQ 5: A couple is expecting a baby. What is the sample space for this random experiment?

- ☐ {boy, girl}
- ☐ {boy}
- ☐ {girl}

A.2 EVENTS

A.2.1 FINDING EVENTS FOR DIE-ROLLING EVENTS

MCQ 6: If you roll a die, what is the set of outcomes for the event "getting a 3"?

- ☐ $\{1,3,5\}$
- ☐ $\{2,3,4\}$
- ☐ $\{1,2,3\}$
- ☐ $\{3\}$

MCQ 7: If you roll a die, what is the set of outcomes for the event "getting a 5 or 6"?

- ☐ $\{5,6\}$
- ☐ $\{4,5,6\}$
- ☐ $\{1,2,3\}$
- ☐ $\{3,4,5\}$

MCQ 8: If you roll a die, what is the set of outcomes for the event "getting a number greater than or equal to 4"?

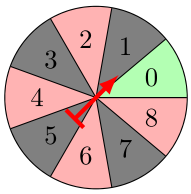
- ☐ $\{1,2,3\}$
- ☐ $\{4,5,6\}$
- ☐ $\{3,4,5\}$
- ☐ $\{2,3,4\}$

MCQ 9: If you roll a die, what is the set of outcomes for the event "even number"?

- ☐ $\{1,3,5\}$
- ☐ $\{2,4,6\}$
- ☐ $\{1,2,3,4,5,6\}$
- ☐ $\{2,3,4,5\}$

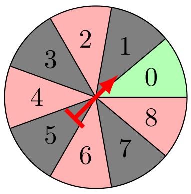
A.2.2 FINDING EVENTS IN A CASINO SPINNER

MCQ 10: If you spin the spinner below, what is the set of outcomes for the event "getting a 2"?



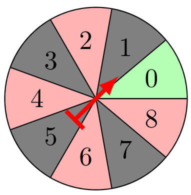
- ☐ $\{2\}$
- ☐ $\{1, 2, 3\}$
- ☐ $\{2, 4, 6\}$
- ☐ $\{0, 1, 2\}$

MCQ 11: If you spin the spinner below, what is the set of outcomes for the event "red"?



- ☐ $\{1, 3, 5, 7\}$
- ☐ $\{0\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{1, 2, 3, 4\}$

MCQ 12: If you spin the spinner below, what is the set of outcomes for the event "getting an odd number"?



- ☐ $\{0, 1, 3\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{1, 2, 3, 4\}$
- ☐ $\{1, 3, 5, 7\}$

A.3 COMPLEMENTARY EVENTS

A.3.1 FINDING THE COMPLEMENTARY EVENTS

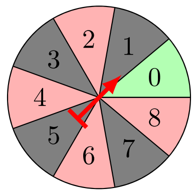
MCQ 13: If you roll a die, what is the set of outcomes for the event "not getting a 6"?

- ☐ $\{2, 3, 4\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$
- ☐ $\{1, 2, 3, 4, 5\}$
- ☐ $\{1, 3, 5\}$

MCQ 14: If you roll a die, what is the set of outcomes for the event "not getting an odd number"?

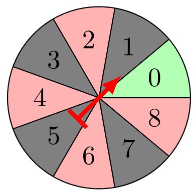
- ☐ $\{2, 4, 6\}$
- ☐ $\{1, 2, 3, 4, 5, 6\}$
- ☐ $\{1, 2, 3\}$
- ☐ $\{1, 3, 5\}$

MCQ 15: If you spin the spinner below, what is the set of outcomes for the event "not getting a 4"?



- ☐ $\{1, 2, 3, 4\}$
- ☐ $\{0, 1, 2, 3, 5, 6, 7, 8\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{4, 5, 6\}$

MCQ 16: If you spin the spinner below, what is the set of outcomes for the event "not getting red"?



- ☐ $\{0, 1, 3, 5, 7\}$
- ☐ $\{2, 4, 6, 8\}$
- ☐ $\{1, 2, 3, 4, 5, 6, 7, 8\}$
- ☐ $\{0\}$

A.4 MULTI-STEP RANDOM EXPERIMENTS

A.4.1 FINDING OUTCOME IN A TABLE

MCQ 17: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child / first child	B	G
B	BB	?
G	GB	GG

Find the missing outcome.

- ☐ BB
- ☐ BG
- ☐ GB

MCQ 18: The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 letter 1	M	A	T
M	MM	MA	MT
A	AM	AA	AT
T	TM	?	TT

Find the missing outcome.

- ☐ TT
- ☐ TA
- ☐ AT

MCQ 19: The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 letter 1	C	O	D	E
C	CC	CO	CD	CE
O	OC	OO	OD	OE
D	DC	?	DD	DE
E	EC	EO	ED	EE

Find the missing outcome.

- ☐ DO
- ☐ OD
- ☐ DC

MCQ 20: The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

letter 2 letter 1	N	O	D	E
N	**X**	?	ND	NE
O	ON	**X**	OD	OE
D	DN	DO	**X**	DE
E	EN	EO	ED	**X**

Find the missing outcome.

- ☐ NN
- ☐ NO
- ☐ ON

MCQ 21: The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

Player 2 Player 1	A	B	C	D
A	**X**	?	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Find the missing outcome.

- ☐ AB
- ☐ BA
- ☐ CA

A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

Ex 22: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child first child	B	G
B	BB	BG
G	GB	GG

Count the number of possible outcomes.

☐ possible outcomes.

Ex 23: There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

position 2 position 1	C	D
A	AC	AD
B	BC	BD

Count the number of possible outcomes.

☐ possible outcomes.

Ex 24: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of possible outcomes.

☐ possible outcomes.

Ex 25: There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

Tuesday Monday	X	Y	Z
X	**X**	XY	XZ
Y	YX	**X**	YZ
Z	ZX	ZY	**X**

Count the number of possible outcomes.

☐ possible outcomes.

A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

Ex 26: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of outcomes for the event that player A is selected.

☐ outcomes.

Ex 27: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "double" (both dice show the same number).

☐ outcomes.

Ex 28: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

☐ outcomes.

Ex 29: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 11."

☐ outcomes.

Ex 30: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 7."

☐ outcomes.

Ex 31: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

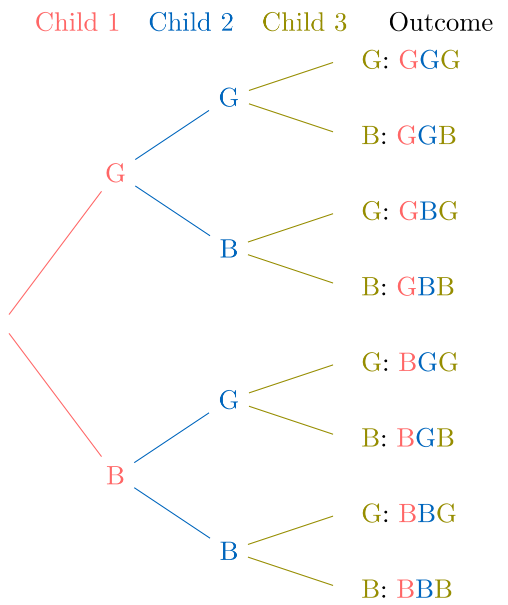
blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is less than or equal to 3."

☐ outcomes.

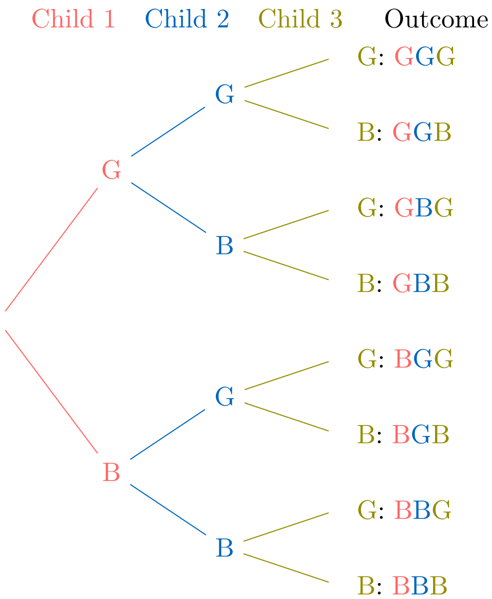
A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TREE DIAGRAM

Ex 32: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



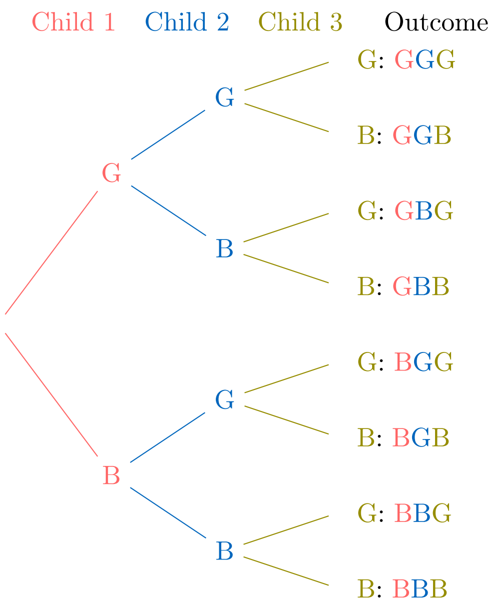
Count the number of possible outcomes for the event where the first child is a boy.

Ex 33: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



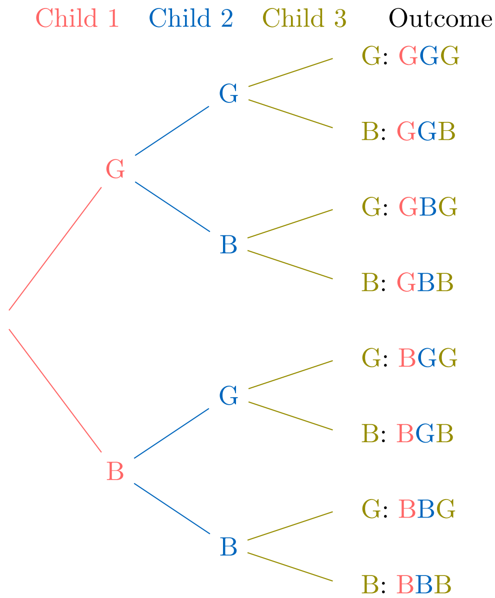
Count the number of possible outcomes for the event where there are exactly two girls.

Ex 34: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible sex outcomes for the three children.



Count the number of possible outcomes for the event where there are at least two girls.

Ex 35: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible sex outcomes for the three children.



Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

A.5 E OR F

A.5.1 FINDING THE UNION OF TWO EVENTS IN DICE EXPERIMENT

MCQ 36: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Find E or F.

- ☐ E or $F = \{2\}$
- ☐ E or $F = \{1, 2, 3, 4, 6\}$
- ☐ E or $F = \{1, 2, 3, 4, 5, 6\}$
- ☐ E or $F = \{1, 2, 3\}$

MCQ 37: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Find G or H.

- ☐ G or $H = \{2, 3, 4, 5, 6\}$
- ☐ G or $H = \{4, 5, 6\}$
- ☐ G or $H = \{2, 3, 5\}$
- ☐ G or $H = \{1, 2, 3, 4, 5, 6\}$

MCQ 38: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Find I or J.

- ☐ I or $J = \{3, 6\}$
- ☐ I or $J = \{1, 2, 3, 4\}$
- ☐ I or $J = \{1, 2, 3, 4, 5, 6\}$
- ☐ I or $J = \{1, 2, 3, 4, 6\}$

A.5.2 FINDING THE UNION OF TWO EVENTS IN FAMILY EXPERIMENT

MCQ 39: Consider a family with three children, assuming each child is equally likely to be a boy or a girl and the genders are independent. Let event A be the event of having only boys, and let event B be the event of having only girls. Find A or B.

- ☐ A or $B = \emptyset$
- ☐ A or $B = \{\text{BBB}, \text{GGG}\}$
- ☐ A or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ A or $B = \{\text{BBB}\}$

MCQ 40: Consider a family with three children. Let event A be the event that the first two children are girls, and let event B be the event of having only girls. Find A or B.

- ☐ A or $B = \emptyset$
- ☐ A or $B = \{\text{GGB}, \text{GGG}\}$
- ☐ A or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ A or $B = \{\text{GGG}\}$

MCQ 41: Consider a family with three children. Let event A be the event of having exactly two girls, and let event B be the event that the first two children are girls. Find A or B.

- ☐ A or $B = \emptyset$
- ☐ A or $B = \{\text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ A or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$
- ☐ A or $B = \{\text{BGG}, \text{GBG}, \text{GGB}\}$

A.5.3 FINDING THE UNION OF TWO EVENTS FROM A TABLE

MCQ 42: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Table: Students

Name	Age	Gender
A	15	Female
B	17	Male
C	16	Female
D	15	Male
E	14	Female
F	17	Female

Let event A be the event of selecting a girl, and event B be the event of selecting a student aged 17 or older.
Find A or B.

- ☐ A or B = {A, B, C, D, E, F}
- ☐ A or B = {C, F}
- ☐ A or B = {A, B, C, D, F}
- ☐ A or B = {A, B, C, E, F}

MCQ 43: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Find X or Y.

- ☐ X or Y = {G, I, J, L}
- ☐ X or Y = {G, H, I, J, L}
- ☐ X or Y = {G, L}
- ☐ X or Y = {I, J, L}

MCQ 44: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than or equal to 180 cm. Find U or V.

- ☐ U or V = {M, O, R}
- ☐ U or V = {O, R}
- ☐ U or V = {M, N, O, R}
- ☐ U or V = {M, O, P, R}

A.6 E AND F

A.6.1 FINDING THE INTERSECTION OF TWO EVENTS IN DICE EXPERIMENT

MCQ 45: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Find E and F.

- ☐ E and $F = \{1, 2, 3, 4, 6\}$
- ☐ E and $F = \{1, 2, 3\}$
- ☐ E and $F = \{2, 4, 6\}$
- ☐ E and $F = \{2\}$

MCQ 46: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Find G and H.

- ☐ G and $H = \{2, 3, 4, 5, 6\}$

☐ G and $H = \{2, 3, 5\}$

☐ G and $H = \{5\}$

☐ G and $H = \{4, 5, 6\}$

MCQ 47: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Find I and J.

☐ I and $J = \{1, 2, 3, 4, 6\}$

☐ I and $J = \{1, 2, 3, 4\}$

☐ I and $J = \{3\}$

☐ I and $J = \{3, 6\}$

A.6.2 FINDING THE INTERSECTION OF TWO EVENTS IN FAMILY EXPERIMENT

MCQ 48: Consider a family with three children. Let event A be the event of having only boys, and let event B be the event of having only girls. Find A and B.

☐ A and $B = \{\text{BBB}, \text{GGG}\}$

☐ A and $B = \{\text{BBB}\}$

☐ A and $B = \{\text{GGG}\}$

☐ A and $B = \emptyset$

MCQ 49: Consider a family with three children. Let event A be the event that the first two children are girls, and let event B be the event of having only girls. Find A and B.

☐ A and $B = \{\text{GGB}, \text{GGG}\}$

☐ A and $B = \emptyset$

☐ A and $B = \{\text{GGB}\}$

☐ A and $B = \{\text{GGG}\}$

MCQ 50: Consider a family with three children. Let event A be the event of having exactly two girls, and let event B be the event that the first two children are girls. Find A and B.

☐ A and $B = \{\text{GGB}\}$

☐ A and $B = \{\text{BGG}, \text{GBG}, \text{GGB}\}$

☐ A and $B = \{\text{GGG}\}$

☐ A and $B = \{\text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$

A.6.3 FINDING THE INTERSECTION OF TWO EVENTS FROM A TABLE

MCQ 51: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Table: Students

Name	Age	Gender
A	15	Female
B	17	Male
C	16	Female
D	15	Male
E	14	Female
F	17	Female

Let event A be the event of selecting a girl, and event B be the event of selecting a student older or equal than 17 years. Find A and B.

☐ A and $B = \{\text{A, B, C, E, F}\}$

☐ A and $B = \{\text{A, C, E, F}\}$

☐ A and $B = \{\text{B, F}\}$

☐ A and $B = \{\text{F}\}$

MCQ 52: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Find X and Y.

☐ X and $Y = \{\text{G, I, J, L}\}$

☐ X and $Y = \{\text{G, L}\}$

☐ X and $Y = \{\text{I, J, L}\}$

☐ X and $Y = \{\text{L}\}$

MCQ 53: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than or equal to 180 cm. Find U and V.

☐ U and $V = \{\text{M, O, R}\}$

☐ U and $V = \{\text{O, R}\}$

☐ U and $V = \emptyset$

☐ U and $V = \{\text{O}\}$

A.7 MUTUALLY EXCLUSIVE

A.7.1 DETERMINING MUTUAL EXCLUSIVITY

MCQ 54: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling an even number.
- Event F: Rolling an odd number.

Are the events E and F mutually exclusive?

☐ Yes

☐ No

MCQ 55: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling a prime number.
- Event F: Rolling an even number.

Are the events E and F mutually exclusive?

☐ Yes

☐ No

MCQ 56: Consider a standard deck of 52 playing cards (no jokers). Two events are defined as follows:

- Event E: Drawing a Queen.
- Event F: Drawing a Heart.

Are the events E and F mutually exclusive?

☐ Yes

☐ No

MCQ 57: Consider a family with exactly two children, where each child is a boy (B) or a girl (G). Two events are defined as follows:

- Event E: The family has only boys.
- Event F: The family has only girls.

Are the events E and F mutually exclusive?

☐ Yes

☐ No

A.7.2 DETERMINING MUTUAL EXCLUSIVITY FROM TABLES

MCQ 58: Consider a sports club where members are listed with their preferred sport and age group. A member is selected at random from the following table:

Table: Sports Club Members

Name	Preferred Sport	Age Group
S	Football	Under 16
T	Basketball	16-18
U	Tennis	Over 18
V	Swimming	Under 16
W	Football	16-18
X	Basketball	Over 18

Let event C be selecting a member whose preferred sport is Football, and event D be selecting a member from the Over 18 age group. Are events C and D mutually exclusive?

☐ Yes

☐ No

MCQ 59: Consider a bakery where items are listed with their type and topping. An item is selected at random from the following table:

Table: Bakery Items

Item	Type	Topping
Muffin	Pastry	Chocolate
Cookie	Cookie	Sprinkles
Cake	Cake	Chocolate
Donut	Pastry	Glaze
Brownie	Cake	None
Croissant	Pastry	None

Let event T be selecting a Pastry, and event U be selecting an item with Chocolate topping. Are events T and U mutually exclusive?

☐ Yes

☐ No

MCQ 60: Consider a library where books are listed with their genre and checkout status. A book is selected at random from the following table:

Table: Library Books

Title	Genre	Status
A	Mystery	Checked Out
B	Fantasy	Available
C	Mystery	Available
D	Romance	Checked Out
E	Fantasy	Checked Out
F	Romance	Available

Let event P be selecting a Mystery book, and event Q be selecting a Checked Out book. Are events P and Q mutually exclusive?

☐ Yes

☐ No

MCQ 61: Consider a zoo where animals are listed with their type and habitat. An animal is selected at random from the following table:

Table: Zoo Animals

Name	Type	Habitat
Lion	Mammal	Savanna
Penguin	Bird	Arctic
Crocodile	Reptile	Swamp
Elephant	Mammal	Savanna
Parrot	Bird	Jungle
Snake	Reptile	Jungle

Let event R be selecting a Mammal, and event S be selecting an animal from the Arctic habitat. Are events R and S mutually exclusive?

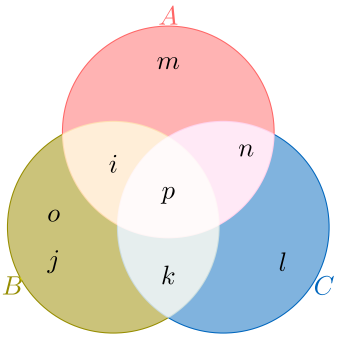
☐ Yes

☐ No

A.8 VENN DIAGRAM

A.8.1 FINDING THE UNION OF TWO EVENTS IN A VENN DIAGRAM

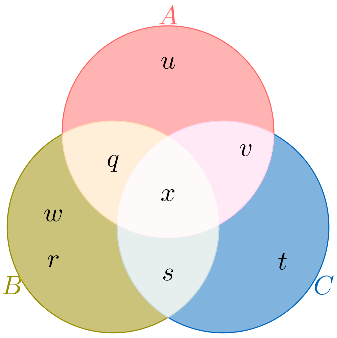
MCQ 62: You are given this populated Venn diagram representing events A, B, and C:



Find A or B.

- ☐ A or $B = \{i, j, k, l, m, n\}$
- ☐ A or $B = \{i, j, k, m, n, o, p\}$
- ☐ A or $B = \{i, j, k, l, m\}$
- ☐ A or $B = \{i, j, k, l, m, o, p\}$

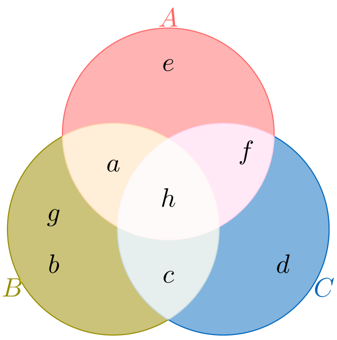
MCQ 63: You are given this populated Venn diagram representing events A, B, and C:



Find A or B.

- ☐ A or $B = \{q, r, s, t, u, v\}$
- ☐ A or $B = \{q, r, s, u, w, x\}$
- ☐ A or $B = \{q, r, s, t, u\}$
- ☐ A or $B = \{q, r, s, t, u, w, x\}$

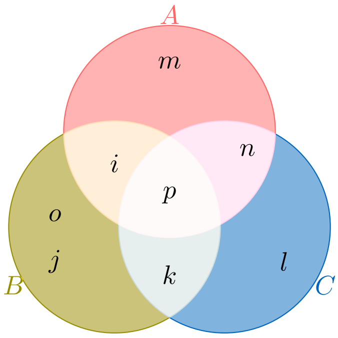
MCQ 64: You are given this populated Venn diagram representing events A, B, and C:



Find A or C.

- ☐ A or $C = \{a, c, d, e, f, h\}$
- ☐ A or $C = \{a, b, e, g, h\}$
- ☐ A or $C = \{a, b, c, d, e\}$
- ☐ A or $C = \{a, b, c, d, e, g, h\}$

MCQ 65: You are given this populated Venn diagram representing events A, B, and C:

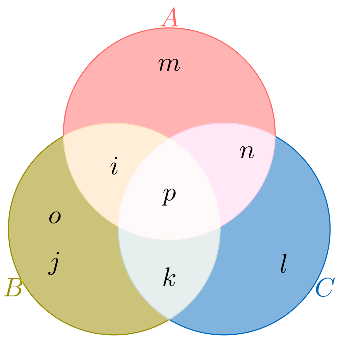


Find B or C.

- ☐ B or $C = \{i, j, k, l, m, n\}$
- ☐ B or $C = \{i, j, k, l, o\}$
- ☐ B or $C = \{i, j, k, l, n, o, p\}$
- ☐ B or $C = \{i, j, k, o, p\}$

A.8.2 FINDING THE INTERSECTION OF TWO EVENTS IN A VENN DIAGRAM

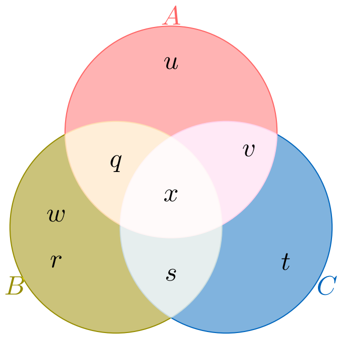
MCQ 66: You are given this populated Venn diagram representing events A, B, and C:



Find the intersection of A and B.

- ☐ A and $B = \{i, p\}$
- ☐ A and $B = \{i, j, k, m, o, p\}$
- ☐ A and $B = \{i, j, k, l, m\}$
- ☐ A and $B = \{j, k, o\}$

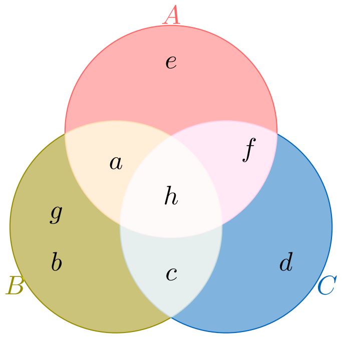
MCQ 67: You are given this populated Venn diagram representing events A, B, and C:



Find the intersection of A and B.

- ☐ A and $B = \{q, x\}$
- ☐ A and $B = \{q, r, s, u, w, x\}$
- ☐ A and $B = \{r, s, w\}$
- ☐ A and $B = \{q, r, s, t, u\}$

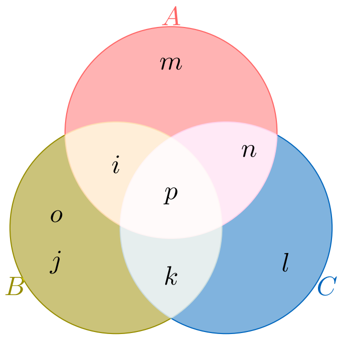
MCQ 68: You are given this populated Venn diagram representing events A, B, and C:



Find the intersection of A and C.

- ☐ A and $C = \{a, c, d, e, f, h\}$
- ☐ A and $C = \{a, e\}$
- ☐ A and $C = \{f, h\}$
- ☐ A and $C = \{c, d, f\}$

MCQ 69: You are given this populated Venn diagram representing events A, B, and C:



Find the intersection of B and C.

- ☐ B and $C = \{i, j, k, l, o, p\}$
- ☐ B and $C = \{k, p\}$
- ☐ B and $C = \{j, k, o\}$
- ☐ B and $C = \{l, n, p\}$

B AXIOMS AND RULES OF PROBABILITY

B.1 AXIOMS OF PROBABILITY

B.1.1 DESCRIBING PROBABILITIES WITH WORDS

MCQ 70: The probability of winning a game is $\frac{1}{10}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

MCQ 71: The probability of winning a game is $\frac{4}{5}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

MCQ 72: The probability of winning a game is $\frac{1}{2}$. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

MCQ 73: The probability of winning a game is 0. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

MCQ 74: The probability of winning a game is 1. Find the word to describe this probability.

- ☐ Impossible
- ☐ Less Likely
- ☐ Even Chance
- ☐ More Likely
- ☐ Certain

B.1.2 MAKING DECISIONS USING PROBABILITIES

MCQ 75: Louis advises you to play because the probability of winning this game is $\frac{3}{4}$. Do you follow his advice?

- ☐ Yes
- ☐ No

MCQ 76: Louis advises you to play because the probability of winning this game is $\frac{1}{4}$. Do you follow his advice?

- ☐ Yes
- ☐ No

MCQ 77: The probability of scoring a penalty is $\frac{1}{2}$ for Louis and $\frac{3}{4}$ for Hugo. Which player do you choose to take the penalty?

- ☐ Louis
- ☐ Hugo

MCQ 78: The probability of scoring a penalty is $\frac{1}{4}$ for Louis and $\frac{3}{5}$ for Hugo. Which player do you choose to take the penalty?

- ☐ Louis
- ☐ Hugo

B.1.3 FINDING PROBABILITY FOR MUTUALLY EXCLUSIVE EVENTS

Ex 79: Let $P(G) = 0.6$ and $P(H) = 0.2$. Assume that events G and H are mutually exclusive.
Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \boxed{}$$

Ex 80: Let $P(C) = 0.4$ and $P(D) = 0.5$. Assume that events C and D are mutually exclusive.
Calculate $P(C \text{ or } D)$.

$$P(C \text{ or } D) = \boxed{}$$

Ex 81: Let $P(A) = 0.5$ and $P(B) = 0.3$. Assume that events A and B are mutually exclusive.
Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \boxed{}$$

B.2 FUNDAMENTAL PROBABILITY RULES

B.2.1 APPLYING THE COMPLEMENT RULE

Ex 82: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$.
Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \boxed{}$$

Ex 83: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{}\%$$

Ex 84: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \boxed{}$$

Ex 85: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

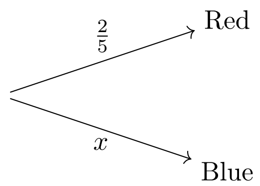
$$P(\text{"No rain"}) = \boxed{}\%$$

Ex 86: In a loto game, the probability of winning is $\frac{1}{100}$.
Find the probability of losing.

$$P(\text{"Losing"}) = \boxed{}$$

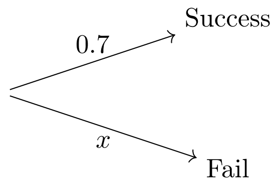
B.2.2 COMPLETING A PROBABILITY TREE DIAGRAM

Ex 87: From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability x of choosing a blue ball.



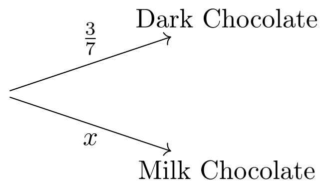
$$x = \frac{\boxed{}}{\boxed{}}$$

Ex 88: Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability x that he misses his first shot.



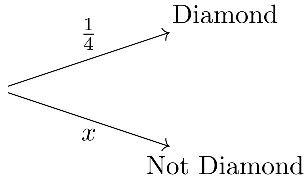
$$x = \boxed{}$$

Ex 89: In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability x of picking a milk chocolate.



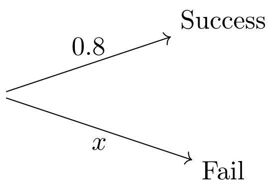
$$x = \frac{\boxed{}}{\boxed{}}$$

Ex 90: In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability x of drawing a card that is not a diamond.



$$x = \frac{\square}{\square}$$

Ex 91: Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability x that she fails to complete the level.



$$x = \square$$

B.2.3 CALCULATING PROBABILITIES FOR UNION OF EVENTS

Ex 92: Let $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \square$$

Ex 93: Let $P(E) = 0.8$, $P(F) = 0.3$ and $P(E \text{ and } F) = 0.2$. Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \square$$

Ex 94: Let $P(G) = 0.6$, $P(H) = 0.2$ and $P(G \text{ and } H) = 0.1$. Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \square$$

Ex 95: Let $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \text{ and } Y) = 0.2$. Calculate $P(X \text{ or } Y)$.

$$P(X \text{ or } Y) = \square$$

B.2.4 CALCULATING PROBABILITIES FOR UNION OF EVENTS IN REAL-WORLD PROBLEMS

Ex 96: In a school survey, the probability of a student liking math is 0.6, and the probability of liking science is 0.4. The probability of a student liking both math and science is 0.25.
What is the probability that a randomly selected student likes either math or science?

$$P(\text{Math or Science}) = \square$$

Ex 97: In a city survey, the probability of a resident using public transportation is 0.7, and the probability of using a bicycle is 0.3. The probability of a resident using both public transportation and a bicycle is 0.15.
What is the probability that a randomly selected resident uses either public transportation or a bicycle?

$$P(\text{Public Transport or Bicycle}) = \square$$

Ex 98: In a company survey, the probability of an employee enjoying team meetings is 0.5, and the probability of enjoying training sessions is 0.4. The probability of an employee enjoying both team meetings and training sessions is 0.2.
What is the probability that a randomly selected employee enjoys either team meetings or training sessions?

$$P(\text{Team Meetings or Training}) = \square$$

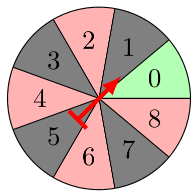
Ex 99: In a neighborhood survey, the probability of a household owning a dog is 0.5, and the probability of owning a cat is 0.35. The probability of a household owning both a dog and a cat is 0.2.
What is the probability that a randomly selected household owns either a dog or a cat?

$$P(\text{Dog or Cat}) = \square$$

B.3 EQUALLY LIKELY

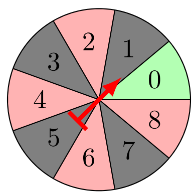
B.3.1 FINDING PROBABILITIES IN A CASINO SPINNER

Ex 100: You spin the casino spinner shown below. Calculate the probability of the event "getting a 2".



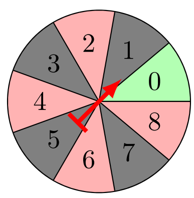
$$P(\text{"getting a 2"}) = \square$$

Ex 101: You spin the casino spinner shown below. Calculate the probability of the event "not getting a 4".



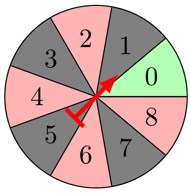
$$P(\text{"not getting a 4"}) = \square$$

Ex 102: You spin the casino spinner shown below. Calculate the probability of the event "red".



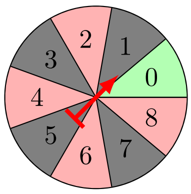
$$P(\text{"red"}) = \square$$

Ex 103: You spin the casino spinner shown below. Calculate the probability of the event "getting an odd number".



$P(\text{"getting an odd number"}) = \boxed{}$

Ex 104: You spin the casino spinner shown below. Calculate the probability of the event "not getting red".



$P(\text{"not getting red"}) = \boxed{}$

B.3.2 FINDING PROBABILITIES IN A DICE EXPERIMENT

Ex 105: If you roll a die, what is the probability of the event "getting a 3"?

$P(\text{"getting a 3"}) = \boxed{}$

Ex 106: If you roll a die, what is the probability of the event "getting a 5 or 6"?

$P(\text{"getting a 5 or 6"}) = \boxed{}$

Ex 107: If you roll a die, what is the probability of the event "getting a number greater than or equal to 4"?

$P(\text{number} \geq 4) = \boxed{}$

Ex 108: If you roll a die, what is the probability of the event "even number"?

$P(\text{"even number"}) = \boxed{}$

Ex 109: If you roll a die, what is the probability of the event "not getting a 6"?

$P(\text{"not getting a 6"}) = \boxed{}$

Ex 110: If you roll a die, what is the probability of the event "not getting an odd number"?

$P(\text{"not getting an odd number"}) = \boxed{}$

B.3.3 CALCULATING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

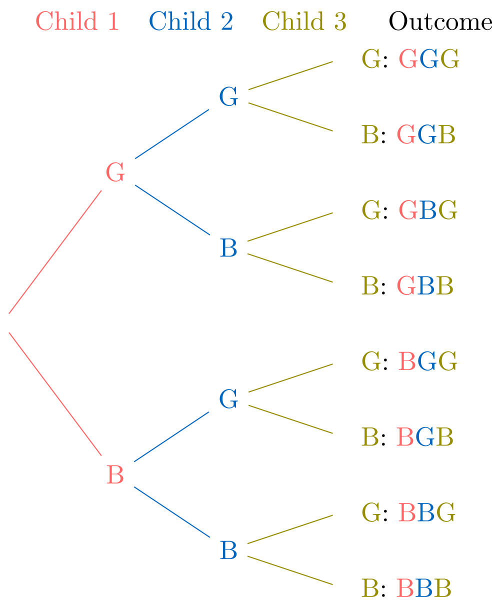
Ex 111: A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

Player 1 \ Player 2	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Calculate the probability that player C is selected as either Player 1 or Player 2.

$P(\text{"selecting player C"}) = \boxed{}$

Ex 112: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.Calculate the probability that the family has at least two girls.



$P(\text{"at least two girls"}) = \boxed{}$

Ex 113: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

red die \ blue die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 7.

$P(\text{"sum is 7"}) = \boxed{}$

Ex 114: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is greater than or equal to 11.

$P(\text{"sum } \geq 11\text{"}) = \boxed{}$

Ex 115: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 6 or 8.

$P(\text{"sum is 6 or 8"}) = \boxed{}$

B.4 PROBABILITY OF INDEPENDENT EVENTS

B.4.1 DRAW A PROBABILITY TREE FOR TWO INDEPENDENT EVENTS

Ex 116: Let E be the event "drawing a red marble" from a bag of 10 marbles with 6 red marbles. Let F be the event "spinning a 1" on a spinner numbered 1 to 10.
Suppose the two events are independent.
Draw the probability tree diagram.

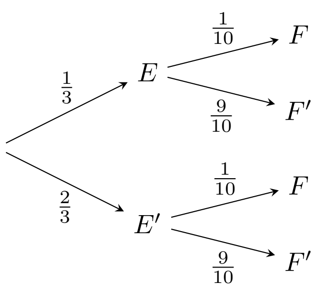
Ex 117: Let A be the event "drawing an ace" from a standard deck of 52 cards. Let B be the event "rolling a 6" on a regular six-sided die.
Suppose the two events are independent.
Draw the probability tree diagram.

Ex 118: Let E be the event "player A succeeds his basketball shot" ($P(E) = 60\%$).
Let F be the event "player B succeeds his basketball shot" ($P(F) = 70\%$).
Assume the two shots are independent.
Draw the probability tree diagram.

Ex 119: Let E be the event "Julia passes her computer test" ($P(E) = 80\%$).
Let F be the event "Julia passes her English test" ($P(F) = 90\%$).
Assume the two tests are independent.
Draw the probability tree diagram.

B.4.2 CALCULATING PROBABILITIES FROM A TREE DIAGRAM

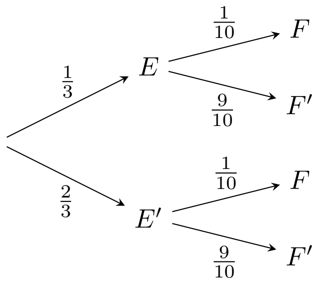
Ex 120: Consider the following probability tree diagram. The two events E and F are independent.



Calculate the probability that both E and F occur

$P(E \text{ and } F) = \boxed{}$

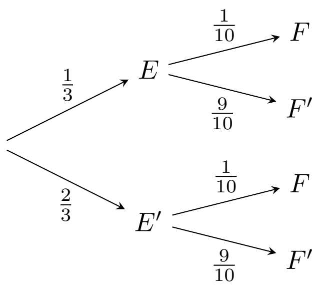
Ex 121: Consider the following probability tree diagram. The two events E and F are independent.



Calculate the probability that neither E nor F occur (i.e., both the complement events E' and F' happen):

$$P(E' \text{ and } F') = \boxed{}$$

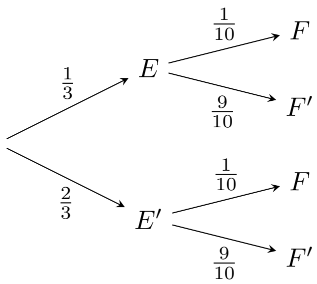
Ex 122: Consider the following probability tree diagram. The two events E and F are independent.



Calculate the probability that E occurs and F does not occur:

$$P(E \text{ and } F') = \boxed{}$$

Ex 123: Consider the following probability tree diagram. The two events E and F are independent.

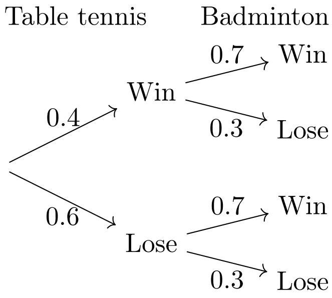


Calculate the probability that E' occurs and F occurs:

$$P(E' \text{ and } F) = \boxed{}$$

B.4.3 CALCULATING PROBABILITIES FROM A TREE DIAGRAM

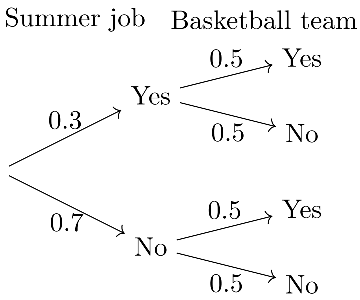
Ex 124: Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability of winning table tennis is 0.4 and the probability of winning badminton is 0.7. The two events are independent. The probability tree is shown below:



Calculate the probability that Niamh wins both games.

$$P(\text{"Win both"}) = \boxed{}$$

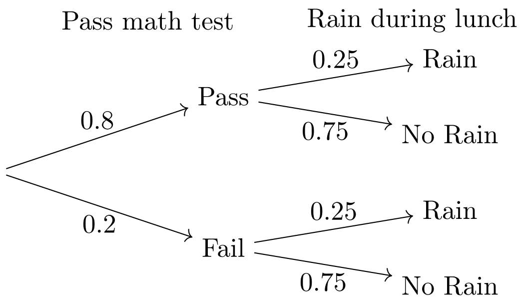
Ex 125: Noah is applying for a summer job and also trying out for a basketball team. The probability that Noah gets the job is 0.3, and the probability that Noah is selected for the basketball team is 0.5. The two events are independent. The probability tree is shown below:



Calculate the probability that Noah does **not** get the job **and** is **not** selected for the basketball team.

$$P(\text{"No job and not selected for team"}) = \boxed{}$$

Ex 126: Maria has a math test and there is a chance of rain during lunch. The probability that Maria passes her math test is 0.8, and the probability that it rains during lunch is 0.25. The two events are independent. The probability tree is shown below:



Calculate the probability that Maria passes her math test **and** it rains during lunch.

$$P(\text{"Pass and Rain"}) = \boxed{}$$

B.4.4 CALCULATING THE PROBABILITY OF TWO INDEPENDENT EVENTS

Ex 127: Let A be the event "drawing an ace" from a standard deck of 52 cards. Let B be the event "rolling a 6" on a regular six-sided die.
Suppose the two events are independent.

1. Draw the probability tree diagram.
2. Calculate the probability of "drawing an ace" and "rolling a 6".

Ex 128: Let E be the event "player A succeeds in their basketball shot" ($P(E) = 40\%$). Let F be the event "player B succeeds in their basketball shot" ($P(F) = 40\%$).
Suppose the two events are independent.

1. Draw the probability tree diagram.
2. Calculate the probability that both players fail their shot.

Ex 129: Let E be the event "drawing a red marble" from a bag of 10 marbles with 6 red marbles. Let F be the event "spinning a 1" on a spinner numbered 1 to 10.
Suppose the two events are independent.

1. Draw the probability tree diagram.
2. Calculate the probability of "drawing a red marble" and "spinning a 1".

Ex 130: Let E be the event "the machine produces a defective item" ($P(E) = 15\%$). Let F be the event "the item is selected for quality control" ($P(F) = 20\%$).
Suppose the two events are independent.

1. Draw the probability tree diagram.
2. Calculate the probability that the machine produces a non-defective item and the item is not selected for quality control.

B.5 EXPERIMENTAL PROBABILITY

B.5.1 CALCULATING EXPERIMENTAL PROBABILITIES IN PERCENTAGE FORM

Ex 131: During a classroom experiment, Ethan flips a coin 50 times and records that it lands on heads 30 times. Calculate the experimental probability that the coin lands on heads, and express the result in percentage form.

$$P(\text{"landing on heads"}) \approx \boxed{}\ \%$$

Ex 132: During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt, and express the result in percentage form.

$$P(\text{"making the next attempt"}) \approx \boxed{\quad\quad}\%$$

Ex 133: During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the experimental probability that the next student will choose a vegetarian meal, and express the result in percentage form.

$$P(\text{choosing a vegetarian meal}) \approx \boxed{\quad\quad}\%$$

Ex 134: Over the course of a year, it rained on 146 days out of 365 recorded days. Estimate the experimental probability that it will rain, and express the result in percentage form.

$$P(\text{"raining"}) \approx \boxed{\quad\quad}\%$$

B.5.2 CONDUCTING EXPERIMENTS TO ESTIMATE PROBABILITIES

Ex 135: In an experiment, you are asked to toss a fair coin at least 30 times. Follow these steps:

1. Note the number of times the coin lands on heads.
2. Note the total number of trials (tosses).
3. Calculate the experimental probability that the coin lands on heads, and express the result in decimal form.

Ex 136: In a classroom experiment, you are asked to ask at least 10 friends to randomly choose a single number from 1 to 5. Follow these steps:

1. Note the number of times the answer is 5.
2. Note the total number of trials (friends asked).
3. Calculate the experimental probability that a friend chooses the number 5, and express the result in decimal form.

C CONDITIONAL PROBABILITY

C.1 DEFINITION

C.1.1 EXPLORING PROBABILITIES WITH TWO-WAY TABLES

Ex 137: Consider a two-way table showing students' preferences about math (love / don't love), categorized by gender:

	Loves Math	Don't Love Math	Total
Girls	35	16	51
Boys	30	19	49
Total	65	35	100

A student is randomly selected from the class. Find the probability that the selected student is a girl.

$$P(\text{"Girl"}) = \boxed{\quad}$$

Ex 138: Consider a two-way table showing students' preferences for participating in a school drama club, categorized by gender:

	Likes Drama Club	Dislikes Drama Club	Total
Girls	28	12	40
Boys	32	18	50
Total	60	30	90

A student is randomly selected from the group. Find the probability that the selected student likes the drama club.

$$P(\text{"Likes Drama Club"}) = \boxed{\quad}$$

Ex 139: Consider a two-way table showing students' preferences for a short school trip, categorized by grade level:

	Likes Trip	Dislikes Trip	Total
Grade 9	15	5	20
Grade 10	25	15	40
Total	40	20	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9 and likes the trip.

$$P(\text{"Grade 9 and Likes Trip"}) = \boxed{\quad}$$

Ex 140: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student is a boy and likes science.

$$P(\text{"Boy and Likes Science"}) = \boxed{\quad}$$

Ex 141: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student likes science, given that the student is a boy.

$$P(\text{"Likes Science"} \mid \text{"Boy"}) = \boxed{}$$

Ex 142: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student likes music, given that the student is in Grade 9.

$$P(\text{"Likes Music"} \mid \text{"Grade 9"}) = \boxed{}$$

Ex 143: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes art.

$$P(\text{"Grade 9"} \mid \text{"Likes Art"}) = \boxed{}$$

Ex 144: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student likes art, given that the student is in Grade 9.

$$P(\text{"Likes Art"} \mid \text{"Grade 9"}) = \boxed{}$$

Ex 145: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes music.

$$P(\text{"Grade 9"} \mid \text{"Likes Music"}) = \boxed{}$$

C.1.2 CALCULATING CONDITIONAL PROBABILITIES

Ex 146: Given that $P(E \text{ and } F) = 0.1$ and $P(F) = 0.4$, find :

$$P(E \mid F) = \boxed{}$$

Ex 147: Given that $P(A \text{ and } B) = 0.15$ and $P(B) = 0.5$, find :

$$P(A \mid B) = \boxed{}$$

Ex 148: Given that $P(X \text{ and } Y) = 0.12$ and $P(Y) = 0.3$, find :

$$P(X \mid Y) = \boxed{}$$

Ex 149: Given that $P(M \text{ and } N) = 0.25$ and $P(N) = 0.8$, find :

$$P(M \mid N) = \boxed{}$$

C.1.3 CALCULATING CONDITIONAL PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 150: In a certain town, it is found that 30% of the families own a pet dog, and 15% of the families own both a pet dog and a cat. If a randomly selected family owns a dog, what is the probability that they also own a cat?

$$P(\text{"Own a cat"} \mid \text{"Own a dog"}) = \boxed{}$$

Ex 151: In a school, it is found that 40% of the students enjoy reading books, and 20% of the students enjoy both reading books and watching movies. If a randomly selected student enjoys reading books, what is the probability that they also enjoy watching movies?

$$P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) = \boxed{}$$

Ex 152: In a neighborhood, it is found that 60% of the households own a car, and 24% of the households own both a car and a bicycle. If a randomly selected household owns a car, what is the probability that they also own a bicycle?

$$P(\text{"Own a bicycle"} \mid \text{"Own a car"}) = \boxed{}$$

Ex 153: In a club, it is found that 25% of the members play soccer, and 10% of the members play both soccer and basketball. If a randomly selected member plays soccer, what is the probability that they also play basketball?

$$P(\text{"Play basketball"} \mid \text{"Play soccer"}) = \boxed{}$$

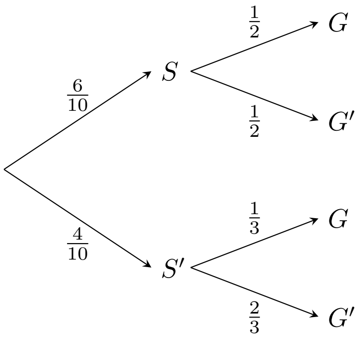
C.2 CONDITIONAL PROBABILITY TREE DIAGRAMS

C.2.1 IDENTIFYING CONDITIONAL PROBABILITY TREE DIAGRAMS

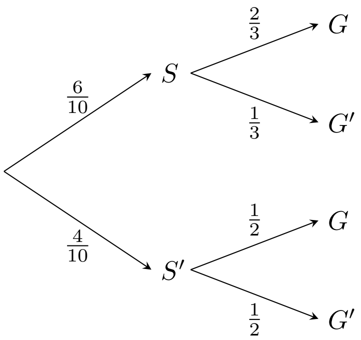
MCQ 154: The probability that Sam is coaching a game is $\frac{6}{10}$, and the probability that Alex is coaching is $\frac{4}{10}$.

- With Coach Sam, the probability of being Goalkeeper is $\frac{1}{2}$.
- With Coach Alex, the probability of being Goalkeeper is $\frac{2}{3}$.

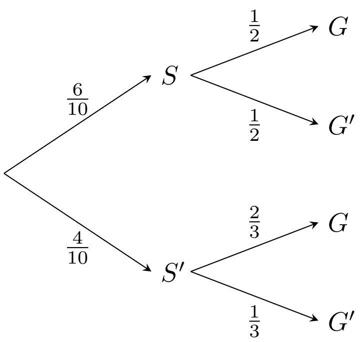
Let S be the event that Sam is the coach.
Let G be the event of being the goalkeeper.
Choose the correct probability tree diagram:



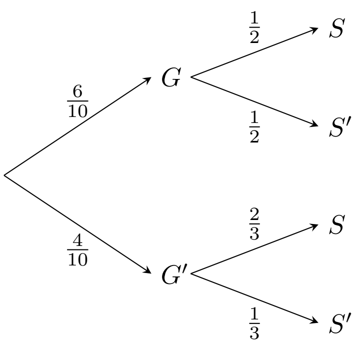
☐



☐



☐

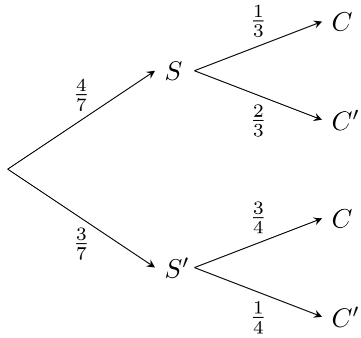


☐

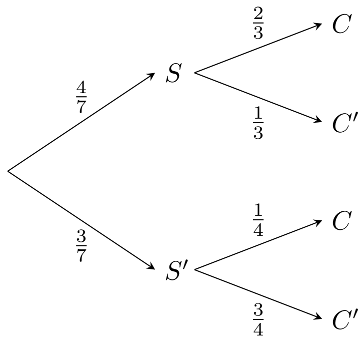
MCQ 155: In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$, and the probability of it being rainy is $\frac{3}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.
- On a rainy day, the probability that the park is crowded is $\frac{1}{4}$.

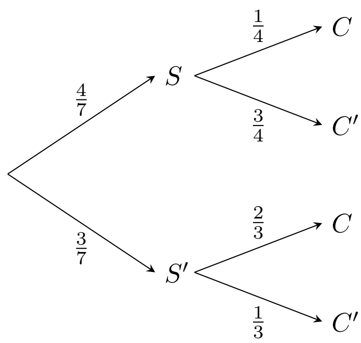
Let S be the event that it's a sunny day.
Let C be the event of the park being crowded.
Choose the correct probability tree diagram:



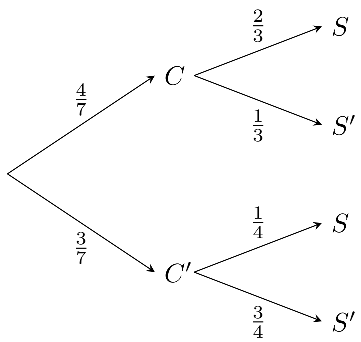
☐



☐



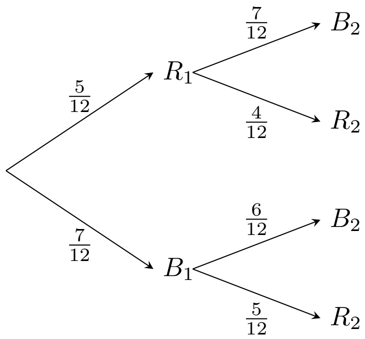
☐

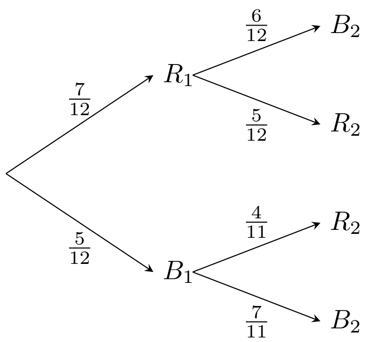


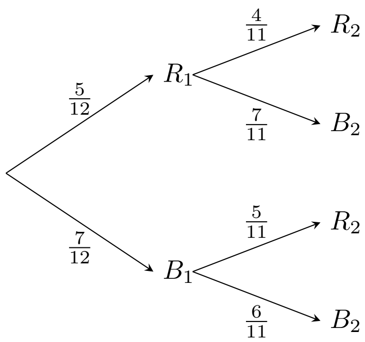
☐

MCQ 156: A bag contains 5 red (R) and 7 blue (B) marbles. One marble is drawn and not replaced, and then a second marble is drawn.
Let R_1 be the event of drawing a red marble first, and let B_1 be

the event of drawing a blue marble first.
Let R_2 be the event of drawing a red marble second, and let B_2 be the event of drawing a blue marble second.
Choose the correct probability tree diagram:

☐ 

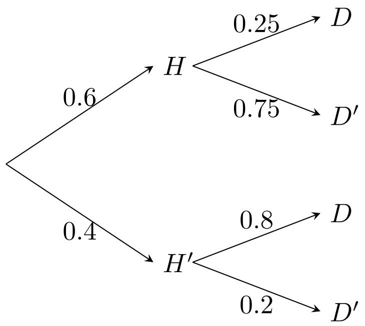
☐ 

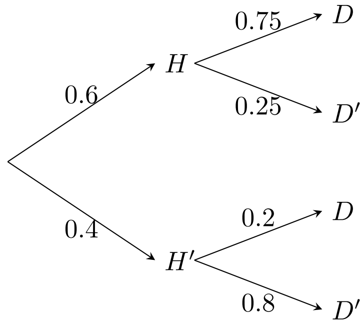
☐ 

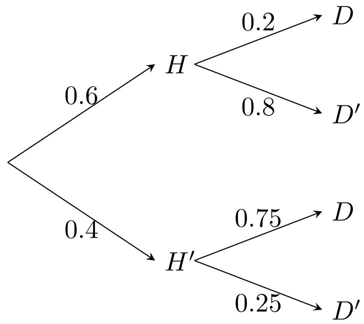
MCQ 157: In a town, the probability that a randomly chosen morning has heavy traffic is 0.6, and the probability that it has light traffic is 0.4.

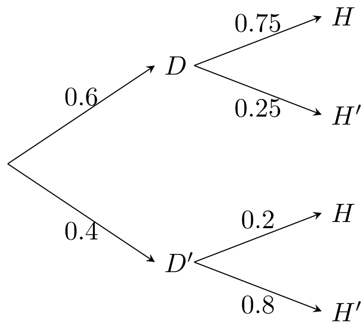
- On a morning with heavy traffic, the probability of a bus being delayed is 0.75.
- On a morning with light traffic, the probability of a bus being delayed is 0.2.

Let H be the event that the morning has heavy traffic.
Let D be the event that the bus is delayed.
Choose the correct probability tree diagram:

☐ 

☐ 

☐ 

☐ 

C.2.2 DRAWING CONDITIONAL PROBABILITY TREE DIAGRAMS

Ex 158: The probability that a team wins the coin toss in a cricket match is $\frac{1}{2}$.

- If the team wins the toss, the probability that it wins the match is $\frac{3}{5}$.
- If the team loses the toss, the probability that it wins the match is $\frac{2}{5}$.

Let T be the event that the team wins the toss.
Let W be the event that the team wins the match.
Draw the probability tree diagram

Ex 159: In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.
- On a non-sunny day, the probability that the park is crowded is $\frac{1}{4}$.

Let S be the event that it's a sunny day.
Let C be the event that the park is crowded.
Draw the probability tree diagram

Ex 160: The probability that a person has a rare disease is $\frac{1}{100}$, and the probability that the person does not have the disease is $\frac{99}{100}$.

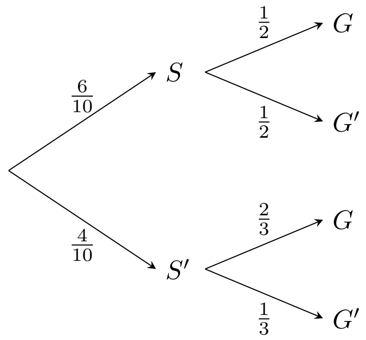
- If the person has the disease, the probability that the test detects it (positive result) is $\frac{99}{100}$.
- If the person does not have the disease, the probability that the test falsely detects it (positive result) is $\frac{1}{100}$.

Let D be the event that the person has the disease.
Let T^+ be the event that the test is positive.
Draw the probability tree diagram

C.3 JOINT PROBABILITY: $P(E \cap F)$

C.3.1 CALCULATING JOINT PROBABILITIES WITH TREES

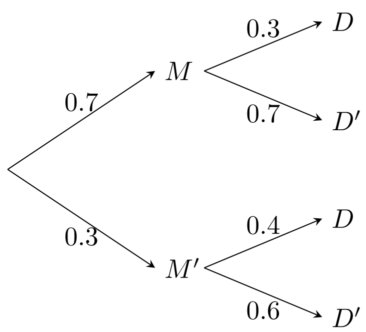
Ex 161: For this probability tree diagram:



find the probability :

$$P(S \text{ and } G) = \boxed{}$$

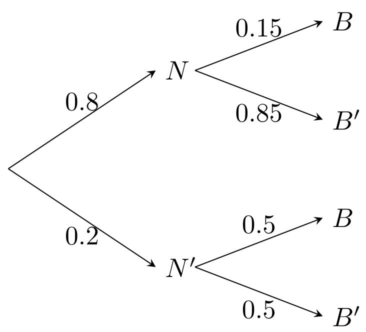
Ex 162: For this probability tree diagram:



find the probability :

$$P(M' \text{ and } D') = \boxed{}$$

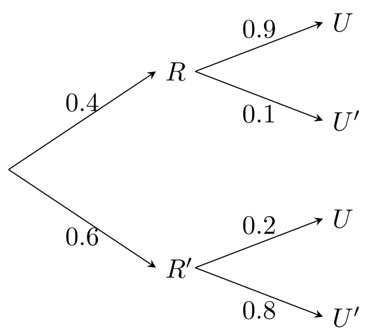
Ex 163: For this probability tree diagram:



find the probability :

$$P(N' \text{ and } B) = \boxed{}$$

Ex 164: For this probability tree diagram:



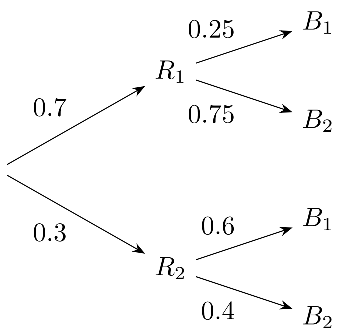
find the probability :

$$P(R \text{ and } U) = \boxed{}$$

C.4 LAW OF TOTAL PROBABILITY

C.4.1 CALCULATING PROBABILITIES WITH TREES

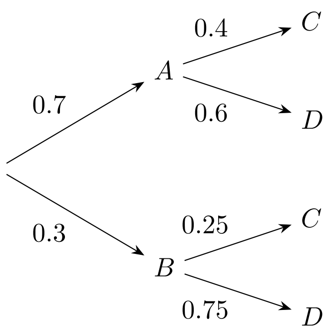
Ex 165: For this probability tree:



calculate the probability :

$$P(B_1) = \boxed{}$$

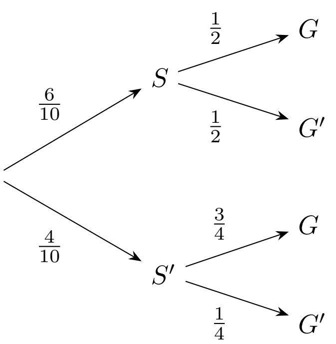
Ex 166: For this probability tree:



calculate the probability :

$$P(D) = \boxed{}$$

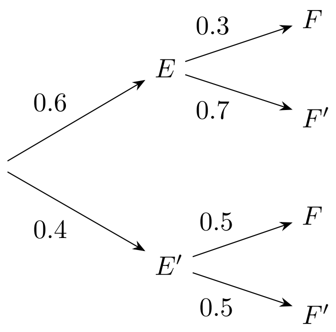
Ex 167: For this probability tree,



calculate the probability :

$$P(G) = \boxed{}$$

Ex 168: For this probability tree:



calculate the probability :

$$P(F') = \boxed{}$$

C.4.2 CALCULATING PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 169: A company produces two types of parts: A and B. 20% of parts are type A and 80% are type B. The probability that a part is defective given type A is 0.02, and the probability that a part is defective given type B is 0.01.
Find the probability that a part is defective :

$$P(\text{"Defective"}) = \boxed{}$$

Ex 170: A meteorologist observes cloud conditions to predict rain. On a given day, 40% of the time the sky is cloudy, and 60% of the time it is clear. The probability of rain given a cloudy sky is 0.75, and the probability of rain given a clear sky is 0.15.
Find the probability that it rains :

$$P(\text{Rain}) = \boxed{}$$

Ex 171: An urn contains 1 red ball and 4 blue balls. A first ball is drawn without replacement. Then a second ball is drawn from the remaining balls.
Find the probability that the second ball drawn is red :

$$P(R_2) = \boxed{}$$

Ex 172: A population is tested for a disease. 30% of the population has the disease. The probability that a test is positive given the person has the disease is 0.95, and the probability that a test is positive given the person does not have the disease is 0.10.
Find the probability that a test is positive :

$$P(\text{Positive}) = \boxed{}$$

C.5 BAYES' THEOREM

C.5.1 UNVEILING THE HIDDEN CAUSE: BAYES' THEOREM IN RARE EVENT DETECTION

Ex 173: Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

Find the probability in percent that a person actually has the disease if their test result is positive (round to 1 decimal place):

$$P(\text{Disease} \mid \text{Test positive}) = \boxed{}\%$$

Ex 174: Consider a rare alien signal that is present in approximately 1 out of every 10,000 radio scans conducted by a space observatory. A signal detector has the following characteristics:

- Sensitivity: If an alien signal is present, the detector correctly identifies it as positive 98 % of the time.
- Specificity: If no alien signal is present, the detector correctly identifies it as negative 96 % of the time.

Find the probability in percent that an alien signal is actually present if the detector returns a positive result (round to 1 decimal place):

$$P(\text{Signal} \mid \text{Positive}) = \boxed{}\%$$

Ex 175: In a city, 1 out of every 100 drivers drives with alcohol in their system. The probability of having an accident given that a driver has alcohol is $1/2$, and the probability of having an accident given that a driver has no alcohol is $1/1000$. Find the probability in percent that a driver has alcohol in their system if they have had an accident (round to 1 decimal place):

$$P(\text{Alcohol} \mid \text{Accident}) = \boxed{}\%$$

Ex 176: In a futuristic society, 1 out of every 500 devices contains a rare quantum crystal as its power source. A crystal detector has been invented with the following properties:

- Sensitivity: If a device has a quantum crystal, the detector correctly registers it as active 90% of the time.
- Specificity: If a device does not have a quantum crystal, the detector correctly registers it as inactive 97% of the time.

Find the probability in percent that a device actually has a quantum crystal if the detector registers it as active (round to 1 decimal place):

$$P(\text{Crystal} \mid \text{Active}) = \boxed{}\%$$