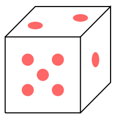
A ALGEBRA OF EVENTS

A.1 SAMPLE SPACE

A.1.1 FINDING THE SAMPLE SPACES

MCQ 1: A fair six-sided die is rolled once.



Find the sample space.

☐ $\{1, 2, 3, 4, 5\}$

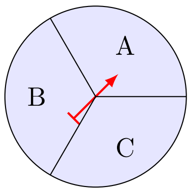
☐ $\{1, 2, 3, 4, 5, 6, 7\}$

☒ $\{1, 2, 3, 4, 5, 6\}$

Answer:

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

MCQ 2: Find the sample space that the spinner can land on:



☒ $\{A, B, C\}$

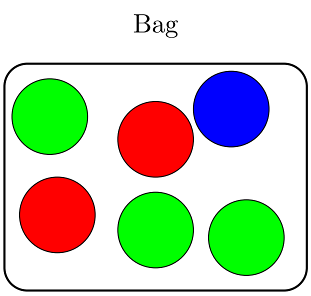
☐ $\{A, B\}$

☐ $\{A, C\}$

Answer:

- The sample space is all possible outcomes.
- The spinner has three distinct regions labeled A, B, and C.
- So, the sample space is: $\{A, B, C\}$.

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.



Find the sample space.

☒ {Red, Blue, Green}

☐ {2 Red, 1 Blue, 3 Green}

☐ {Red, Red, Blue, Green, Green, Green}

Answer:

- When choosing a ball randomly from the bag containing 2 red balls, 1 blue ball, and 3 green balls, the balls are identical in color, so we do not distinguish between them based on quantity.
- So, the sample space (all possible outcomes) is: {Red, Blue, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

☒ {B, N, A}

☐ {B, A, N, A, N, A}

☐ {A, B, N, A, B, N}

Answer:

- When choosing a letter randomly from the word "BANANA", the possible outcomes are the distinct letters in the word.
- So, the sample space (all possible outcomes) is: {B, A, N} or {B, N, A}. The order in which the letters are listed does not matter.

A.2 EVENTS

A.2.1 FINDING THE EVENTS

MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

☐ {O, R, A, N, G, E}

☒ {O, A, E}

☐ {R, G, N}

☐ {A, G, E}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When choosing a letter randomly from the word "ORANGE", the event where the chosen letter is a vowel consists of the vowels in the word.
- So, the event is: {O, A, E}.

MCQ 6: A fair six-sided dice is rolled once.
Find the event where the outcome is an even number.

☐ {1, 3, 5}

☒ {2, 4, 6 }

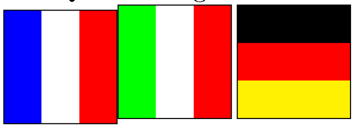
☐ {1, 2, 3, 4, 5, 6}

☐ {2, 3, 4, 5}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When rolling a fair six-sided dice, the event where the outcome is an even number consists of the even numbers on the dice.
- So, the event is: {2, 4, 6}.

MCQ 7: A flag is chosen randomly from:



France Italy Germany

Find the event where the outcome is a flag with blue in them.

☒ {France }

☐ {Italy, France}

☐ {Italy, France, Germany}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, only the French flag has blue in it.
- So, the correct answer is: {France}.

MCQ 8: A flag is chosen randomly from:

France Italy Germany Japan

Find the event where the outcome is a flag with red in them.

☐ {France, Japan}

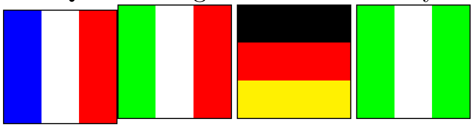
☐ {Italy, France}

☒ {Italy, France, Germany, Japan}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, France, Italy, Germany, and Japan have red in them.
- So, the correct answer is: {Italy, France, Germany, Japan}.

MCQ 9: A flag is chosen randomly from:



France Italy Germany Nigeria

Find the event where the outcome is a flag with green in them.

☐ {France, Nigeria}

☒ {Italy, Nigeria}

☐ {Italy, France, Germany}

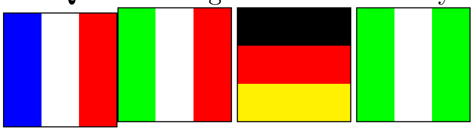
Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, Italy and Nigeria have green in them.
- So, the correct answer is: {Italy, Nigeria}.

A.3 COMPLEMENTARY EVENT

A.3.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:



France Italy Germany Nigeria

Let E be the event where the selected flag contains green. Find the complement of event E, denoted as E'.

☒ E' = {France, Germany}

☐ E' = {Italy, Nigeria}

☐ E' = {Italy, France, Germany}

Answer:

- The event E includes flags with the color green: {Italy, Nigeria}.
- The complement event E' consists of flags that do not have green.
- So, the complement event E' = {France, Germany}.

MCQ 11: A flag is chosen at random from the following set:

France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red. Find the complement of event E, denoted E'.

☐ E' = {France, Germany}

☒ E' = {Nigeria}

☐ E' = {Italy, France, Germany}

Answer:

- The event E consists of flags that contain the color red: {France, Italy, Germany}.
- The complement of event E, denoted E', consists of flags that do not contain the color red.
- Since Nigeria is the only flag without the color red, $E' =$ {Nigeria}.
- So, the correct answer is $E' =$ {Nigeria}.

MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name.
Find the complement of event E, denoted as E'.

☒ $E' =$ {Emily, Ava, Sophia}

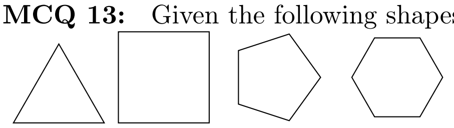
☐ $E' =$ {James}

☐ $E' =$ {James, Ava}

Answer:

- The event E includes boy's names: {James}.
- The complement event E' consists of names that are not boy's names (i.e., girl's names).
- So, the complement event $E' =$ {Emily, Ava, Sophia}.

MCQ 13: Given the following shapes:



Triangle Square Pentagon Hexagon

Let E be the event where a polygon with an even number of sides is chosen.
Find the complement of event E, denoted as E'.

☐ $E' =$ {Square, Hexagon}

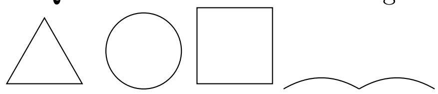
☒ $E' =$ {Triangle, Pentagon}

☐ $E' =$ {Triangle, Square, Pentagon, Hexagon}

Answer:

- Triangle: three sides. Square: four sides. Pentagon: five sides. Hexagon: six sides.
- The event E includes polygons with an even number of sides: {Square, Hexagon}.
- The complementary event E' consists of polygons that do not have an even number of sides (have an odd number of sides).
- Therefore, the complementary event $E' =$ {Triangle, Pentagon}.

MCQ 14: Consider the following shapes:



Triangle Circle Square Curve

Let E be the event where the shape is a polygon.
Find the complement of event E, denoted as E'.

☐ $E' =$ {Triangle, Square}

☐ $E' =$ {Triangle, Circle, Square, Curve}

☒ $E' =$ {Circle, Curve}

Answer:

- The event E includes shapes that are polygons: {Triangle, Square}.
- The complementary event E' consists of shapes that are not polygons.
- Therefore, the complementary event $E' =$ {Circle, Curve}.

A.4 MULTI-STEP RANDOM EXPERIMENTS

A.4.1 FINDING OUTCOME IN A TABLE

MCQ 15: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child / first child	B	G
B	BB	?
G	GB	GG

Find the missing outcome.

☐ BB

☒ BG

☐ GB

Answer:

- The first child is represented by the row ("first child"), and the second child by the column ("second child").
- The missing outcome is BG.

MCQ 16: The table below shows the possible outcomes when selecting two letters at random from the word "MAT" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 / letter 1	M	A	T
M	MM	MA	MT
A	AM	AA	AT
T	TM	?	TT

Find the missing outcome.

☐ TT

☒ TA

☐ AT

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is TA.

MCQ 17: The table below shows the possible outcomes when selecting two letters at random from the word "CODE" **with replacement** (after choosing a letter, it is put back before the next selection).

letter 2 / letter 1	C	O	D	E
C	CC	CO	CD	CE
O	OC	OO	OD	OE
D	DC	?	DD	DE
E	EC	EO	ED	EE

Find the missing outcome.

☒ DO

☐ OD

☐ DC

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is DO.

MCQ 18: The table below shows the possible outcomes when selecting two letters at random from the word "NODE" **without replacement** (after choosing a letter, it is not put back before the next selection). An "X" means no outcome is possible.

letter 2 / letter 1	N	O	D	E
N	**X**	?	ND	NE
O	ON	**X**	OD	OE
D	DN	DO	**X**	DE
E	EN	EO	ED	**X**

Find the missing outcome.

☐ NN

☒ NO

☐ ON

Answer:

- The first letter is "Letter 1" (row), and the second letter is "Letter 2" (column).
- The missing outcome is NO.

MCQ 19: The table below shows the possible outcomes when a coach selects two players at random from four players (A, B, C, D) **without replacement** (after choosing a player, they are not put back before the next selection). An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	?	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Find the missing outcome.

☒ AB

☐ BA

☐ CA

Answer:

- The first player is "Player 1" (row), and the second player is "Player 2" (column).
- The missing outcome is AB.

A.4.2 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN A TABLE

Ex 20: The table below shows the possible outcomes for the sexes of two children, first and second, where each can be a Boy (B) or a Girl (G).

second child / first child	B	G
B	BB	BG
G	GB	GG

Count the number of possible outcomes.

[4] possibles outcomes.

Answer:

- The sample space (the possible outcomes) is $\{GG, GB, BP, FF\}$.
- The number of possible outcomes is 4.

Ex 21: There are four players: A, B, C, and D. For position 1, only players A and B are eligible. For position 2, only players C and D are eligible. The table below shows the possible selections for the two positions.

position 2 / position 1	C	D
A	AC	AD
B	BC	BD

Count the number of possible outcomes.

[4] possible outcomes.

Answer:

- The sample space (the possible outcomes) is {AC, AD, BC, BD}.
- The number of possible outcomes is 4.

Ex 22: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of possible outcomes.

12 possible outcomes.

Answer:

- The sample space (the possible outcomes) is $\{AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC\}$.
- The number of possible outcomes is 12.

Ex 23: There are three students: X, Y, and Z. A teacher selects one student each day, on Monday and Tuesday, to recite a poem. The selection is made without replacement, meaning the same student cannot be chosen both days. The table below shows the possible selections for the two days.

Tuesday / Monday	X	Y	Z
X	**X**	XY	XZ
Y	YX	**X**	YZ
Z	ZX	ZY	**X**

Count the number of possible outcomes.

6 possible outcomes.

Answer:

- The sample space (the possible outcomes) is $\{XY, XZ, YX, YZ, ZX, ZY\}$.
- The number of possible outcomes is 6.

A.4.3 COUNTING THE NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT

Ex 24: There are four players: A, B, C, and D. A coach selects two players at random without replacement. The table below shows the possible selections for the two positions. An "X" means no outcome is possible.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Count the number of outcomes for the event that player A is selected.

6 outcomes.

Answer:

- The event that player A is selected includes all outcomes where A is either Player 1 or Player 2.
- From the table:
  - When A is Player 1 (row A): AB, AC, AD (3 outcomes).
  - When A is Player 2 (column A): BA, CA, DA (3 outcomes).
- Total outcomes: $\{AB, AC, AD, BA, CA, DA\}$, which is 6 outcomes.

Ex 25: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "double digit" (both dice show the same number).

6 outcomes.

Answer:

- The event "double digit" includes all outcomes where the red die and blue die show the same number.
- From the table:
  - Possible doubles: 11, 22, 33, 44, 55, 66 (6 outcomes).
- Total outcomes: {11, 22, 33, 44, 55, 66}, which is 6 outcomes.

Ex 26: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "at least one 6" (at least one die shows a 6).

11 outcomes.

Answer:

- The event "at least one 6" includes all outcomes where at least one of the red die or blue die shows a 6.
- From the table:
  - Red die = 6 (row 6): 61, 62, 63, 64, 65, 66 (6 outcomes).
  - Blue die = 6 (column 6), excluding (6,6) already counted: 16, 26, 36, 46, 56 (5 outcomes).
- Total outcomes: $\{16, 26, 36, 46, 56, 61, 62, 63, 64, 65, 66\}$, which is 11 outcomes.

Ex 27: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 11."

[2] outcomes.

Answer:

- The event "the sum of the dice is equal to 11" includes all outcomes where the red die and blue die sum to 11.
- From the table:
  - Possible pairs: 56 ($5+6=11$), 65 ($6+5=11$) (2 outcomes).
- Total outcomes: $\{56, 65\}$, which is 2 outcomes.

Ex 28: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "the sum of the dice is equal to 7."

[6] outcomes.

Answer:

- The event "the sum of the dice is equal to 7" includes all outcomes where the red die and blue die sum to 7.
- From the table:
  - Possible pairs: 16 ($1+6=7$), 25 ($2+5=7$), 34 ($3+4=7$), 43 ($4+3=7$), 52 ($5+2=7$), 61 ($6+1=7$) (6 outcomes).
- Total outcomes: $\{16, 25, 34, 43, 52, 61\}$, which is 6 outcomes.

Ex 29: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Count the number of outcomes for the event "a result inferior or equal to 3" (the sum of the dice is less than or equal to 3).

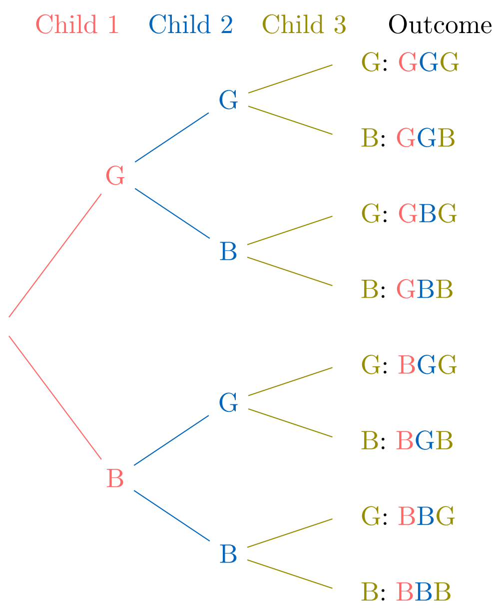
[3] outcomes.

Answer:

- The event "a result inferior or equal to 3" includes all outcomes where the sum of the red die and blue die is less than or equal to 3.
- From the table:
  - Possible pairs: 11 ($1+1=2$), 12 ($1+2=3$), 21 ($2+1=3$) (3 outcomes).
- Total outcomes: $\{11, 12, 21\}$, which is 3 outcomes.

A.4.4 COUNTING THE NUMBER OF POSSIBLE OUTCOMES IN AN TREE DIAGRAM

Ex 30: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



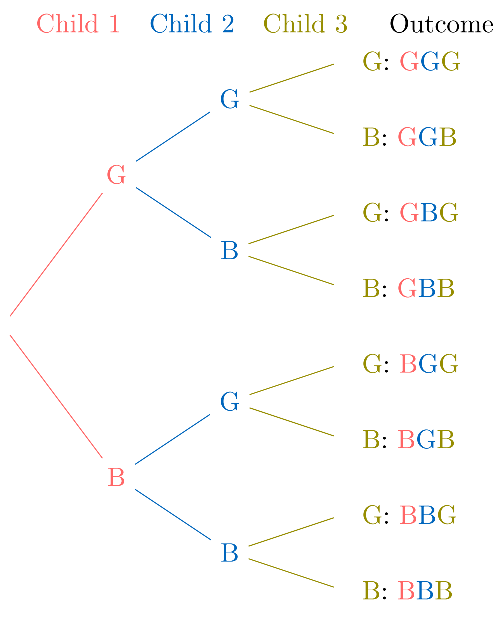
Count the number of possible outcomes for the event where the first child is a boy.

4

Answer:

- The event where the first child is a boy includes all outcomes starting with B, represented as $B**$.
- These outcomes are: BBB, BBG, BGB, BGG.
- The number of possible outcomes is 4.

Ex 31: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



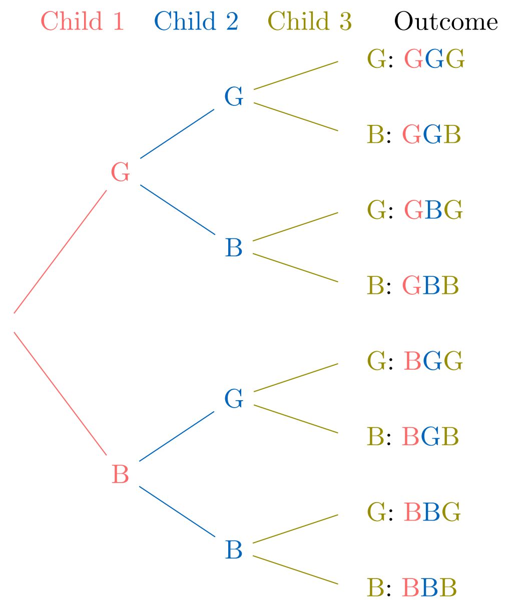
Count the number of possible outcomes for the event where there are exactly two girls.

3

Answer:

- The event where there are exactly two girls includes all outcomes with exactly two G's and one B.
- These outcomes are: BGG, GBG, GGB.
- The number of possible outcomes is 3.

Ex 32: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below shows all 8 possible gender outcomes for the three children.



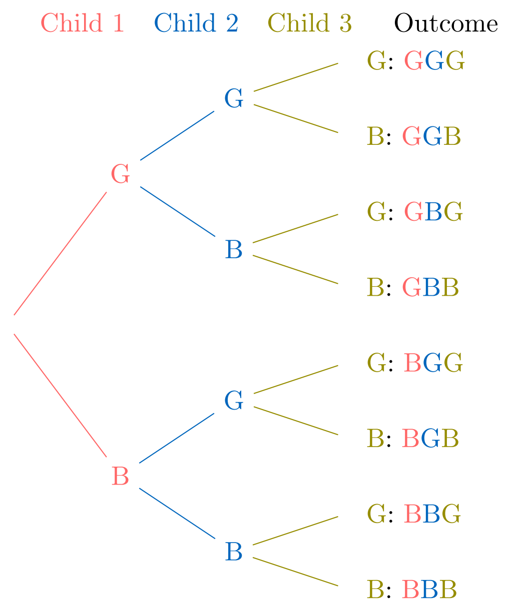
Count the number of possible outcomes for the event where there are at least two girls.

4

Answer:

- The event where there are at least two girls includes all outcomes with two or three girls.
- These outcomes are: BGG, GBG, GGB, GGG.
- The number of possible outcomes is 4.

Ex 33: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Count the number of possible outcomes for the event where the family has mixed-sex children (at least one boy and one girl).

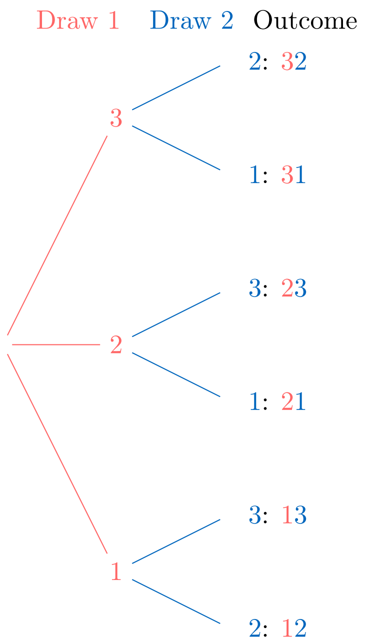
6

Answer:

- The event where the family has mixed-sex children includes all outcomes with at least one boy (B) and one girl (G), excluding all-boys (BBB) and all-girls (GGG).

- These outcomes are: BBG, BGB, GBB, GGB, GBG, BGG.
- The number of possible outcomes is 6.

Ex 34: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below shows all possible outcomes for these two selections.



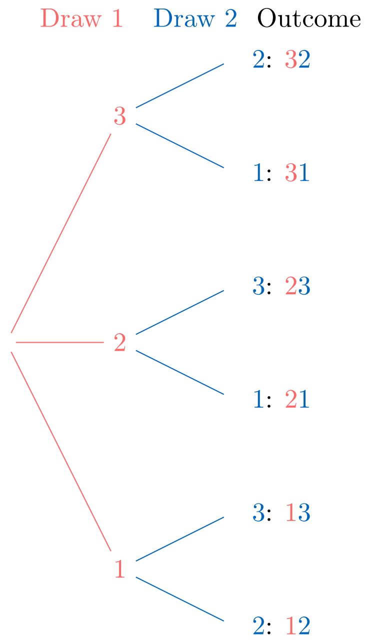
Count the number of possible outcomes in the sample space for the two ticket selections.

6

Answer:

- The sample space includes all possible pairs of tickets drawn, where the first number is the first ticket and the second is the ticket drawn afterward. Since a ticket cannot be drawn twice, pairs with repeated numbers (e.g., 11, 22, 33) are excluded.
- These outcomes are: 12, 13, 21, 23, 31, 32.
- The number of possible outcomes is 6.

Ex 35: Tickets numbered 1, 2, and 3 are placed in a bag. One ticket is drawn and set aside, then a second ticket is drawn. The tree diagram below displays all possible outcomes for these two selections.



Count the number of possible outcomes for the event where the ticket numbered 1 is drawn (either as the first or second ticket).

4

Answer:

- The event where ticket numbered 1 is drawn includes outcomes where 1 is either the first ticket ($1*$) or the second ticket ($*1$). Repeated numbers are impossible due to the selection process.
- These outcomes are: 12, 13, 21, 31.
- The number of possible outcomes is 4.

A.5 E OR F

A.5.1 FINDING THE UNION OF TWO EVENTS

MCQ 36: Let $E = \{A, C, D\}$ and $F = \{E, F, D\}$. Which option below correctly represents the event E or F?

Choose one true answer:

☐ E or $F = \{D\}$

☐ E or $F = \{A, C, D, E, F, H\}$

☒ E or $F = \{A, C, D, E, F\}$

☐ E or $F = \{A, C, E, F\}$

Answer:

- The event E or F (the union $E \cup F$) includes all outcomes that are in E, in F, or in both.
- Given:
  - $E = \{A, C, D\}$,
  - $F = \{E, F, D\}$.
- Combining these, we take all unique elements: $\{A, C, D, E, F\}$.

MCQ 37: A standard deck of 52 playing cards is used. Let event A be the event of drawing a red card, and let event B be the event of drawing a face card (Jack, Queen, or King). Which option below correctly represents the event A or B?

Choose one true answer:

☒ A or $B = \{\text{all red cards and all face cards}\}$

☐ A or $B = \{\text{all red face cards}\}$

☐ A or $B = \{\text{all red cards and all black face cards}\}$

☐ A or $B = \{\text{all cards}\}$

Answer:

- The event A or B (the union $A \cup B$) includes all outcomes that are in A, in B, or in both.
- Given:
  - $A = \{\text{all red cards}\}$, which includes 26 cards (13 hearts and 13 diamonds),
  - $B = \{\text{all face cards}\}$, which includes 12 cards (3 Jacks, 3 Queens, 3 Kings per suit, across 4 suits).
- Combining these, A or B includes all red cards (26) and all face cards (12), counting overlaps (6 red face cards) only once. Thus, it's the set of all red cards and all face cards.

MCQ 38: A weather forecast for a week lists possible conditions each day. Let event $W = \{\text{Monday, Tuesday, Wednesday}\}$ be the event of it being windy, and let event $R = \{\text{Wednesday, Thursday, Friday}\}$ be the event of it raining. Which option below correctly represents the event W or R?

Choose one true answer:

☐ W or $R = \{\text{Monday}\}$

☐ W or $R = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$

☐ W or $R = \{\text{Tuesday, Wednesday}\}$

☒ W or $R = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}$

Answer:

- The event W or R (the union $W \cup R$) includes all outcomes that are in W, in R, or in both.
- Combining these, we take all unique days: $\{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}$.

MCQ 39: A fruit basket contains various fruits. Let event $P = \{\text{cherry, peach, plum}\}$ be the event of selecting a pitted fruit, and let event $S = \{\text{mango, orange, peach}\}$ be the event of selecting a sweet fruit. Which option below correctly represents the event P or S?

Choose one true answer:

☒ P or $S = \{\text{cherry, peach, plum, mango, orange}\}$

☐ P or $S = \{\text{cherry, peach, plum}\}$

☐ P or $S = \{\text{mango, orange}\}$

☐ P or $S = \{\text{cherry, mango}\}$

Answer:

- The event P or S (the union $P \cup S$) includes all outcomes that are in P, in S, or in both.
- Combining $P = \{\text{cherry, peach, plum}\}$ and $S = \{\text{mango, orange, peach}\}$, we take all unique fruits: $\{\text{cherry, peach, plum, mango, orange}\}$.

A.5.2 FINDING THE UNION OF TWO EVENTS

MCQ 40: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Which option below correctly represents the event E or F?

Choose one true answer:

☐ E or $F = \{2\}$

☒ E or $F = \{1, 2, 3, 4, 6\}$

☐ E or $F = \{1, 2, 3, 4, 5, 6\}$

☐ E or $F = \{1, 2, 3\}$

Answer:

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event E, rolling an even number, is $\{2, 4, 6\}$.
- Event F, rolling a number less than 4, is $\{1, 2, 3\}$.
- The event E or F (the union $E \cup F$) includes all outcomes that are in E, in F, or in both. Combining these, we get $\{1, 2, 3, 4, 6\}$.

MCQ 41: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Which option below correctly represents the event G or H?

Choose one true answer:

☒ G or $H = \{2, 3, 4, 5, 6\}$

☐ G or $H = \{4, 5, 6\}$

☐ G or $H = \{2, 3, 5\}$

☐ G or $H = \{1, 2, 3, 4, 5, 6\}$

Answer:

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event G, rolling a number greater than 3, is $\{4, 5, 6\}$.
- Event H, rolling a prime number, is $\{2, 3, 5\}$.
- The event G or H (the union $G \cup H$) includes all outcomes that are in G, in H, or in both. Combining these, we get $\{2, 3, 4, 5, 6\}$.

MCQ 42: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Which option below correctly represents the event I or J?

Choose one true answer:

☐ I or $J = \{3, 6\}$

☐ I or $J = \{1, 2, 3, 4\}$

☐ I or $J = \{1, 2, 3, 4, 5, 6\}$

☒ I or $J = \{1, 2, 3, 4, 6\}$

Answer:

- The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.
- Event I, rolling a number divisible by 3, is $\{3, 6\}$.
- Event J, rolling a number less than 5, is $\{1, 2, 3, 4\}$.
- The event I or J (the union $I \cup J$) includes all outcomes that are in I, in J, or in both. Combining these, we get $\{1, 2, 3, 4, 6\}$.

MCQ 43: A family has three children. Let event A be the event of having at least two boys, and let event B be the event of having at least one girl. Which option below correctly represents the event A or B? (Use B for boy and G for girl.)

Choose one true answer:

☐ A or $B = \{\text{BBB}\}$

☒ A or $B = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$

☐ A or $B = \{\text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}\}$

☐ A or $B = \{\text{GGG}\}$

Answer:

- The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- Event A, having at least two boys, is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{GBB}\}$.
- Event B, having at least one girl, is $\{\text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$.
- The event A or B (the union $A \cup B$) includes all outcomes that are in A, in B, or in both. Combining these, we get $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$.

A.5.3 FINDING THE UNION OF TWO EVENTS FROM A TABLE

MCQ 44: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Tableau : Élèves

Nom	Âge	Genre
A	15	Féminin
B	17	Masculin
C	16	Féminin
D	15	Masculin
E	14	Féminin
F	17	Féminin

Let event A be the event of selecting a girl, and event B be the event of selecting a student older or equal than 17 years. Which option below correctly represents the event A or B?

Choose one true answer:

☐ A or $B = \{$A, B, C, D, E, F$\}$

☐ A or $B = \{$C, F$\}$

☐ A or $B = \{$A, B, C, D, F$\}$

☒ A or $B = \{$A, B, C, E, F$\}$

Answer:

- The sample space is $\{$A, B, C, D, E, F$\}$.
- Event A, selecting a girl, is $\{$A, C, E, F$\}$.
- Event B, selecting a student older or equal than 16, is $\{$B, F$\}$.
- The event A or B (the union $A \cup B$) includes all outcomes that are in A, in B, or in both. Combining these, we get $\{$A, B, C, E, F$\}$.

MCQ 45: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Which option below correctly represents the event X or Y?

Choose one true answer:

☐ X or $Y = \{$G, I, J, L$\}$

☒ X or $Y = \{$G, H, I, J, L$\}$

☐ X or $Y = \{$G, L$\}$

☐ X or $Y = \{$I, J, L$\}$

Answer:

- The sample space is $\{$G, H, I, J, K, L$\}$.
- Event X, selecting a student who plays the violin, is $\{$G, L$\}$.
- Event Y, selecting a student aged 16 or older, is $\{$I, J, L$\}$.
- The event X or Y (the union $X \cup Y$) includes all outcomes that are in X, in Y, or in both. Combining these, we get $\{$G, I, J, L$\}$. Note: The correct answer should be $\{$G, I, J, L$\}$, indicating a possible error in the provided options.

MCQ 46: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than 180 cm. Which option below correctly represents the event U or V?

Choose one true answer:

- ☐ U or $V = \{$M, O, R$\}$
- ☐ U or $V = \{$O, R$\}$
- ☐ U or $V = \{$M, N, O, R$\}$
- ☒ U or $V = \{$M, O, R$\}$

Answer:

- The sample space is $\{$M, N, O, P, Q, R$\}$.
- Event U, selecting a defender, is $\{$O, R$\}$.
- Event V, selecting a player taller than 180 cm, is $\{$O, R$\}$. (Note: M is 180 cm, not taller than 180 cm.)
- The event U or V (the union $U \cup V$) includes all outcomes that are in U, in V, or in both. Combining these, we get $\{$O, R$\}$. Note: The provided correct answer $\{$M, O, R$\}$ seems incorrect unless "taller than 180 cm" includes 180 cm, adjusting $V = \{$M, O, R$\}$, making U or $V = \{$M, O, R$\}$.

A.6 E AND F

A.6.1 FINDING THE INTERSECTION OF TWO EVENTS

MCQ 47: Let $E = \{A, C, D\}$ and $F = \{E, F, D\}$. Which option below correctly represents the event E and F?

Choose one true answer:

- ☒ E and $F = \{D\}$
- ☐ E and $F = \{A, C, D, E, F\}$
- ☐ E and $F = \{A, C, D\}$
- ☐ E and $F = \{E, F, D\}$

Answer:

1) The event E and F (the intersection $E \cap F$) includes all outcomes that are in both E and F.
2) Given:
   - $E = \{A, C, D\}$,
   - $F = \{E, F, D\}$.
3) The common element between E and F is D. Thus, E and $F = \{D\}$.

MCQ 48: A standard deck of 52 playing cards is used. Let event A be the event of drawing a red card, and let event B be the event of drawing a face card (Jack, Queen, or King). Which option below correctly represents the event A and B?

Choose one true answer:

- ☐ A and $B = \{$all red cards$\}$
- ☒ A and $B = \{$all red face cards$\}$
- ☐ A and $B = \{$all face cards$\}$
- ☐ A and $B = \{$all cards$\}$

Answer:

1) The event A and B (the intersection $A \cap B$) includes all outcomes that are in both A and B.
2) Given:
   - $A = \{$all red cards$\}$, which includes 26 cards (13 hearts and 13 diamonds),
   - $B = \{$all face cards$\}$, which includes 12 cards (3 Jacks, 3 Queens, 3 Kings per suit, across 4 suits).
3) The common outcomes are the red face cards (Jack, Queen, King of hearts and diamonds), totaling 6 cards. Thus, A and $B = \{$all red face cards$\}$.

MCQ 49: A weather forecast for a week lists possible conditions each day. Let event $W = \{$Monday, Tuesday, Wednesday$\}$ be the event of it being windy, and let event $R = \{$Wednesday, Thursday, Friday$\}$ be the event of it raining. Which option below correctly represents the event W and R?

Choose one true answer:

- ☐ W and $R = \{$Monday$\}$
- ☐ W and $R = \{$Monday, Tuesday, Wednesday, Thursday, Friday$\}$
- ☒ W and $R = \{$Wednesday$\}$
- ☐ W and $R = \{$Tuesday, Wednesday$\}$

Answer:

1) The event W and R (the intersection $W \cap R$) includes all outcomes that are in both W and R.
2) Given:
   - $W = \{$Monday, Tuesday, Wednesday$\}$,
   - $R = \{$Wednesday, Thursday, Friday$\}$.
3) The common day between W and R is Wednesday. Thus, W and $R = \{$Wednesday$\}$.

MCQ 50: A fruit basket contains various fruits. Let event $P = \{$cherry, peach, plum$\}$ be the event of selecting a pitted fruit, and let event $S = \{$mango, orange, peach$\}$ be the event of selecting a sweet fruit. Which option below correctly represents the event P and S?

Choose one true answer:

- ☐ P and $S = \{$cherry, peach, plum, mango, orange$\}$
- ☒ P and $S = \{$peach$\}$

☐ P and $S = \{\text{mango, orange}\}$

☐ P and $S = \{\text{cherry, mango}\}$

Answer:

1) The event P and S (the intersection $P \cap S$) includes all outcomes that are in both P and S.

2) Given:

   - $P = \{\text{cherry, peach, plum}\}$,
   - $S = \{\text{mango, orange, peach}\}$.

3) The common fruit between P and S is peach. Thus, P and $S = \{\text{peach}\}$.

A.6.2 FINDING THE INTERSECTION OF TWO EVENTS

MCQ 51: Consider the roll of a standard six-sided die. Let event E be the event of rolling an even number, and let event F be the event of rolling a number less than 4. Which option below correctly represents the event E and F?

Choose one true answer:

☒ E and $F = \{2\}$

☐ E and $F = \{1, 2, 3, 4, 6\}$

☐ E and $F = \{1, 2, 3\}$

☐ E and $F = \{2, 4, 6\}$

Answer:

1) The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.

2) Event E, rolling an even number, is $\{2, 4, 6\}$.

3) Event F, rolling a number less than 4, is $\{1, 2, 3\}$.

4) The event E and F (the intersection $E \cap F$) includes all outcomes that are in both E and F. The common element is 2. Thus, E and $F = \{2\}$.

MCQ 52: Consider the roll of a standard six-sided die. Let event G be the event of rolling a number greater than 3, and let event H be the event of rolling a prime number. Which option below correctly represents the event G and H?

Choose one true answer:

☐ G and $H = \{2, 3, 4, 5, 6\}$

☐ G and $H = \{4, 5, 6\}$

☒ G and $H = \{5\}$

☐ G and $H = \{2, 3, 5\}$

Answer:

1) The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.

2) Event G, rolling a number greater than 3, is $\{4, 5, 6\}$.

3) Event H, rolling a prime number, is $\{2, 3, 5\}$.

4) The event G and H (the intersection $G \cap H$) includes all outcomes that are in both G and H. The common element is 5. Thus, G and $H = \{5\}$.

MCQ 53: Consider the roll of a standard six-sided die. Let event I be the event of rolling a number divisible by 3, and let event J be the event of rolling a number less than 5. Which option below correctly represents the event I and J?

Choose one true answer:

☒ I and $J = \{3\}$

☐ I and $J = \{1, 2, 3, 4\}$

☐ I and $J = \{3, 6\}$

☐ I and $J = \{1, 2, 3, 4, 6\}$

Answer:

1) The sample space for rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$.

2) Event I, rolling a number divisible by 3, is $\{3, 6\}$.

3) Event J, rolling a number less than 5, is $\{1, 2, 3, 4\}$.

4) The event I and J (the intersection $I \cap J$) includes all outcomes that are in both I and J. The common element is 3. Thus, I and $J = \{3\}$.

MCQ 54: A family has three children. Let event A be the event of having at least two boys, and let event B be the event of having at least one girl. Which option below correctly represents the event A and B? (Use B for boy and G for girl.)

Choose one true answer:

☐ A and $B = \{\text{BBB}\}$

☐ A and $B = \{\text{BBG}, \text{BGB}, \text{GBB}\}$

☒ A and $B = \{\text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$

☐ A and $B = \{\text{GGG}\}$

Answer:

1) The sample space for a family with three children is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$.

2) Event A, having at least two boys, is $\{\text{BBB}, \text{BBG}, \text{BGB}, \text{GBB}\}$.

3) Event B, having at least one girl, is $\{\text{BBG}, \text{BGB}, \text{GBB}, \text{BGG}, \text{GBG}, \text{GGB}, \text{GGG}\}$.

4) The event A and B (the intersection $A \cap B$) includes all outcomes that are in both A and B. The common outcomes are $\{\text{BBG}, \text{BGB}, \text{GBB}\}$. Thus, A and $B = \{\text{BBG}, \text{BGB}, \text{GBB}\}$.

A.6.3 FINDING THE INTERSECTION OF TWO EVENTS FROM A TABLE

MCQ 55: In a classroom, students are listed in a table with their names, ages, and genders. Consider a random selection of one student from this table:

Tableau : Élèves

Nom	Âge	Genre
A	15	Féminin
B	17	Masculin
C	16	Féminin
D	15	Masculin
E	14	Féminin
F	17	Féminin

Let event A be the event of selecting a girl, and event B be the event of selecting a student older than or equal to 17 years. Which option below correctly represents the event A and B?

Choose one true answer:

- ☐ A and $B = \{A, C, E, F\}$
- ☒ A and $B = \{B, F\}$
- ☒ A and $B = \{F\}$
- ☐ A and $B = \{A, C, F\}$

Answer:

1) The sample space is {A, B, C, D, E, F}.
2) Event A, selecting a girl, is {A, C, E, F}.
3) Event B, selecting a student older than or equal to 17 years, is {B, F}.
4) The event A and B (the intersection $A \cap B$) includes all outcomes that are in both A and B. The common element is {F}. Thus, A and B = {F}.

MCQ 56: In a music class, students are listed in a table with their names, ages, and preferred instruments. Consider a random selection of one student from this table:

Table: Students

Name	Age	Instrument
G	14	Violin
H	15	Piano
I	17	Guitar
J	16	Drums
K	15	Flute
L	18	Violin

Let event X be the event of selecting a student who plays the violin, and event Y be the event of selecting a student aged 16 or older. Which option below correctly represents the event X and Y?

Choose one true answer:

- ☐ X and Y = {G, I, J, L}
- ☒ X and Y = {L}
- ☐ X and Y = {G, L}
- ☐ X and Y = {I, J, L}

Answer:

1) The sample space is {G, H, I, J, K, L}.
2) Event X, selecting a student who plays the violin, is {G, L}.
3) Event Y, selecting a student aged 16 or older, is {I, J, L}.
4) The event X and Y (the intersection $X \cap Y$) includes all outcomes that are in both X and Y. The common element is {L}. Thus, X and Y = {L}.

MCQ 57: In a sports team, players are listed in a table with their names, heights, and positions. Consider a random selection of one player from this team:

Table: Players

Name	Height (cm)	Position
M	180	Forward
N	170	Goalkeeper
O	185	Defender
P	175	Midfielder
Q	165	Forward
R	190	Defender

Let event U be the event of selecting a defender, and event V be the event of selecting a player taller than 180 cm. Which option below correctly represents the event U and V?

Choose one true answer:

- ☒ U and V = {O, R}
- ☐ U and V = {M, O, R}
- ☐ U and V = {R}
- ☐ U and V = {M, N, O, R}

Answer:

1) The sample space is {M, N, O, P, Q, R}.
2) Event U, selecting a defender, is {O, R}.
3) Event V, selecting a player taller than 180 cm, is {O, R} (Note: M is 180 cm, not taller than 180 cm).
4) The event U and V (the intersection $U \cap V$) includes all outcomes that are in both U and V. The common elements are {O, R}. Thus, U and V = {O, R}.

A.7 MUTUALLY EXCLUSIVE

A.7.1 DETERMINING MUTUAL EXCLUSIVITY

MCQ 58: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling an even number.
- Event F: Rolling an odd number.

Are the events E and F mutually exclusive?

Choose one true answer:

- ☒ Yes, the events are mutually exclusive.
- ☐ No, the events are not mutually exclusive.
- ☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).
2) Event E, rolling an even number, includes: 2, 4, 6 (3 outcomes).
3) Event F, rolling an odd number, includes: 1, 3, 5 (3 outcomes).
4) There is no number that is both even and odd, so the intersection $E \cap F = \emptyset$ (empty set).

5) Since there are no common outcomes, events E and F are mutually exclusive.

6) Therefore, the correct answer is: Yes, the events are mutually exclusive.

MCQ 59: Consider rolling a standard six-sided die (numbered 1 to 6). Two events are defined as follows:

- Event E: Rolling a prime number.
- Event F: Rolling an even number.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☒ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).

2) Event E, rolling a prime number, includes: 2, 3, 5 (3 outcomes). Note: 1 is not a prime number.

3) Event F, rolling an even number, includes: 2, 4, 6 (3 outcomes).

4) The number 2 is both a prime number (in E) and an even number (in F), so it is a common outcome.

5) Since $E \cap F = \{2\}$ is not empty, events E and F are not mutually exclusive.

6) Therefore, the correct answer is: No, the events are not mutually exclusive.

MCQ 60: Consider a standard deck of 52 playing cards (no jokers). Two events are defined as follows:

- Event E: Drawing a Queen.
- Event F: Drawing a Heart.

Are the events E and F mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☒ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($E \cap F$) is empty—there are no outcomes common to both.

2) Event E, drawing a Queen, includes the cards: Queen of Hearts, Queen of Spades, Queen of Clubs, and Queen of Diamonds (4 outcomes).

3) Event F, drawing a Heart, includes all 13 Hearts: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King of Hearts.

4) The Queen of Hearts is both a Queen (in E) and a Heart (in F), so it is a common outcome.

5) Since $E \cap F = \{\text{Queen of Hearts}\}$ is not empty, events E and F are not mutually exclusive.

6) Therefore, the correct answer is: No, the events are not mutually exclusive.

MCQ 61: Consider a family with exactly two children, where each child is a boy (B) or a girl (G). Two events are defined as follows:

- Event E: The family has only boys.
- Event F: The family has only girls.

Are the events E and F mutually exclusive?

Choose one true answer:

☒ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur at the same time, meaning they have no outcomes in common ($E \cap F = \emptyset$).

2) The sample space for a family with two children is: $\{BB, BG, GB, GG\}$, where B = boy and G = girl.

3) Event E, having only boys, is: $\{BB\}$ (1 outcome).

4) Event F, having only girls, is: $\{GG\}$ (1 outcome).

5) There is no family outcome that has only boys and only girls at the same time, so the intersection $E \cap F = \emptyset$ (empty set).

6) Since there are no common outcomes, events E and F are mutually exclusive.

7) Therefore, the correct answer is: Yes, the events are mutually exclusive.

A.7.2 DETERMINING MUTUAL EXCLUSIVITY FROM TABLES

MCQ 62: Consider a sports club where members are listed with their preferred sport and age group. A member is selected at random from the following table:

Table: Sports Club Members

Name	Preferred Sport	Age Group
S	Football	Under 16
T	Basketball	16-18
U	Tennis	Over 18
V	Swimming	Under 16
W	Football	16-18
X	Basketball	Over 18

Let event C be selecting a member whose preferred sport is Football, and event D be selecting a member from the Over 18 age group. Are events C and D mutually exclusive?

Choose one true answer:

☒ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($C \cap D$) is empty—there are no common outcomes.

2) Event C, selecting a member whose preferred sport is Football, includes: S (Under 16), W (16-18) — 2 outcomes.

3) Event D, selecting a member from the Over 18 age group, includes: U (Tennis), X (Basketball) — 2 outcomes.

4) From the table, no member prefers Football and is in the Over 18 age group. The outcomes for C (S, W) and D (U, X) are distinct.

5) Since $C \cap D = \emptyset$ (empty set), events C and D are mutually exclusive.

6) Therefore, the correct answer is: Yes, the events are mutually exclusive.

MCQ 63: Consider a bakery where items are listed with their type and topping. An item is selected at random from the following table:

Table: Bakery Items

Item	Type	Topping
Muffin	Pastry	Chocolate
Cookie	Cookie	Sprinkles
Cake	Cake	Chocolate
Donut	Pastry	Glaze
Brownie	Cake	None
Croissant	Pastry	None

Let event T be selecting a Pastry, and event U be selecting an item with Chocolate topping. Are events T and U mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☒ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($T \cap U$) is empty—no common outcomes.

2) Event T, selecting a Pastry, includes: Muffin (Chocolate), Donut (Glaze), Croissant (None) — 3 outcomes.

3) Event U, selecting an item with Chocolate topping, includes: Muffin (Pastry), Cake (Cake) — 2 outcomes.

4) The Muffin is both a Pastry (in T) and has Chocolate topping (in U), so there is a common outcome.

5) Since $T \cap U = \{\text{Muffin}\}$ is not empty, events T and U are not mutually exclusive.

6) Therefore, the correct answer is: No, the events are not mutually exclusive.

MCQ 64: Consider a library where books are listed with their genre and checkout status. A book is selected at random from the following table:

Table: Library Books

Title	Genre	Status
A	Mystery	Checked Out
B	Fantasy	Available
C	Mystery	Available
D	Romance	Checked Out
E	Fantasy	Checked Out
F	Romance	Available

Let event P be selecting a Mystery book, and event Q be selecting a Checked Out book. Are events P and Q mutually exclusive?

Choose one true answer:

☐ Yes, the events are mutually exclusive.

☒ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($P \cap Q$) is empty—no common outcomes.

2) Event P, selecting a Mystery book, includes: A (Checked Out), C (Available) — 2 outcomes.

3) Event Q, selecting a Checked Out book, includes: A (Mystery), D (Romance), E (Fantasy) — 3 outcomes.

4) The book A is both a Mystery (in P) and Checked Out (in Q), so there is a common outcome.

5) Since $P \cap Q = \{\text{A}\}$ is not empty, events P and Q are not mutually exclusive.

6) Therefore, the correct answer is: No, the events are not mutually exclusive.

MCQ 65: Consider a zoo where animals are listed with their type and habitat. An animal is selected at random from the following table:

Table: Zoo Animals

Name	Type	Habitat
Lion	Mammal	Savanna
Penguin	Bird	Arctic
Crocodile	Reptile	Swamp
Elephant	Mammal	Savanna
Parrot	Bird	Jungle
Snake	Reptile	Jungle

Let event R be selecting a Mammal, and event S be selecting an animal from the Arctic habitat. Are events R and S mutually exclusive?

Choose one true answer:

☒ Yes, the events are mutually exclusive.

☐ No, the events are not mutually exclusive.

☐ It cannot be determined from the information given.

Answer:

1) Two events are mutually exclusive if they cannot occur simultaneously, meaning their intersection ($R \cap S$) is empty—no common outcomes.

2) Event R, selecting a Mammal, includes: Lion (Savanna), Elephant (Savanna) — 2 outcomes.

3) Event S, selecting an animal from the Arctic habitat, includes: Penguin (Bird) — 1 outcome.

4) From the table, no animal is both a Mammal and from the Arctic habitat. The outcomes for R (Lion, Elephant) and S (Penguin) are distinct.

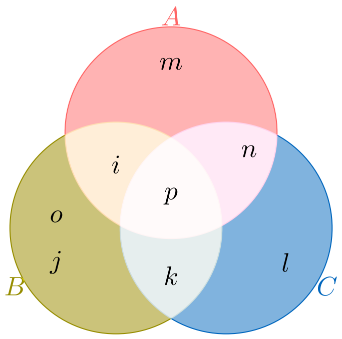
5) Since $R \cap S = \emptyset$ (empty set), events R and S are mutually exclusive.

6) Therefore, the correct answer is: Yes, the events are mutually exclusive.

A.8 VENN DIAGRAM

A.8.1 FINDING THE UNION OF TWO EVENTS IN A VENN DIAGRAM

MCQ 66: You are given this populated Venn diagram representing events A, B, and C:



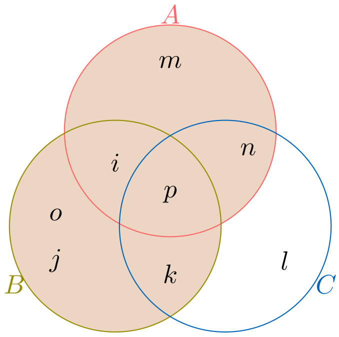
Which option below correctly shows the union of A and B?

Choose one true answer:

- ☐ A or $B = \{i, j, k, l, m, n\}$
- ☒ A or $B = \{i, j, k, m, o, p\}$
- ☐ A or $B = \{i, j, k, l, m\}$
- ☐ A or $B = \{i, j, k, l, m, o, p\}$

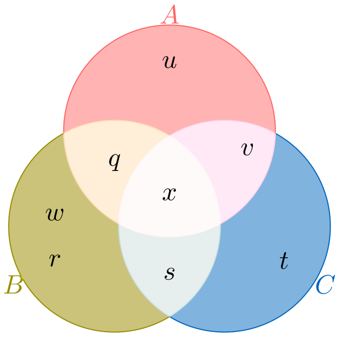
Answer:

- The union of events A and B ($A \cup B$) includes all elements that are in A, in B, or in both.
- From the Venn diagram:
  - Event $A = \{i, m, n, p\}$ (elements in the top circle).
  - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
  - Combining these, A or $B = \{i, j, k, m, o, p\}$.
- The shaded area represents



A or B:

MCQ 67: You are given this populated Venn diagram representing events A, B, and C:



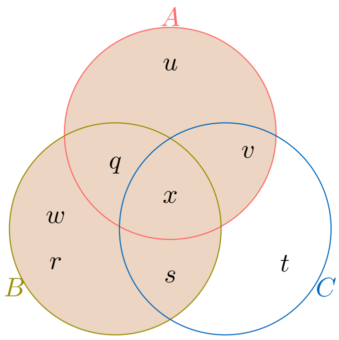
Which option below correctly shows the union of A and B?

Choose one true answer:

- ☐ A or $B = \{q, r, s, t, u, v\}$
- ☒ A or $B = \{q, r, s, u, w, x\}$
- ☐ A or $B = \{q, r, s, t, u\}$
- ☐ A or $B = \{q, r, s, t, u, w, x\}$

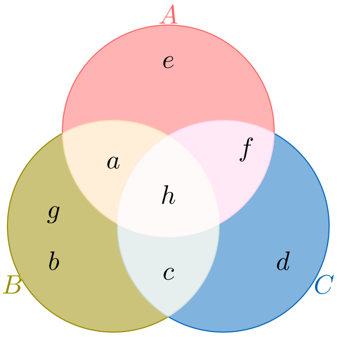
Answer:

- The union of events A and B ($A \cup B$) includes all elements that are in A, in B, or in both.
- From the Venn diagram:
  - Event $A = \{q, u, v, x\}$ (elements in the top circle).
  - Event $B = \{q, r, s, w, x\}$ (elements in the left circle).
  - Combining these, A or $B = \{q, r, s, u, w, x\}$.
- The shaded area represents



A or B:

MCQ 68: You are given this populated Venn diagram representing events A, B, and C:



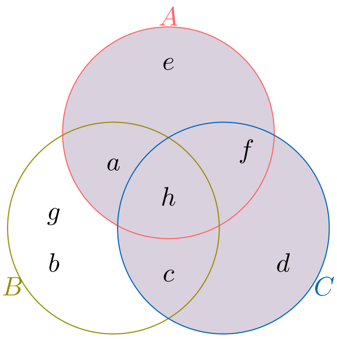
Which option below correctly shows the union of A and C?

Choose one true answer:

- ☒ A or $C = \{a, c, d, e, f, h\}$
- ☐ A or $C = \{a, b, e, g, h\}$
- ☐ A or $C = \{a, b, c, d, e\}$
- ☐ A or $C = \{a, b, c, d, e, g, h\}$

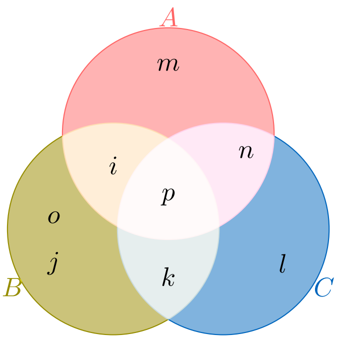
Answer:

- The union of events A and C ($A \cup C$) includes all elements that are in A, in C, or in both.
- From the Venn diagram:
  - Event $A = \{a, e, f, h\}$ (elements in the top circle).
  - Event $C = \{c, d, f, h\}$ (elements in the right circle).
  - Combining these, A or $C = \{a, c, d, e, f, h\}$.
- The shaded area represents



A or C:

MCQ 69: You are given this populated Venn diagram representing events A, B, and C:



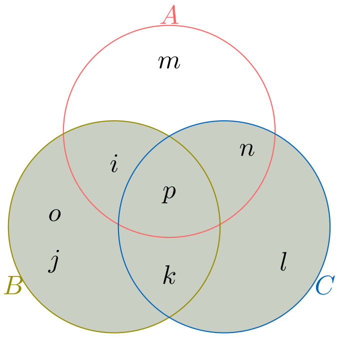
Which option below correctly shows the union of B and C?

Choose one true answer:

- ☐ B or $C = \{i, j, k, l, m, n\}$
- ☐ B or $C = \{i, j, k, l, o, p\}$
- ☒ B or $C = \{i, j, k, l, o, p\}$
- ☐ B or $C = \{i, j, k, l, o, p\}$

Answer:

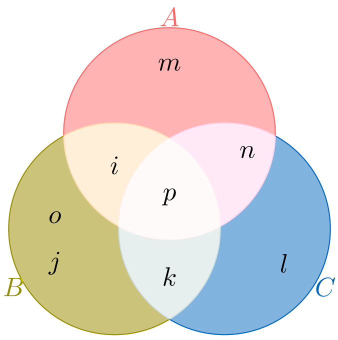
- The union of events B and C ($B \cup C$) includes all elements that are in B, in C, or in both.
- From the Venn diagram:
  - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
  - Event $C = \{k, l, n, p\}$ (elements in the right circle).
  - Combining these, B or $C = \{i, j, k, l, o, p\}$.
- The shaded area represents



B or C:

A.8.2 FINDING THE INTERSECTION OF TWO EVENTS IN A VENN DIAGRAM

MCQ 70: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the intersection of A and B?

Choose one true answer:

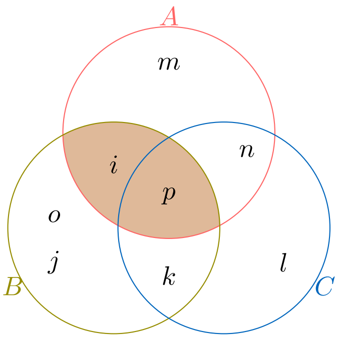
- ☒ A and $B = \{i, p\}$
- ☐ A and $B = \{i, j, k, m, o, p\}$
- ☐ A and $B = \{i, j, k, l, m\}$
- ☐ A and $B = \{j, k, o\}$

Answer:

1) The intersection of events A and B ($A \cap B$) includes all elements that are in both A and B.
2) From the Venn diagram:
   - Event $A = \{i, m, n, p\}$ (elements in the top circle).
   - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).

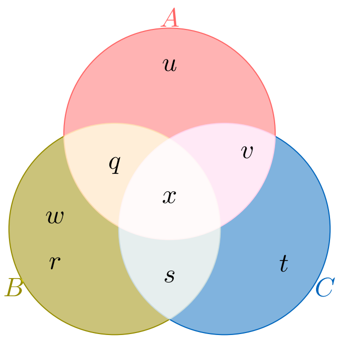
- The common elements are $\{i, p\}$. Thus, A and $B = \{i, p\}$.

3) The shaded area represents



A and B:

MCQ 71: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the intersection of A and B?

Choose one true answer:

☒ A and $B = \{q, x\}$

☐ A and $B = \{q, r, s, u, w, x\}$

☐ A and $B = \{r, s, w\}$

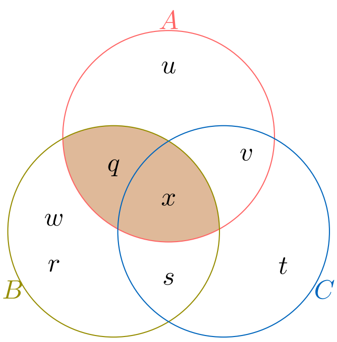
☐ A and $B = \{q, r, s, t, u\}$

Answer:

1) The intersection of events A and B ($A \cap B$) includes all elements that are in both A and B.

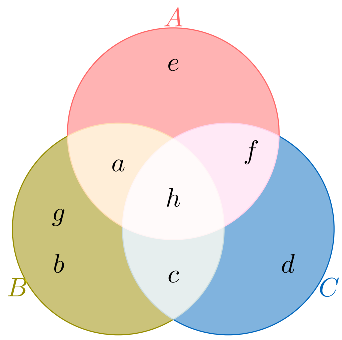
2) From the Venn diagram:
   - Event $A = \{q, u, v, x\}$ (elements in the top circle).
   - Event $B = \{q, r, s, w, x\}$ (elements in the left circle).
   - The common elements are $\{q, x\}$. Thus, A and $B = \{q, x\}$.

3) The shaded area represents



A and B:

MCQ 72: You are given this populated Venn diagram representing events A, B, and C:



Which option below correctly shows the intersection of A and C?

Choose one true answer:

☐ A and $C = \{a, c, d, e, f, h\}$

☐ A and $C = \{a, e\}$

☒ A and $C = \{f, h\}$

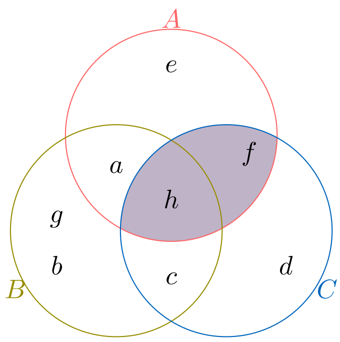
☐ A and $C = \{c, d, f\}$

Answer:

1) The intersection of events A and C ($A \cap C$) includes all elements that are in both A and C.

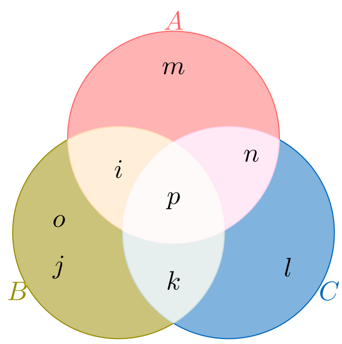
2) From the Venn diagram:
   - Event $A = \{a, e, f, h\}$ (elements in the top circle).
   - Event $C = \{c, d, f, h\}$ (elements in the right circle).
   - The common elements are $\{f, h\}$. Thus, A and $C = \{f, h\}$.

3) The shaded area represents



A and C:

MCQ 73: You are given this populated Venn diagram representing events A, B, and C:



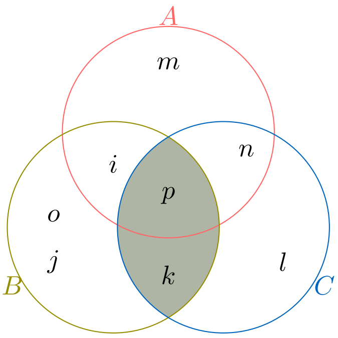
Which option below correctly shows the intersection of B and C?

Choose one true answer:

- ☐ B and $C = \{i, j, k, l, o, p\}$
- ☒ B and $C = \{k, p\}$
- ☐ B and $C = \{j, k, o\}$
- ☐ B and $C = \{l, n, p\}$

Answer:

1) The intersection of events B and C ($B \cap C$) includes all elements that are in both B and C.
2) From the Venn diagram:
    - Event $B = \{i, j, k, o, p\}$ (elements in the left circle).
    - Event $C = \{k, l, n, p\}$ (elements in the right circle).
    - The common elements are $\{k, p\}$. Thus, B and $C = \{k, p\}$.
3) The shaded area represents B and C:



B PROBABILITY

B.1 DEFINITION

B.1.1 DETERMINING THE PROBABILITY

MCQ 74: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find $P(E)$, the probability that Keziah eats rice this week.

- ☐ $P(E) = 1\%$
- ☐ $P(E) = 50\%$
- ☒ $P(E) = 99\%$

Answer:

- Since he eats rice often, it's very likely.
- So, the probability that Keziah eats rice this week is $P(E) = 99\%$.

MCQ 75: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find $P(E)$, the probability that Emily drinks water tomorrow.

- ☐ $P(E) = 50\%$
- ☐ $P(E) = 90\%$
- ☒ $P(E) = 100\%$

Answer:

- She drinks water every day, so it's certain.
- So, the probability that Emily drinks water tomorrow is $P(E) = 100\%$.

MCQ 76: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find $P(E)$, the probability that it snows this July.

- ☒ $P(E) = 0.01\%$
- ☐ $P(E) = 5\%$
- ☐ $P(E) = 99.9\%$

Answer:

- It's extremely rare, so very unlikely.
- So, the probability that it snows this July is $P(E) = 0.01\%$.

MCQ 77: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find $P(E)$, the probability that Samuel plays this weekend.

- ☐ $P(E) = 5\%$
- ☐ $P(E) = 20\%$
- ☒ $P(E) = 90\%$

Answer:

- He loves it, so it's very likely.
- So, the probability that Samuel plays this weekend is $P(E) = 90\%$.

MCQ 78: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find $P(E)$, the probability that Benjamin rolls a number bigger than 7.

- ☒ $P(E) = 0\%$
- ☐ $P(E) = 50\%$
- ☐ $P(E) = 100\%$

Answer:

- A die only has numbers 1 to 6, so it's impossible.
- So, the probability that Benjamin rolls a number bigger than 7 is $P(E) = 0\%$.

B.1.2 FINDING PROBABILITY FOR MUTUALLY EXCLUSIVE EVENTS

Ex 79: Let $P(G) = 0.6$ and $P(H) = 0.2$. Assume that events G and H are mutually exclusive.
Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \boxed{0.8}$$

Answer:

1) Since G and H are mutually exclusive events, they cannot occur at the same time.

2) The probability of either event G or event H occurring is the sum of their individual probabilities.

3) We are given $P(G) = 0.6$ and $P(H) = 0.2$.

4) Therefore,
$$\begin{aligned} P(G \text{ or } H) &= P(G) + P(H) \\ &= 0.6 + 0.2 \\ &= 0.8. \end{aligned}$$

5) So, the probability of either event G or event H occurring is 0.8.

Ex 80: Let $P(A) = 0.5$ and $P(B) = 0.3$. Assume that events A and B are mutually exclusive.
Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \boxed{0.8}$$

Answer:

1) Since A and B are mutually exclusive events, they cannot occur at the same time.

2) The probability of either event A or event B occurring is the sum of their individual probabilities.

3) We are given $P(A) = 0.5$ and $P(B) = 0.3$.

4) Therefore,
$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ &= 0.5 + 0.3 \\ &= 0.8. \end{aligned}$$

5) So, the probability of either event A or event B occurring is 0.8.

Ex 81: Let $P(C) = 0.4$ and $P(D) = 0.5$. Assume that events C and D are mutually exclusive.
Calculate $P(C \text{ or } D)$.

$$P(C \text{ or } D) = \boxed{0.9}$$

Answer:

1) Since C and D are mutually exclusive events, they cannot occur at the same time.

2) The probability of either event C or event D occurring is the sum of their individual probabilities.

3) We are given $P(C) = 0.4$ and $P(D) = 0.5$.

4) Therefore,
$$\begin{aligned} P(C \text{ or } D) &= P(C) + P(D) \\ &= 0.4 + 0.5 \\ &= 0.9. \end{aligned}$$

5) So, the probability of either event C or event D occurring is 0.9.

Ex 82: Let $P(E) = 0.7$ and $P(F) = 0.1$. Assume that events E and F are mutually exclusive.
Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \boxed{0.8}$$

Answer:

1) Since E and F are mutually exclusive events, they cannot occur at the same time.

2) The probability of either event E or event F occurring is the sum of their individual probabilities.

3) We are given $P(E) = 0.7$ and $P(F) = 0.1$.

4) Therefore,
$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) \\ &= 0.7 + 0.1 \\ &= 0.8. \end{aligned}$$

5) So, the probability of either event E or event F occurring is 0.8.

B.2 PROBABILITY RULES

B.2.1 APPLYING THE COMPLEMENT RULE

Ex 83: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$.
Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- The probability of getting heads is $\frac{1}{2}$.
- The event "Getting tails" is the complement of "Getting heads."
- Using the complement rule:
$$\begin{aligned} P(\text{"Getting tails"}) &= 1 - P(\text{"Getting heads"}) \\ &= 1 - \frac{1}{2} \\ &= \frac{2}{2} - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$
- So, the probability of getting tails is $\frac{1}{2} = 50\%$.

Ex 84: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{30}\%$$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing."
- Using the complement rule:
$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not laugh at the joke is 30%.

Ex 85: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \frac{\boxed{1}}{\boxed{10}}$$

Answer:

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:

$$\begin{aligned} P(\text{"Selecting a boy"}) &= 1 - P(\text{"Selecting a girl"}) \\ &= 1 - \frac{9}{10} \\ &= \frac{10}{10} - \frac{9}{10} \\ &= \frac{1}{10} \end{aligned}$$

- So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

Ex 86: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \boxed{30}\%$$

Answer:

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of the event "Rain".
- Using the complement rule:

$$\begin{aligned} P(\text{"No rain"}) &= 1 - P(\text{"Rain"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that it will not rain tomorrow is 30%.

Ex 87: A survey shows that 70% of the students in a school love Math.
Find the probability that a randomly chosen student does not love Math.

$$P(\text{"Not loving Math"}) = \boxed{30}\%$$

Answer:

- The probability that a student loves Math is 70%.
- The event "Not loving Math" is the complement event of "Loving Math."
- Using the complement rule:

$$\begin{aligned} P(\text{"Not loving Math"}) &= 1 - P(\text{"Loving Math"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not love Math is 30%.

MCQ 88: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

- ☒ $P(\text{"Not laughing"}) = 30\%$
- ☐ $P(\text{"Not laughing"}) = 70\%$
- ☐ $P(\text{"Not laughing"}) = 50\%$

Answer:

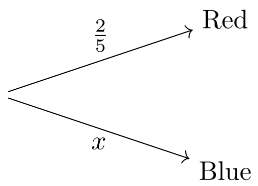
- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing".
- Using the complement rule:

$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- So, the probability that a student does not laugh at the joke is 30%.

B.2.2 COMPLETING A PROBABILITY TREE DIAGRAM

Ex 89: From a bag containing red balls and blue balls, the probability of choosing a red ball is $\frac{2}{5}$. Find the probability x of choosing a blue ball.



$$x = \frac{\boxed{3}}{\boxed{5}}$$

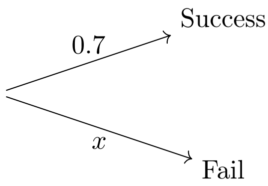
Answer:

- The probability of choosing a red ball from the bag is given as $\frac{2}{5}$.
- Since the only other option is choosing a blue ball, the probability of choosing a blue ball is calculated as follows:

$$\begin{aligned} P(\text{"Blue"}) &= 1 - P(\text{"Red"}) \\ &= 1 - \frac{2}{5} \\ &= \frac{5}{5} - \frac{2}{5} \\ &= \frac{3}{5} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{5}$.

Ex 90: Jasper is playing basketball. The probability that he makes his first shot is 0.7. Find the probability x that he misses his first shot.



$$x = \boxed{0.3}$$

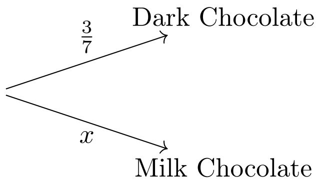
Answer: Let's figure this out step by step:

- The probability that Jasper makes his first shot is 0.7.
- Since the complement event is missing the shot, the probability x is calculated as follows:

$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.7 \\ &= 0.3 \end{aligned}$$

- So, the correct answer is $x = 0.3$.

Ex 91: In a box of assorted chocolates, the probability of picking a dark chocolate is $\frac{3}{7}$. Find the probability x of picking a milk chocolate.



$$x = \frac{\boxed{4}}{\boxed{7}}$$

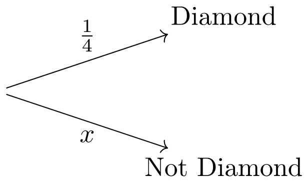
Answer:

- The probability of picking a dark chocolate from the box is $\frac{3}{7}$.
- Since the only other option is picking a milk chocolate, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Milk chocolate"}) &= 1 - P(\text{"Dark chocolate"}) \\ &= 1 - \frac{3}{7} \\ &= \frac{7}{7} - \frac{3}{7} \\ &= \frac{4}{7} \end{aligned}$$

- So, the correct answer is $x = \frac{4}{7}$.

Ex 92: In a deck of cards, the probability of drawing a card from the suit of diamonds is $\frac{1}{4}$. Find the probability x of drawing a card that is not a diamond.



$$x = \frac{\boxed{3}}{\boxed{4}}$$

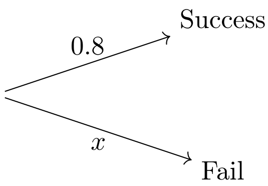
Answer:

- The probability of drawing a diamond from a deck of cards is $\frac{1}{4}$.
- Since the only other option is drawing a card that is not a diamond, the probability is calculated as follows:

$$\begin{aligned} P(\text{"Not diamond"}) &= 1 - P(\text{"Diamond"}) \\ &= 1 - \frac{1}{4} \\ &= \frac{4}{4} - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

- So, the correct answer is $x = \frac{3}{4}$.

Ex 93: Emma is playing a video game. The probability that she completes a level is 0.8. Find the probability x that she fails to complete the level.



$$x = \boxed{0.2}$$

Answer: Let's figure this out step by step:

- The probability that Emma completes the level is 0.8.
- Since the complement event is failing to complete the level, the probability x is calculated as follows:

$$\begin{aligned} P(\text{"Fail"}) &= 1 - P(\text{"Success"}) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

- So, the correct answer is $x = 0.2$.

B.2.3 CALCULATING PROBABILITIES FOR UNION OF EVENTS

Ex 94: Let $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Calculate $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \boxed{0.7}$$

Answer:

1) To calculate $P(A \text{ or } B)$, we use the general probability addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

2) We are given:

$$P(A) = 0.5, \quad P(B) = 0.3, \quad \text{and} \quad P(A \text{ and } B) = 0.1.$$

3) Substitute the given values into the formula:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.5 + 0.3 - 0.1 \\ &= 0.7. \end{aligned}$$

4) So, the probability of either event A or event B occurring is 0.7.

Ex 95: Let $P(E) = 0.8$, $P(F) = 0.3$ and $P(E \text{ and } F) = 0.2$. Calculate $P(E \text{ or } F)$.

$$P(E \text{ or } F) = \boxed{0.9}$$

Answer:

1) To calculate $P(E \text{ or } F)$, we use the general probability addition rule:

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F).$$

2) We are given:

$$P(E) = 0.8, \quad P(F) = 0.3, \quad \text{and} \quad P(E \text{ and } F) = 0.2.$$

3) Substitute the given values into the formula:

$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= 0.8 + 0.3 - 0.2 \\ &= 0.9. \end{aligned}$$

4) So, the probability of either event E or event F occurring is 0.9.

Ex 96: Let $P(G) = 0.6$, $P(H) = 0.2$ and $P(G \text{ and } H) = 0.1$ Calculate $P(G \text{ or } H)$.

$$P(G \text{ or } H) = \boxed{0.7}$$

Answer:

1) To calculate $P(G \text{ or } H)$, we use the general probability addition rule:

$$P(G \text{ or } H) = P(G) + P(H) - P(G \text{ and } H).$$

2) We are given:

$$P(G) = 0.6, \quad P(H) = 0.2, \quad \text{and} \quad P(G \text{ and } H) = 0.1.$$

3) Substitute the given values into the formula:

$$\begin{aligned} P(G \text{ or } H) &= P(G) + P(H) - P(G \text{ and } H) \\ &= 0.6 + 0.2 - 0.1 \\ &= 0.7. \end{aligned}$$

4) So, the probability of either event G or event H occurring is 0.7.

Ex 97: Let $P(X) = 0.7$, $P(Y) = 0.4$ and $P(X \text{ and } Y) = 0.2$. Calculate $P(X \text{ or } Y)$.

$$P(X \text{ or } Y) = \boxed{0.9}$$

Answer:

1) To calculate $P(X \text{ or } Y)$, we use the general probability addition rule:

$$P(X \text{ or } Y) = P(X) + P(Y) - P(X \text{ and } Y).$$

2) We are given:

$$P(X) = 0.7, \quad P(Y) = 0.4, \quad \text{and} \quad P(X \text{ and } Y) = 0.2.$$

3) Substitute the given values into the formula:

$$\begin{aligned} P(X \text{ or } Y) &= P(X) + P(Y) - P(X \text{ and } Y) \\ &= 0.7 + 0.4 - 0.2 \\ &= 0.9. \end{aligned}$$

4) So, the probability of either event X or event Y occurring is 0.9.

B.2.4 CALCULATING PROBABILITIES FOR UNION OF EVENTS IN REAL-WORLD PROBLEMS

Ex 98: In a school survey, the probability of a student liking math is 0.6, and the probability of liking science is 0.4. The probability of a student liking both math and science is 0.25. What is the probability that a randomly selected student likes either math or science?

$$P(\text{Math or Science}) = \boxed{0.75}$$

Answer:

1) Let M be the event of liking math, and let S be the event of liking science. We are given:

$$P(M) = 0.6, \quad P(S) = 0.4, \quad \text{and} \quad P(M \text{ and } S) = 0.25.$$

2) The probability of a student liking either math or science is given by $P(M \text{ or } S)$.

3) Using the addition rule of probability:

$$P(M \text{ or } S) = P(M) + P(S) - P(M \text{ and } S).$$

4) Substituting the given values:

$$\begin{aligned} P(M \text{ or } S) &= 0.6 + 0.4 - 0.25 \\ &= 0.75. \end{aligned}$$

5) Therefore, the probability that a student likes either math or science is 0.75.

Ex 99: In a city survey, the probability of a resident using public transportation is 0.7, and the probability of using a bicycle is 0.3. The probability of a resident using both public transportation and a bicycle is 0.15.
What is the probability that a randomly selected resident uses either public transportation or a bicycle?

$$P(\text{Public Transport or Bicycle}) = \boxed{0.85}$$

Answer:

1) Let T be the event of using public transportation, and let B be the event of using a bicycle. We are given:

$$P(T) = 0.7, \quad P(B) = 0.3, \quad \text{and} \quad P(T \text{ and } B) = 0.15.$$

2) The probability of a resident using either public transportation or a bicycle is given by $P(T \text{ or } B)$.

3) Using the addition rule of probability:

$$P(T \text{ or } B) = P(T) + P(B) - P(T \text{ and } B).$$

4) Substituting the given values:

$$\begin{aligned} P(T \text{ or } B) &= 0.7 + 0.3 - 0.15 \\ &= 0.85. \end{aligned}$$

5) Therefore, the probability that a resident uses either public transportation or a bicycle is 0.85.

Ex 100: In a company survey, the probability of an employee enjoying team meetings is 0.5, and the probability of enjoying training sessions is 0.4. The probability of an employee enjoying both team meetings and training sessions is 0.2.
What is the probability that a randomly selected employee enjoys either team meetings or training sessions?

$$P(\text{Team Meetings or Training}) = \boxed{0.7}$$

Answer:

1) Let R be the event of enjoying team meetings, and let F be the event of enjoying training sessions. We are given:

$$P(R) = 0.5, \quad P(F) = 0.4, \quad \text{and} \quad P(R \text{ and } F) = 0.2.$$

2) The probability of an employee enjoying either team meetings or training sessions is given by $P(R \text{ or } F)$.

3) Using the addition rule of probability:

$$P(R \text{ or } F) = P(R) + P(F) - P(R \text{ and } F).$$

4) Substituting the given values:

$$\begin{aligned} P(R \text{ or } F) &= 0.5 + 0.4 - 0.2 \\ &= 0.7. \end{aligned}$$

5) Therefore, the probability that an employee enjoys either team meetings or training sessions is 0.7.

Ex 101: In a neighborhood survey, the probability of a household owning a dog is 0.5, and the probability of owning a cat is 0.35. The probability of a household owning both a dog and a cat is 0.2.
What is the probability that a randomly selected household owns either a dog or a cat?

$$P(\text{Dog or Cat}) = \boxed{0.65}$$

Answer:

1) Let D be the event of owning a dog, and let C be the event of owning a cat. We are given:

$$P(D) = 0.5, \quad P(C) = 0.35, \quad \text{and} \quad P(D \text{ and } C) = 0.2.$$

2) The probability of a household owning either a dog or a cat is given by $P(D \text{ or } C)$.

3) Using the addition rule of probability:

$$P(D \text{ or } C) = P(D) + P(C) - P(D \text{ and } C).$$

4) Substituting the given values:

$$\begin{aligned} P(D \text{ or } C) &= 0.5 + 0.35 - 0.2 \\ &= 0.65. \end{aligned}$$

5) Therefore, the probability that a household owns either a dog or a cat is 0.65.

B.3 EQUALLY LIKELY

B.3.1 DETERMINING THE PROBABILITY

Ex 102: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.
Find the probability that we choose a red ball.

$$P(\text{"choosing a red ball"}) = \frac{\boxed{2}}{\boxed{5}}$$

Answer:

- To find the probability of choosing a red ball, divide the number of red balls by the total number of balls.
- $\begin{aligned} P(\text{"choosing a red ball"}) &= \frac{\text{number of red balls}}{\text{total number of balls}} \\ &= \frac{2}{5} \end{aligned}$

Ex 103: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P(\text{"drawing an Ace"}) = \frac{\boxed{1}}{\boxed{13}}$$

Answer:

- To find the probability of drawing an Ace, divide the number of Aces by the total number of cards.
- There are 4 Aces in a standard deck of 52 playing cards.
- $\begin{aligned} P(\text{"drawing an Ace"}) &= \frac{\text{number of Aces}}{\text{total number of cards}} \\ &= \frac{4}{52} \\ &= \frac{1 \times \cancel{4}}{13 \times \cancel{4}} \\ &= \frac{1}{13} \end{aligned}$

Ex 104: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P(\text{"rolling an even number"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- To find the probability of rolling an even number, divide the number of even numbers by the total number of sides.
- There are 3 even numbers on a six-sided die (2, 4, and 6).
- $$\begin{aligned} P(\text{rolling an even number}) &= \frac{\text{number of even numbers}}{\text{total number of sides}} \\ &= \frac{3}{6} \\ &= \frac{1 \times \cancel{3}}{2 \times \cancel{3}} \\ &= \frac{1}{2} \end{aligned}$$

MCQ 105: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.
Find the probability that the selected fruit is an orange (simplify the fraction).

$$P(\text{"selecting an orange"}) = \frac{\boxed{1}}{\boxed{5}}$$

Answer:

- To find the probability of selecting an orange, divide the number of oranges by the total number of fruits.
- There are 2 oranges out of 10 fruits in total.
- $$\begin{aligned} P(\text{"selecting an orange"}) &= \frac{\text{number of oranges}}{\text{total number of fruits}} \\ &= \frac{2}{10} \\ &= \frac{1 \times \cancel{2}}{5 \times \cancel{2}} \\ &= \frac{1}{5} \end{aligned}$$

B.3.2 DETERMINING THE PROBABILITY IN MULTI-STEP RANDOM EXPERIMENTS

Ex 106: A coach selects two players at random from a group of four players, labeled A, B, C, and D, without replacement (once a player is chosen, they are not available for the next selection). The table below shows all possible outcomes for selecting Player 1 and Player 2, where an "X" indicates an impossible outcome due to the same player being selected twice.

Player 2 / Player 1	A	B	C	D
A	**X**	AB	AC	AD
B	BA	**X**	BC	BD
C	CA	CB	**X**	CD
D	DA	DB	DC	**X**

Calculate the probability that player C is selected as either Player 1 or Player 2, simplifying the fraction.

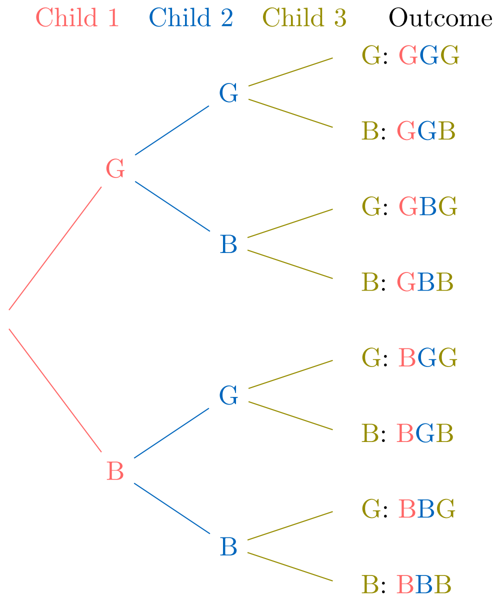
$$P(\text{"selecting player C"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- The sample space consists of all possible pairs of players: AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. This totals 12 outcomes.
- The event that player C is selected includes pairs where C is either Player 1 or Player 2: CA, CB, CD, AC, BC, DC. This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"C is selected"}) &= \frac{\text{number of outcomes where C is selected}}{\text{number of possible outcomes}} \\ &= \frac{6}{12} \\ &= \frac{1 \times 6}{2 \times 6} \\ &= \frac{1}{2} \end{aligned}$$

Ex 107: Parents have three children, each either a boy (B) or a girl (G). The tree diagram below illustrates all 8 possible gender outcomes for the three children.



Calculate the probability that the family has at least two girls, simplifying the fraction.

$$P(\text{"at least two girls"}) = \frac{\boxed{1}}{\boxed{2}}$$

Answer:

- The sample space consists of all possible gender outcomes for three children: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG. This totals 8 outcomes.
- The event that the family has at least two girls includes outcomes with two or three girls: BGG, GBG, GGB, GGG. This totals 4 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"at least 2 girls"}) &= \frac{\text{Number of outcomes with at least 2 gir}}{\text{Total number of outcomes}} \\ &= \frac{4}{8} \\ &= \frac{1 \times 4}{2 \times 4} \\ &= \frac{1}{2} \end{aligned}$$

Ex 108: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 7, simplifying the fraction.

$$P(\text{"sum is 7"}) = \frac{\boxed{1}}{\boxed{6}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for Die 1 times 6 outcomes for Die 2).
- The event that the sum is exactly 7 includes pairs: 16 ($1+6=7$), 25 ($2+5=7$), 34 ($3+4=7$), 43 ($4+3=7$), 52 ($5+2=7$), 61 ($6+1=7$). This totals 6 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 7"}) &= \frac{\text{Number of outcomes with sum 7}}{\text{Total number of outcomes}} \\ &= \frac{6}{36} \\ &= \frac{1\times 6}{6\times 6} \\ &= \frac{1}{6} \end{aligned}$$

Ex 109: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is greater than or equal to 11, simplifying the fraction.

$$P(\text{"sum } \geq 11\text{"}) = \frac{\boxed{1}}{\boxed{12}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is greater than or equal to 11 includes pairs: 56 ($5+6=11$), 65 ($6+5=11$), 66 ($6+6=12$). This totals 3 outcomes.
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum } \geq 11\text{"}) &= \frac{\text{Number of outcomes with sum } \geq 11}{\text{Total number of outcomes}} \\ &= \frac{3}{36} \\ &= \frac{1\times 3}{12\times 3} \\ &= \frac{1}{12} \end{aligned}$$

Ex 110: A pair of colored dice (one red and one blue) is rolled. Each die has faces numbered 1 to 6. The table below shows the possible outcomes for the two dice.

blue die / red die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Calculate the probability that the sum of the two dice is exactly 6 or 8, simplifying the fraction.

$$P(\text{"sum is 6 or 8"}) = \frac{\boxed{5}}{\boxed{18}}$$

Answer:

- The sample space consists of all possible pairs of outcomes from rolling two six-sided dice: 36 outcomes (6 outcomes for the red die times 6 outcomes for the blue die).
- The event that the sum is exactly 6 or 8 includes pairs: 15, 24, 33, 42, 51 (sum 6); 26, 35, 44, 53, 62 (sum 8). This totals 10 outcomes (5 for 6, 5 for 8).
- The probability is calculated as:

$$\begin{aligned} P(\text{"sum is 6 or 8"}) &= \frac{\text{Number of outcomes with sum 6 or 8}}{\text{Total number of outcomes}} \\ &= \frac{10}{36} \\ &= \frac{5\times 2}{18\times 2} \\ &= \frac{5}{18} \end{aligned}$$

B.3.3 CRACKING PROBABILITIES

Ex 111: In a race with 20 horses, you bet on 3 horses to finish first, second, and third in exact order (a "triple forecast"). What's the probability of winning your bet?

$$P(\text{"Winning"}) = \boxed{\frac{1}{6840}}$$

Answer: Let's break it down step by step:

- **Total possible outcomes**: For a triple forecast, the horses must finish in a specific order.

- You have 20 choices for the horse that finishes 1st.
- After that, 19 horses are left to choose from for 2nd place.
- Then, 18 horses remain for 3rd place.
- So, the total number of possible outcomes in the sample space is:

$$n(U) = 20 \times 19 \times 18.$$

- **Favorable outcomes**: Let E be the event of correctly predicting one specific triple forecast in order. Since you're betting on just one exact arrangement of 3 horses (e.g., Horse A first, Horse B second, Horse C third), there's only 1 way to get it right. Thus:

$$n(E) = 1.$$

- **Probability calculation**: The probability of winning the triple forecast is the ratio of favorable outcomes to total outcomes:

$$\begin{aligned} P(E) &= \frac{n(E)}{n(U)} \\ &= \frac{1}{20 \times 19 \times 18} \\ &= \frac{1}{6840}. \end{aligned}$$

So, your chance of winning is $\frac{1}{6840}$—pretty slim odds, but that's what makes it exciting!

Ex 112: In a race with 20 horses, you bet on 3 horses to finish in the top 3 positions (first, second, and third) in any order (a "trio" bet). What's the probability of winning your bet?

$$P(\text{Winning}) = \boxed{\frac{1}{1140}}$$

Answer: Let's break it down step by step:

- **Total possible outcomes**: For a trio bet, you need to select 3 horses out of 20 to finish in the top 3, regardless of their order. The number of ways to choose 3 horses from 20 is given by the combination formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, where order doesn't matter:

$$\binom{20}{3} = \frac{20!}{3!17!} = 1140$$

  So, there are 1140 possible sets of 3 horses that can finish in the top 3.

- **Favorable outcomes**: Let E be the event of winning your trio bet. Since you've chosen one specific set of 3 horses (e.g., Horses A, B, and C), and they must occupy the top 3 positions in any order, there's only 1 favorable set:

$$n(E) = 1$$

- **Probability calculation**: The probability of winning is the ratio of favorable outcomes to total outcomes:

$$\begin{aligned} P(E) &= \frac{1}{\binom{20}{3}} \\ &= \frac{1}{1140} \end{aligned}$$

So, your chance of winning is $\frac{1}{1140}$ — still a long shot, but better odds than predicting the exact order!

Ex 113: A combination lock uses a 3-digit code, where each digit is chosen from 0 to 9 (10 possible digits), and digits cannot repeat. You guess a specific 3-digit code to unlock it. What's the probability of guessing the correct code on your first try?

$$P(\text{Correct Guess}) = \boxed{\frac{1}{720}}$$

Answer: Let's break it down step by step:

- **Total possible outcomes**: For a 3-digit code with no repeating digits, the digits must be chosen in a specific order:
  - You have 10 choices (0–9) for the first digit.
  - After that, 9 digits remain for the second position (since no repeats are allowed).
  - Then, 8 digits remain for the third position.
  - So, the total number of possible codes is:

$$n(U) = 10 \times 9 \times 8 = 720$$

- **Favorable outcomes**: Let E be the event of guessing the correct 3-digit code in exact order. Since you're guessing one specific combination (e.g., 3-7-1), there's only 1 way to get it right:

$$n(E) = 1$$

- **Probability calculation**: The probability of guessing the correct code is the ratio of favorable outcomes to total outcomes:

$$\begin{aligned} P(E) &= \frac{n(E)}{n(U)} \\ &= \frac{1}{10 \times 9 \times 8} \\ &= \frac{1}{720} \end{aligned}$$

So, your chance of guessing the correct code is $\frac{1}{720}$ — a slim chance, but that's the thrill of the challenge!

Ex 114: The slogan associated with an American lottery is *You can't win if you don't play.* It's true, but what's the probability? In Lotto, you pick 6 numbers from a grid of 49. What is the probability of winning the jackpot (i.e., matching all 6 numbers)?

$$P(\text{"Winning"}) = \boxed{\frac{1}{13983816}}$$

Answer: Let's break it down step by step:

- **Total possible outcomes**: In Lotto, you pick 6 numbers from 49, and the order doesn't matter for the jackpot (you just need to match the 6 drawn numbers). The number of ways to choose 6 numbers from 49 is given by the combination formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$:

$$\binom{49}{6} = \frac{49!}{6!(43)!} = 13983816$$

  So, there are 13,983,816 possible sets of 6 numbers.

- **Favorable outcomes**: Let E be the event of winning the jackpot. Since you've picked one specific set of 6 numbers (e.g., 3, 12, 19, 27, 35, 42), and it must match the 6 drawn numbers exactly, there's only 1 favorable outcome:

$$n(E) = 1$$

- **Probability calculation**: The probability of winning is the ratio of favorable outcomes to total outcomes:

$$\begin{aligned} P(E) &= \frac{n(E)}{\binom{49}{6}} \\ &= \frac{1}{13983816} \end{aligned}$$

So, your chance of winning is $\frac{1}{13983816}$ — incredibly slim odds, proving the slogan's point: you've got to play to even have a shot!

Ex 115: The slogan associated with an American lottery is *You can't win if you don't play.* It's true, but what's the probability? In Lotto, you pick 6 numbers from a grid of 49. What is the probability of winning a small prize (i.e., matching exactly 5 out of the 6 numbers drawn)?

$$P(\text{"Small Prize"}) = \boxed{\frac{1}{54201}}$$

Answer: Let's break it down step by step:

- **Total possible outcomes**: In Lotto, you pick 6 numbers from 49, and the total number of possible sets of 6 numbers drawn (order doesn't matter) is given by the combination formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$:

$$\binom{49}{6} = 13983816$$

  So, there are 13,983,816 possible draws.

- **Favorable outcomes**: Let E be the event of winning a small prize by matching exactly 5 of your 6 chosen numbers with the 6 drawn numbers. Suppose your numbers are, e.g., $\{1, 2, 3, 4, 5, 6\}$. To match exactly 5:
  - Choose 5 of your 6 numbers to match: $\binom{6}{5} = 6$ ways (e.g., $\{1, 2, 3, 4, 5\}$, $\{1, 2, 3, 4, 6\}$, etc.).
  - The 6th number drawn must be one of the remaining 43 numbers (49 total minus your 6): $\binom{43}{1} = 43$ ways.
  - Total favorable outcomes: $\binom{6}{5} \times \binom{43}{1} = 6 \times 43 = 258$.

- **Probability calculation**: The probability is the ratio of favorable outcomes to total outcomes:

$$\begin{aligned} P(E) &= \frac{\binom{6}{5} \times \binom{43}{1}}{\binom{49}{6}} \\ &= \frac{6 \times 43}{13983816} \\ &= \frac{258}{13983816} \\ &= \frac{1}{54201} \quad (\text{simplified: } 13983816 \div 258 = 54201) \end{aligned}$$

So, your chance of winning a small prize is $\frac{1}{54201}$ — still tough, but a lot better than the jackpot!

Ex 116: In a class of 30 students, what is the probability that at least two students have the same birthday? Assume a year has 365 days and birthdays are equally likely. (round to 2 decimal places)

$$P(\text{"At least two share a birthday"}) = \boxed{0.71}$$

Answer: Let's break it down step by step:

- **Define the events**: Let E be the event that at least two students share a birthday. The complement, E', is the event that all 30 students have different birthdays.
- **Total possible outcomes**: Each student's birthday can be any of the 365 days.

$$365^{30}$$

- **Total outcomes for E'**:The number of ways all 30 students can have distinct birthdays:

$$365 \times 364 \times 363 \times \cdots \times 336 = \frac{365!}{335!}$$

- **Probability of the complement**: The probability that all 30 students have different birthdays is:

$$P(E') = \frac{\frac{365!}{335!}}{365^{30}}$$

  Using a calculator, this product is approximately:

$$P(E') \approx 0.293683$$

- **Probability calculation**: The probability of at least two sharing a birthday is the complement:

$$\begin{aligned} P(E) &= 1 - P(E') \\ &= 1 - 0.293683 \\ &\approx 0.706317 \\ &\approx 0.71 \quad (\text{rounded to 2 decimal places}) \end{aligned}$$

So, the probability that at least two students share a birthday is about 0.71 — surprisingly high for just 30 people!

C CONDITIONAL PROBABILITY

C.1 DEFINITION

C.1.1 EXPLORING PROBABILITIES WITH TWO-WAY TABLES

Ex 117: Consider a two-way table showing students preferences for loving math, categorized by gender:

	Loves Math	Not Love Math	Total
Girls	35	16	51
Boys	30	19	49
Total	65	35	100

A student is randomly selected from the class. Find the probability that the selected student is a girl.

$$P(\text{"Girl"}) = \boxed{\frac{51}{100}}$$

Answer:

- Total number of girls: 51.
- Total number of students: 100.
- Since all students are equally likely to be selected, the probability of picking a girl is:

$$\begin{aligned}P(\text{"Girl"}) &= \frac{\text{Number of girls}}{\text{Total number of students}}\\ &= \frac{51}{100}.\end{aligned}$$

Ex 118: Consider a two-way table showing students' preferences for participating in a school drama club, categorized by gender:

	Likes Drama Club	Dislikes Drama Club	Total
Girls	28	12	40
Boys	32	18	50
Total	60	30	90

A student is randomly selected from the group. Find the probability that the selected student likes the drama club.

$$P(\text{"Likes Drama Club"}) = \boxed{\frac{60}{90}}$$

Answer:

- Total number of students who like the drama club: 60.
- Total number of students: 90.
- Since all students are equally likely to be selected, the probability that a randomly selected student likes the drama club is:

$$\begin{aligned}P(\text{"Likes Drama Club"}) &= \frac{\text{Number of students who like drama club}}{\text{Total number of students}}\\ &= \frac{60}{90}\\ &= \frac{2}{3}\text{ (simplify)}.\end{aligned}$$

Ex 119: Consider a two-way table showing students' preferences for a short school trip, categorized by grade level:

	Likes Trip	Dislikes Trip	Total
Grade 9	15	5	20
Grade 10	25	15	40
Total	40	20	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9 and likes the trip.

$$P(\text{"Grade 9 and Likes Trip"}) = \boxed{\frac{15}{60}}$$

Answer:

- Number of Grade 9 students who like the trip: 15.
- Total number of students: 60.
- Since all students are equally likely to be selected, the probability that a randomly selected student is in Grade 9 and likes the trip is:

$$\begin{aligned}P(\text{"Grade 9 and Likes Trip"}) &= \frac{\text{Number of Grade 9 students who like the trip}}{\text{Total number of students}}\\ &= \frac{15}{60}\\ &= \frac{1}{4}\text{ (simplify)}.\end{aligned}$$

Ex 120: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student is a boy and likes science.

$$P(\text{"Boy and Likes Science"}) = \boxed{\frac{24}{60}}$$

Answer:

- Number of boys who like science: 24.
- Total number of students: 60.
- Since all students are equally likely to be selected, the probability that a randomly selected student is a boy and likes science is:

$$\begin{aligned}P(\text{"Boy and Likes Science"}) &= \frac{\text{Number of boys who like science}}{\text{Total number of students}}\\ &= \frac{24}{60}\\ &= \frac{2}{5}\text{ (simplify)}.\end{aligned}$$

Ex 121: Consider a two-way table showing students' preferences for science, categorized by gender:

	Likes Science	Dislikes Science	Total
Girls	18	12	30
Boys	24	6	30
Total	42	18	60

A student is randomly selected from the group. Find the probability that the selected student likes science, given that the student is a boy.

$$P(\text{"Likes Science"} \mid \text{"Boy"}) = \boxed{\frac{24}{30}}$$

Answer:

- Number of boys who like science: 24.
- Total number of boys: 30.
- Since all boys are equally likely to be selected, the conditional probability that a student likes science, given that the student is a boy, is:

$$\begin{aligned}P(\text{"Likes Science"} \mid \text{"Boy"}) &= \frac{\text{Number of boys who like science}}{\text{Total number of boys}}\\ &= \frac{24}{30}\\ &= \frac{4}{5}\text{ (simplify)}.\end{aligned}$$

Ex 122: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student likes music, given that the student is in Grade 9.

$$P(\text{"Likes Music"} \mid \text{"Grade 9"}) = \boxed{\frac{20}{30}}$$

Answer:

- Number of Grade 9 students who like music: 20.
- Total number of Grade 9 students: 30.
- Since all Grade 9 students are equally likely to be selected, the conditional probability that a student likes music, given that the student is in Grade 9, is:

$$\begin{aligned} P(\text{"Likes Music"} \mid \text{"Grade 9"}) &= \frac{\text{Number of Grade 9 students who like music}}{\text{Total number of Grade 9 students}} \\ &= \frac{20}{30} \\ &= \frac{2}{3} \text{ (simplify).} \end{aligned}$$

Ex 123: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes art.

$$P(\text{"Grade 9"} \mid \text{"Likes Art"}) = \boxed{\frac{12}{30}}$$

Answer:

- Number of Grade 9 students who like art: 12.
- Total number of students who like art: 30.
- Since all students who like art are equally likely to be selected, the conditional probability that a student is in Grade 9, given that the student likes art, is:

$$\begin{aligned} P(\text{"Grade 9"} \mid \text{"Likes Art"}) &= \frac{\text{Number of Grade 9 students who like art}}{\text{Total number of students who like art}} \\ &= \frac{12}{30} \\ &= \frac{2}{5} \text{ (simplify).} \end{aligned}$$

Ex 124: Consider a two-way table showing students' preferences for art, categorized by grade level:

	Likes Art	Dislikes Art	Total
Grade 9	12	8	20
Grade 10	18	12	30
Total	30	20	50

A student is randomly selected from the group. Find the probability that the selected student likes art, given that the student is in Grade 9.

$$P(\text{"Likes Art"} \mid \text{"Grade 9"}) = \boxed{\frac{12}{20}}$$

Answer:

- Number of Grade 9 students who like art: 12.
- Total number of Grade 9 students: 20.
- Since all Grade 9 students are equally likely to be selected, the conditional probability that a student likes art, given that the student is in Grade 9, is:

$$\begin{aligned} P(\text{"Likes Art"} \mid \text{"Grade 9"}) &= \frac{\text{Number of Grade 9 students who like art}}{\text{Total number of Grade 9 students}} \\ &= \frac{12}{20} \\ &= \frac{3}{5} \text{ (simplify).} \end{aligned}$$

Ex 125: Consider a two-way table showing students' preferences for music, categorized by grade level:

	Likes Music	Dislikes Music	Total
Grade 9	20	10	30
Grade 10	15	15	30
Total	35	25	60

A student is randomly selected from the group. Find the probability that the selected student is in Grade 9, given that the student likes music.

$$P(\text{"Grade 9"} \mid \text{"Likes Music"}) = \boxed{\frac{20}{35}}$$

Answer:

- Number of Grade 9 students who like music: 20.
- Total number of students who like music: 35.
- Since all students who like music are equally likely to be selected, the conditional probability that a student is in Grade 9, given that the student likes music, is:

$$\begin{aligned} P(\text{"Grade 9"} \mid \text{"Likes Music"}) &= \frac{\text{Number of Grade 9 students who like music}}{\text{Total number of students who like music}} \\ &= \frac{20}{35} \\ &= \frac{4}{7} \text{ (simplify).} \end{aligned}$$

C.1.2 CALCULATING CONDITIONAL PROBABILITIES

Ex 126: Given that $P(E \text{ and } F) = 0.1$ and $P(F) = 0.4$, find :

$$P(E \mid F) = \boxed{0.25}$$

Answer: Applying the conditional probability formula,

$$\begin{aligned}P(E \mid F) &= \frac{P(E \text{ and } F)}{P(F)} \\ &= \frac{0.1}{0.4} \\ &= 0.25\end{aligned}$$

Ex 127: Given that $P(A \text{ and } B) = 0.15$ and $P(B) = 0.5$, find :

$$P(A \mid B) = \boxed{0.3}$$

Answer: Applying the conditional probability formula,

$$\begin{aligned}P(A \mid B) &= \frac{P(A \text{ and } B)}{P(B)} \\ &= \frac{0.15}{0.5} \\ &= 0.3\end{aligned}$$

Ex 128: Given that $P(X \text{ and } Y) = 0.12$ and $P(Y) = 0.3$, find :

$$P(X \mid Y) = \boxed{0.4}$$

Answer: Applying the conditional probability formula,

$$\begin{aligned}P(X \mid Y) &= \frac{P(X \text{ and } Y)}{P(Y)} \\ &= \frac{0.12}{0.3} \\ &= 0.4\end{aligned}$$

Ex 129: Given that $P(M \text{ and } N) = 0.25$ and $P(N) = 0.8$, find :

$$P(M \mid N) = \boxed{0.3125}$$

Answer: Applying the conditional probability formula,

$$\begin{aligned}P(M \mid N) &= \frac{P(M \text{ and } N)}{P(N)} \\ &= \frac{0.25}{0.8} \\ &= 0.3125\end{aligned}$$

C.1.3 CALCULATING CONDITIONAL PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 130: In a certain town, it is found that 30% of the families own a pet dog, and 15% of the families own both a pet dog and a cat. If a randomly selected family owns a dog, what is the probability that they also own a cat?

$$P(\text{"Own a cat"} \mid \text{"Own a dog"}) = \boxed{0.5}$$

Answer: To find the conditional probability, use the formula:

$$\begin{aligned}P(\text{"Own a cat"} \mid \text{"Own a dog"}) &= \frac{P(\text{"Own a dog and a cat"})}{P(\text{"Own a dog"})} \\ &= \frac{0.15}{0.3} \\ &= 0.5\end{aligned}$$

This means that if a family owns a dog, there is a 50% chance they also own a cat.

Ex 131: In a school, it is found that 40% of the students enjoy reading books, and 20% of the students enjoy both reading books and watching movies. If a randomly selected student enjoys reading books, what is the probability that they also enjoy watching movies?

$$P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) = \boxed{0.5}$$

Answer: To find the conditional probability, use the formula:

$$\begin{aligned}P(\text{"Enjoy movies"} \mid \text{"Enjoy books"}) &= \frac{P(\text{"Enjoy books and movies"})}{P(\text{"Enjoy books"})} \\ &= \frac{0.20}{0.4} \\ &= 0.5\end{aligned}$$

This means that if a student enjoys reading books, there is a 50% chance they also enjoy watching movies.

Ex 132: In a neighborhood, it is found that 60% of the households own a car, and 24% of the households own both a car and a bicycle. If a randomly selected household owns a car, what is the probability that they also own a bicycle?

$$P(\text{"Own a bicycle"} \mid \text{"Own a car"}) = \boxed{0.4}$$

Answer: To find the conditional probability, use the formula:

$$\begin{aligned}P(\text{"Own a bicycle"} \mid \text{"Own a car"}) &= \frac{P(\text{"Own a car and a bicycle"})}{P(\text{"Own a car"})} \\ &= \frac{0.24}{0.6} \\ &= 0.4\end{aligned}$$

This means that if a household owns a car, there is a 40% chance they also own a bicycle.

Ex 133: In a club, it is found that 25% of the members play soccer, and 10% of the members play both soccer and basketball. If a randomly selected member plays soccer, what is the probability that they also play basketball?

$$P(\text{"Play basketball"} \mid \text{"Play soccer"}) = \boxed{0.4}$$

Answer: To find the conditional probability, use the formula:

$$\begin{aligned}P(\text{"Play basketball"} \mid \text{"Play soccer"}) &= \frac{P(\text{"Play soccer and basketball"})}{P(\text{"Play soccer"})} \\ &= \frac{0.10}{0.25} \\ &= 0.4\end{aligned}$$

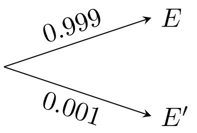
This means that if a member plays soccer, there is a 40% chance they also play basketball.

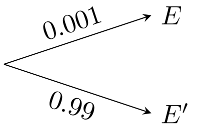
C.2 CONDITIONAL PROBABILITY TREE DIAGRAMS

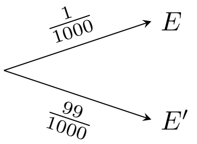
C.2.1 CONDITIONAL PROBABILITY TREE DIAGRAMS: LEVEL 1

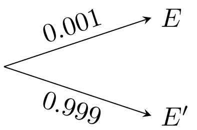
MCQ 134: Consider a rare disease that affects 1 in every 1 000 people.
Let E be the event of having the disease.
A person is randomly selected from the population.

Choose the correct probability tree diagram:

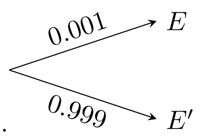
☐ 

☐ 

☐ 

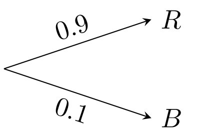
☒ 

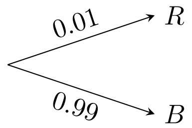
Answer:

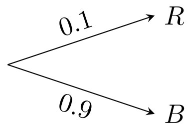
- The probability tree is constructed as follows: the first two branches represent whether a person has the disease (E) or does **not** have the disease (E').
- The probability of having the disease, $P(E)$, is $\frac{1}{1\,000} = 0.001$.
- The probability of **not** having the disease, $P(E')$, is $1 - 0.001 = 0.999$.
- So, the correct probability tree diagram is: 

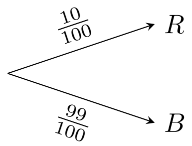
MCQ 135: Imagine a bag containing 100 marbles, of which 10 are red (R) and the rest are blue (B). A marble is randomly drawn from the bag.
Let R be the event of drawing a red marble.

Choose the correct probability tree diagram:

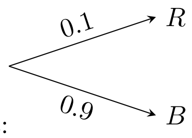
☐ 

☐ 

☒ 

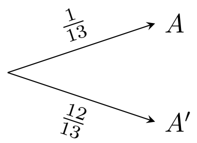
☐ 

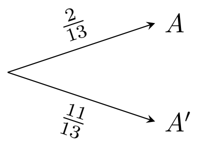
Answer:

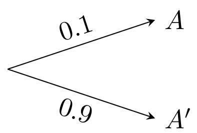
- The probability tree is constructed as follows: the first level shows the probability of drawing a red (R) or blue (B) marble.
- The probability of drawing a red marble, $P(R)$, is $\frac{10}{100} = 0.1$.
- The probability of drawing a blue marble, $P(B)$, is $1 - 0.1 = 0.9$.
- Therefore, the correct probability tree diagram is: 

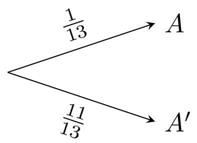
MCQ 136: A deck of cards contains 52 cards, of which 4 are aces. A card is randomly drawn from the deck.
Let A be the event of drawing an ace.

Choose the correct probability tree diagram:

☒ 

☐ 

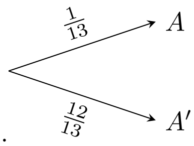
☐ 

☐ 

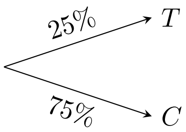
Answer:

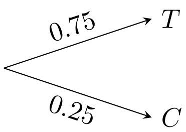
- The probability tree is constructed as follows: the first level shows the probability of drawing an ace (A) or not an ace (A').

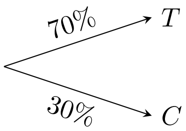
- The probability of drawing an ace, $P(A)$, is $\frac{4}{52} = \frac{1}{13}$.
- The probability of not drawing an ace, $P(A')$, is $1 - \frac{1}{13} = \frac{12}{13}$.
- Therefore, the correct probability tree diagram is:

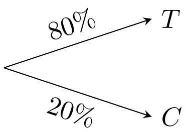


MCQ 137: A survey shows that 75% of a town's population prefers tea (T) over coffee (C). A person is randomly selected from the town's population.
Let T be the event of a person preferring tea.
Choose the correct probability tree diagram:

☐ 

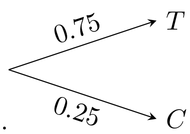
☒ 

☐ 

☐ 

Answer:

- The probability tree is constructed as follows: the first level shows the probability of a person preferring tea (T) or coffee (C).
- The probability of preferring tea, $P(T)$, is $75\% = 0.75$.
- The probability of preferring coffee, $P(C)$, is $1 - 0.75 = 0.25$.
- Therefore, the correct probability tree diagram is:

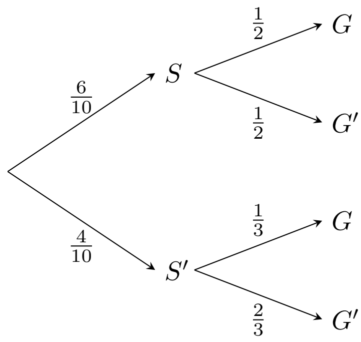


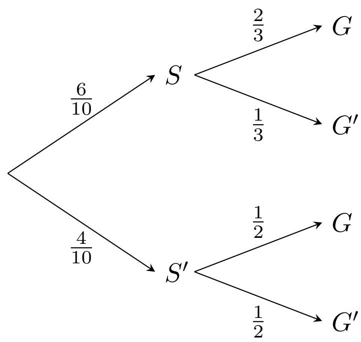
C.2.2 CONDITIONAL PROBABILITY TREE DIAGRAMS: LEVEL 2

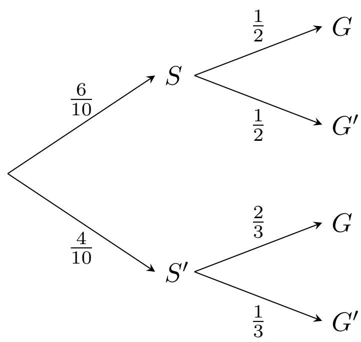
MCQ 138: The probability that Sam is coaching a game is $\frac{6}{10}$, and the probability that Alex is coaching is $\frac{4}{10}$.

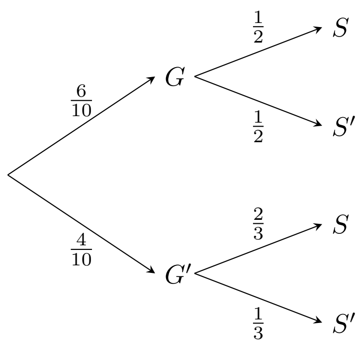
- With Coach Sam, the probability of being Goalkeeper is $\frac{1}{2}$.
- With Coach Alex, the probability of being Goalkeeper is $\frac{2}{3}$.

Let S be the event that Sam is the coach.
Let G be the event of being the goalkeeper.
Choose the correct probability tree diagram:

☐ 

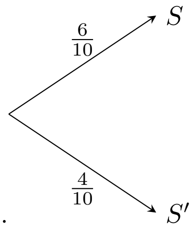
☐ 

☒ 

☐ 

Answer:

- Let S' be the event that Alex is the coach and G' be the event of not being the goalkeeper.
- The probability that Sam is coaching, denoted as $P(S)$, is $\frac{6}{10}$.
- The probability that Alex is coaching, denoted as $P(S')$, is $\frac{4}{10}$.

- So, the first step is: 

- From the Sam and Alex branches, we have two events: being the goalkeeper (G) and not being the goalkeeper (G').

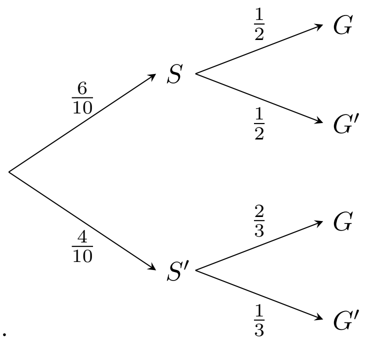
- The probability of being the goalkeeper when Sam is the coach is given as $P(G \mid S) = \frac{1}{2}$.

- The probability of not being the goalkeeper when Sam is the coach is $P(G' \mid S) = 1 - \frac{1}{2} = \frac{1}{2}$.

- The probability of being the goalkeeper when Alex is the coach is given as $P(G \mid S') = \frac{2}{3}$.

- The probability of not being the goalkeeper when Alex is the coach is $P(G' \mid S') = 1 - \frac{2}{3} = \frac{1}{3}$.

- So, the correct probability tree diagram



is:

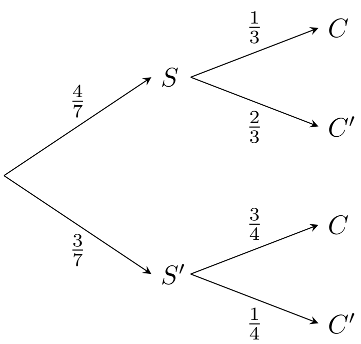
MCQ 139: In a city, the probability of a randomly chosen day being sunny is $\frac{4}{7}$, and the probability of it being rainy is $\frac{3}{7}$.

- On a sunny day, the probability that the park is crowded is $\frac{2}{3}$.

- On a rainy day, the probability that the park is crowded is $\frac{1}{4}$.

Let S be the event that it's a sunny day.
Let C be the event of the park being crowded.

Choose the correct probability tree diagram:



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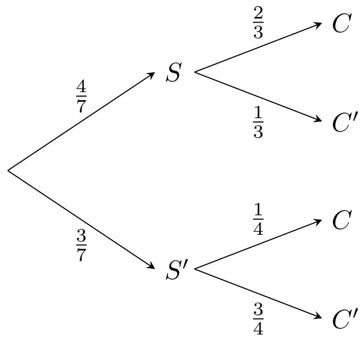

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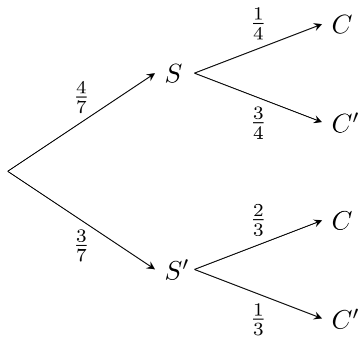

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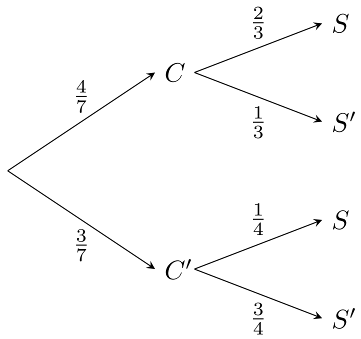


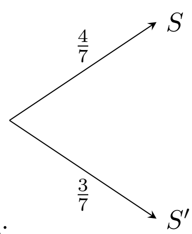
☐

Answer:

- Let S' be the event that it's a rainy day and C' be the event that the park is not crowded.

- The probability that it's a sunny day, denoted as $P(S)$, is $\frac{4}{7}$.

- The probability that it's a rainy day, denoted as $P(S')$, is $\frac{3}{7}$.

- So, the first step is: 

- From the sunny and rainy branches, we have two events: the park being crowded (C) and the park not being crowded (C').

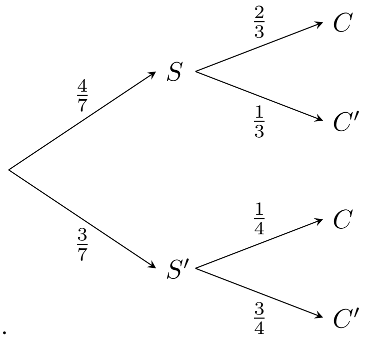
- The probability that the park is crowded on a sunny day is given as $P(C \mid S) = \frac{2}{3}$.

- The probability that the park is not crowded on a sunny day is $P(C' \mid S) = 1 - \frac{2}{3} = \frac{1}{3}$.

- The probability that the park is crowded on a rainy day is given as $P(C \mid S') = \frac{1}{4}$.

- The probability that the park is not crowded on a rainy day is $P(C' \mid S') = 1 - \frac{1}{4} = \frac{3}{4}$.

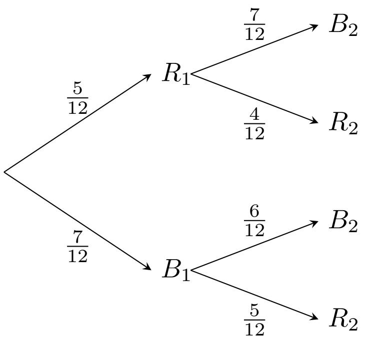
- So, the correct probability tree diagram



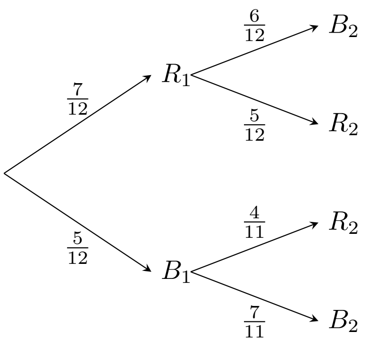
is:

MCQ 140: A bag contains 5 red (R) and 7 blue (B) marbles. One marble is drawn and not replaced, and then a second marble is drawn.
Let R_1 be the event of drawing a red marble first, and let B_1 be the event of drawing a blue marble first.
Let R_2 be the event of drawing a red marble second, and let B_2 be the event of drawing a blue marble second.

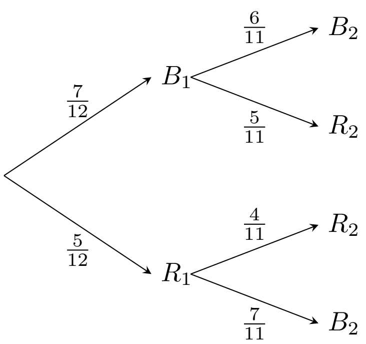
Choose the correct probability tree diagram:



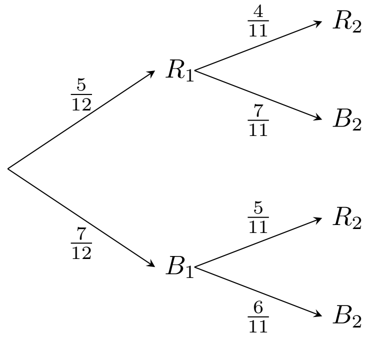
☐



☐



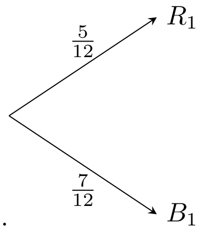
☐



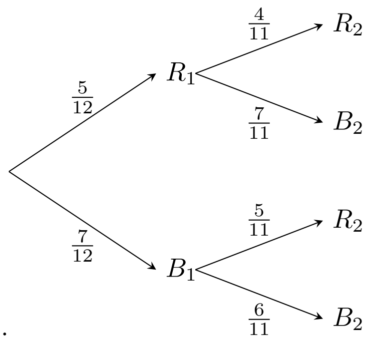
☒

Answer:

- The total number of marbles is $5 + 7 = 12$.
- The probability of drawing a red marble first, denoted as $P(R_1)$, is $\frac{5}{12}$.
- The probability of drawing a blue marble first, denoted as $P(B_1)$, is $\frac{7}{12}$.



- So, the first step is:
- From the red and blue branches, we have two possibilities for the second draw: drawing a red marble (R_2) or a blue marble (B_2).
- After drawing a red marble first (leaving 4 red and 7 blue, total 11), the probability of drawing a red marble second is $P(R_2 \mid R_1) = \frac{4}{11}$.
- The probability of drawing a blue marble second after a red marble first is $P(B_2 \mid R_1) = \frac{7}{11}$.
- After drawing a blue marble first (leaving 5 red and 6 blue, total 11), the probability of drawing a red marble second is $P(R_2 \mid B_1) = \frac{5}{11}$.
- The probability of drawing a blue marble second after a blue marble first is $P(B_2 \mid B_1) = \frac{6}{11}$.
- So, the correct probability tree diagram



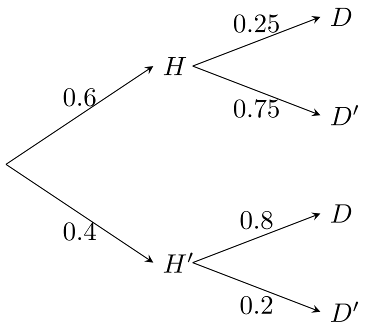
is:

MCQ 141: In a town, the probability that a randomly chosen morning has heavy traffic is 0.6, and the probability that it has light traffic is 0.4.

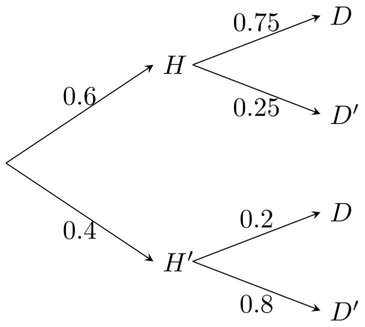
- On a morning with heavy traffic, the probability of a bus being delayed is 0.75.
- On a morning with light traffic, the probability of a bus being delayed is 0.2.

Let H be the event that the morning has heavy traffic.
Let D be the event that the bus is delayed.

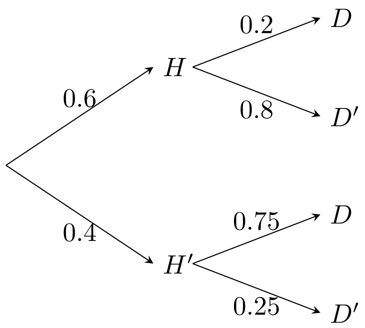
Choose the correct probability tree diagram:



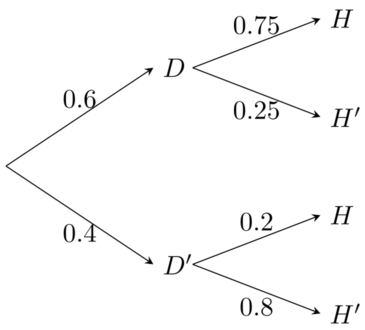
☐



☒



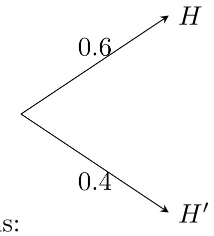
☐



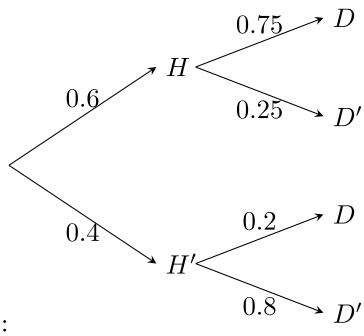
☐

Answer:

- Let H' be the event that the morning has light traffic and D' be the event that the bus is not delayed.
- The probability that the morning has heavy traffic, denoted as $P(H)$, is 0.6.
- The probability that the morning has light traffic, denoted as $P(H')$, is 0.4.



- So, the first step is:
- From the heavy and light traffic branches, we have two events: the bus being delayed (D) and the bus not being delayed (D').
- The probability that the bus is delayed on a morning with heavy traffic is given as $P(D \mid H) = 0.75$.
- The probability that the bus is not delayed on a morning with heavy traffic is $P(D' \mid H) = 1 - 0.75 = 0.25$.
- The probability that the bus is delayed on a morning with light traffic is given as $P(D \mid H') = 0.2$.
- The probability that the bus is not delayed on a morning with light traffic is $P(D' \mid H') = 1 - 0.2 = 0.8$.
- So, the correct probability tree diagram

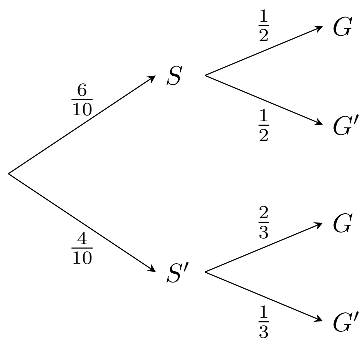


is:

C.3 JOINT PROBABILITY: $P(E$ and $F)$

C.3.1 CALCULATING JOINT PROBABILITIES WITH TREES

Ex 142: For this probability tree diagram:

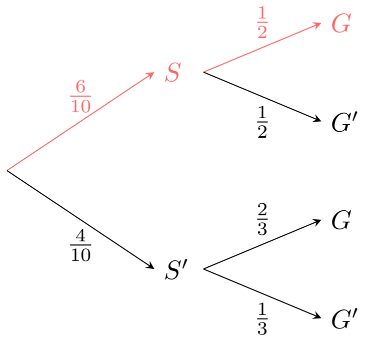


find the probability :

$$P(S \text{ and } G) = \boxed{\frac{3}{10}}$$

Answer:

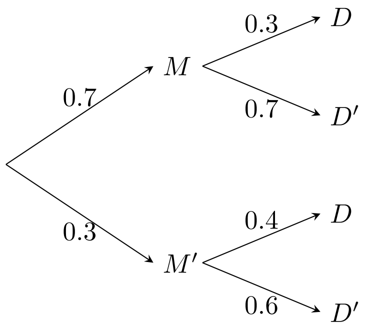
- **Path:** S to G (highlighted):



- **Calculate:**

$$\begin{aligned} P(S \text{ and } G) &= P(S) \times P(G \mid S) \\ &= \frac{6}{10} \times \frac{1}{2} \\ &= \frac{6}{20} \\ &= \frac{3}{10} \end{aligned}$$

Ex 143: For this probability tree diagram:

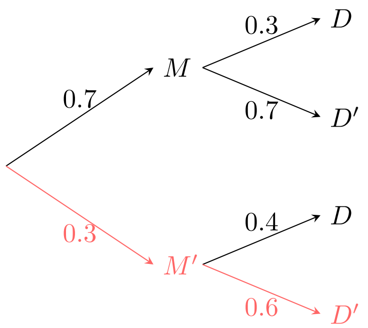


find the probability :

$$P(M' \text{ and } D') = \boxed{0.18}$$

Answer:

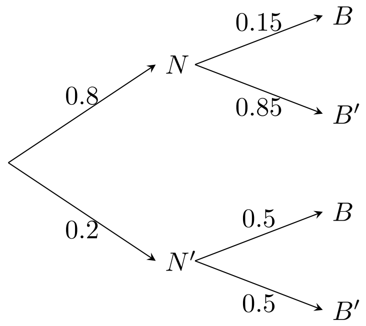
- **Path:** M' to D' (highlighted):



- **Calculate:**

$$\begin{aligned} P(M' \text{ and } D') &= P(M') \times P(D' \mid M') \\ &= 0.3 \times 0.6 \\ &= 0.18 \end{aligned}$$

Ex 144: For this probability tree diagram:

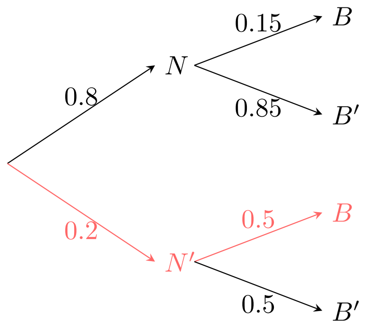


find the probability :

$$P(N' \text{ and } B) = \boxed{0.1}$$

Answer:

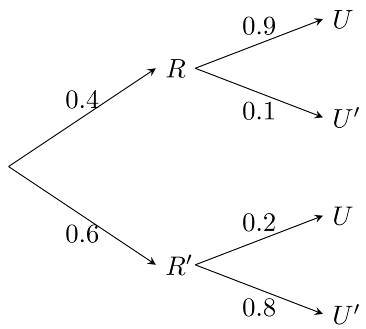
- **Path:** N' to B (highlighted):



- **Calculate:**

$$\begin{aligned} P(N' \text{ and } B) &= P(N') \times P(B \mid N') \\ &= 0.2 \times 0.5 \\ &= 0.1 \end{aligned}$$

Ex 145: For this probability tree diagram:

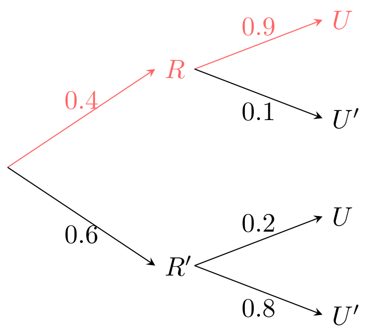


find the probability :

$$P(R \text{ and } U) = \boxed{0.36}$$

Answer:

- **Path:** R to U (highlighted):



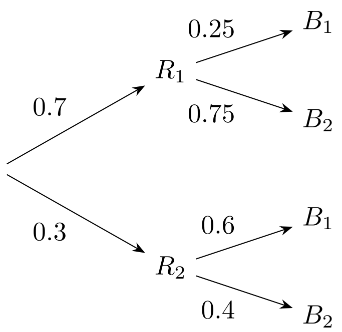
- **Calculate:**

$$\begin{aligned} P(R \text{ and } U) &= P(R) \times P(U \mid R) \\ &= 0.4 \times 0.9 \\ &= 0.36 \end{aligned}$$

C.4 LAW OF TOTAL PROBABILITY

C.4.1 CALCULATING PROBABILITIES WITH TREES

Ex 146: For this probability tree:

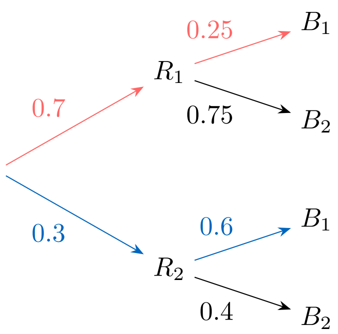


calculate the probability :

$$P(B_1) = \boxed{0.355}$$

Answer:

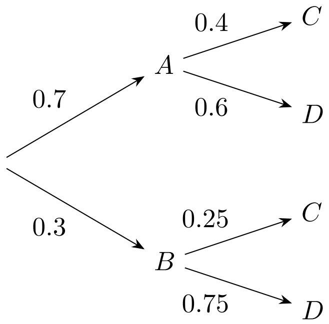
1. Paths to B_1:



2. Calculate:

$$\begin{aligned} P(B_1) &= P(R_1) \times P(B_1 \mid R_1) + P(R_2) \times P(B_1 \mid R_2) \\ &= 0.7 \times 0.25 + 0.3 \times 0.6 \\ &= 0.175 + 0.18 \\ &= 0.355 \end{aligned}$$

Ex 147: For this probability tree:

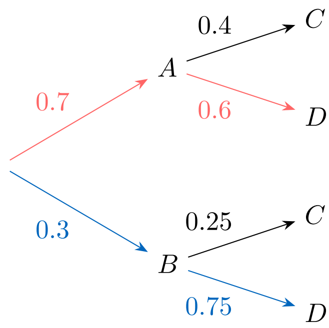


calculate the probability :

$$P(D) = \boxed{0.645}$$

Answer:

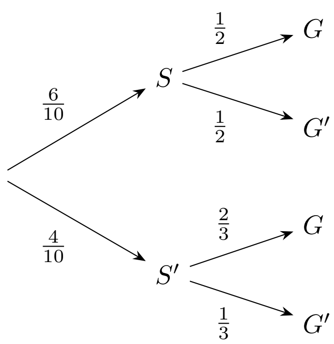
1. Paths to D:



2. Calculate:

$$\begin{aligned} P(D) &= P(A) \times P(D \mid A) + P(B) \times P(D \mid B) \\ &= 0.7 \times 0.6 + 0.3 \times 0.75 \\ &= 0.42 + 0.225 \\ &= 0.645 \end{aligned}$$

Ex 148: For this probability tree,

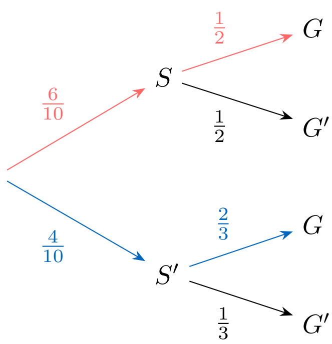


calculate the probability :

$$P(G) = \boxed{\frac{17}{30}}$$

Answer:

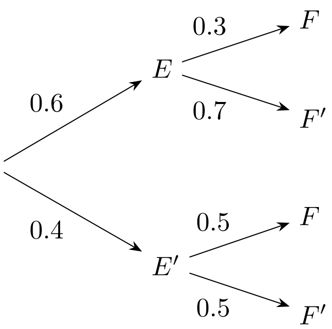
1. Paths to G:



2. Calculate:

$$\begin{aligned}P(G) &= P(S)\times P(G\mid S)+P(S')\times P(G\mid S')\\&=\frac{6}{10}\times\frac{1}{2}+\frac{4}{10}\times\frac{2}{3}\\&=\frac{6}{20}+\frac{8}{30}\\&=\frac{9}{30}+\frac{8}{30}\\&=\frac{17}{30}.\end{aligned}$$

Ex 149: For this probability tree:

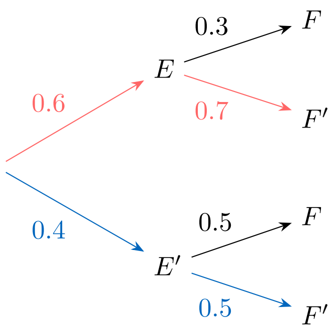


calculate the probability :

$$P(F') = \boxed{0.62}$$

Answer:

1. Paths to F':



2. Calculate:

$$\begin{aligned}P(F') &= P(E)\times P(F'\mid E)+P(E')\times P(F'\mid E')\\&=0.6\times 0.7+0.4\times 0.5\\&=0.42+0.2\\&=0.62\end{aligned}$$

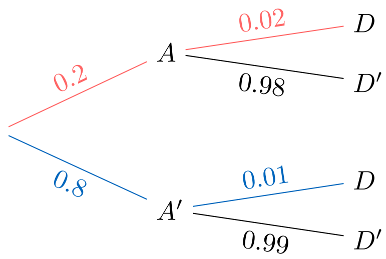
C.4.2 CALCULATING PROBABILITIES IN REAL-WORLD PROBLEMS

Ex 150: A company produces two types of parts: A and B. 20% of parts are type A and 80% are type B. The probability that a part is defective given type A is 0.02, and the probability that a part is defective given type B is 0.01.
Find the probability that a part is defective :

$$P(\text{"Defective"}) = \boxed{0.012}$$

Answer:

- **Define the events:**
  - A: Part is type A.
  - D: Part is defective.
- **Define the probabilities:**
  - $P(A) = 20\% = \frac{20}{100} = 0.2$, and $P(A') = 80\% = \frac{80}{100} = 0.8$.
  - $P(D|A) = 0.02$, and $P(D|A') = 0.01$.
- **Paths to D:**



- **Law of total probability:**

$$\begin{aligned}P(D) &= P(A)\times P(D|A)+P(A')\times P(D|A')\\&=0.2\times 0.02+0.8\times 0.01\\&=0.004+0.008\\&=0.012\end{aligned}$$

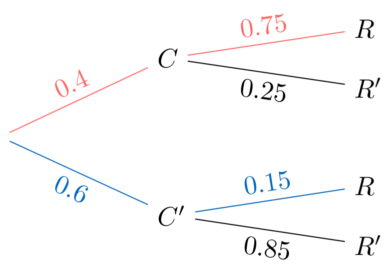
- The probability that a part is defective is 0.012.

Ex 151: A meteorologist observes cloud conditions to predict rain. On a given day, 40% of the time the sky is cloudy, and 60% of the time it is clear. The probability of rain given a cloudy sky is 0.75, and the probability of rain given a clear sky is 0.15.
Find the probability that it rains :

$$P(\text{Rain}) = \boxed{0.39}$$

Answer:

- **Define the events:**
  - C: Sky is cloudy.
  - R: It rains.
- **Define the probabilities:**
  - $P(C) = 40\% = \frac{40}{100} = 0.4$, and $P(C') = 60\% = \frac{60}{100} = 0.6$.
  - $P(R|C) = 0.75$, and $P(R|C') = 0.15$.
- **Paths to R:**



- **Law of total probability:**

$$\begin{aligned} P(R) &= P(C) \times P(R|C) + P(C') \times P(R|C') \\ &= 0.4 \times 0.75 + 0.6 \times 0.15 \\ &= 0.3 + 0.09 \\ &= 0.39 \end{aligned}$$

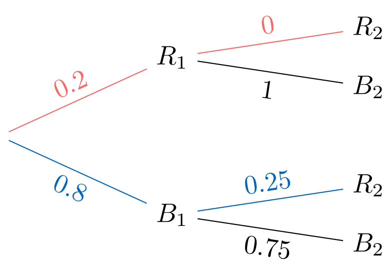
- The probability that it rains is 0.39.

Ex 152: An urn contains 1 red ball and 4 blue balls. A first ball is drawn without replacement. Then a second ball is drawn from the remaining balls.
Find the probability that the second ball drawn is red :

$$P(R_2) = \boxed{0.2}$$

Answer:

- **Define the events:**
  - R_1: First ball drawn is red.
  - B_1: First ball drawn is blue.
  - R_2: Second ball drawn is red.
- **Define the probabilities:**
  - Total balls = 5 (1 red + 4 blue).
  - $P(R_1) = \frac{1}{5} = 0.2$, and $P(B_1) = \frac{4}{5} = 0.8$.
  - $P(R_2|R_1) = \frac{0}{4} = 0$ (if first is red, 0 red remain out of 4).
  - $P(R_2|B_1) = \frac{1}{4} = 0.25$ (if first is blue, 1 red remains out of 4).
- **Paths to R_2:**



- **Law of total probability:**

$$\begin{aligned} P(R_2) &= P(R_1) \times P(R_2|R_1) + P(B_1) \times P(R_2|B_1) \\ &= 0.2 \times 0 + 0.8 \times 0.25 \\ &= 0 + 0.2 \\ &= 0.2 \end{aligned}$$

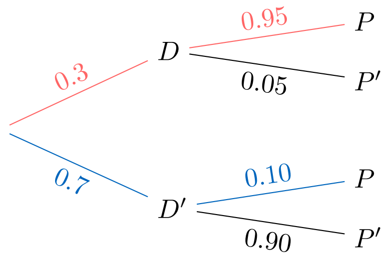
- The probability that the second ball drawn is red is 0.2.

Ex 153: A population is tested for a disease. 30% of the population has the disease. The probability that a test is positive given the person has the disease is 0.95, and the probability that a test is positive given the person does not have the disease is 0.10.
Find the probability that a test is positive :

$$P(\text{Positive}) = \boxed{0.355}$$

Answer:

- **Define the events:**
  - D: Person has the disease.
  - P: Test is positive.
- **Define the probabilities:**
  - $P(D) = 30\% = \frac{30}{100} = 0.3$, and $P(D') = 1 - 0.3 = 0.7$.
  - $P(P|D) = 0.95$, and $P(P|D') = 0.10$.
- **Paths to P:**



- **Law of total probability:**

$$\begin{aligned} P(P) &= P(D) \times P(P|D) + P(D') \times P(P|D') \\ &= 0.3 \times 0.95 + 0.7 \times 0.10 \\ &= 0.285 + 0.07 \\ &= 0.355 \end{aligned}$$

- The probability that a test is positive is 0.355.

C.5 BAYES' THEOREM

C.5.1 UNVEILING THE HIDDEN CAUSE: BAYES' THEOREM IN RARE EVENT DETECTION

Ex 154: Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

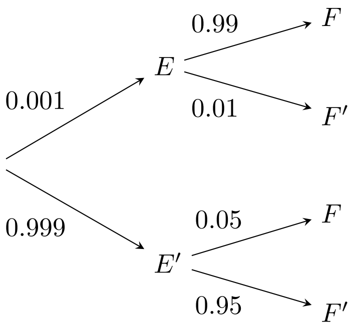
Find the probability in percent that a person actually has the disease if their test result is positive (round to 1 decimal place):

$$P(\text{Disease} \mid \text{Test positive}) = \boxed{1.9}\%$$

Answer:

- **Define the events:**
  - E: The person has the disease.
  - F: The test result is positive.
- **Define the probabilities:**
  - $P(E) = \frac{1}{1000} = 0.001$, thus $P(E') = 1 - 0.001 = 0.999$.
  - $P(F|E) = 0.99$, hence $P(F'|E) = 1 - 0.99 = 0.01$.
  - $P(F'|E') = 0.95$, hence $P(F|E') = 1 - 0.95 = 0.05$.

- **Draw the probability tree:**



- **Calculate** $P(F)$**:**

$$\begin{aligned} P(F) &= P(E) \times P(F|E) + P(E') \times P(F|E') \\ &= (0.001 \times 0.99) + (0.999 \times 0.05) \\ &= 0.00099 + 0.04995 \\ &= 0.05094 \end{aligned}$$

- **Calculate** $P(E \mid F)$**:**Using Bayes' theorem:

$$\begin{aligned} P(E \mid F) &= \frac{P(E) \times P(F|E)}{P(F)} \\ &= \frac{0.001 \times 0.99}{0.05094} \\ &\approx 0.01943 \\ &\approx 1.943\% \quad \text{(convert to percent)} \\ &\approx 1.9\% \quad \text{(round to 1 decimal place)} \end{aligned}$$

- The probability that a person actually has the disease, given a positive test result, is approximately 1.9%. This highlights a key issue with screening tests for rare conditions: even highly accurate tests can yield a significant proportion of false positives.

Ex 155: Consider a rare disease that affects approximately 1 in every 1,000 people. A medical test developed for detecting this disease has the following characteristics:

- Sensitivity: If a person has the disease, the test correctly returns a positive result 99% of the time.
- Specificity: If a person does not have the disease, the test correctly returns a negative result 95% of the time.

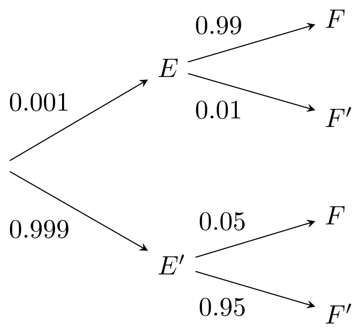
Find the probability in percent that a person actually has the disease if their test result is positive (round at 1 decimal place):

$$P(\text{"Disease"} \mid \text{"Test positive"}) = \boxed{1.9}\%$$

Answer:

- **Define the events**:
  - Event E: The person has the disease.
  - Event F: The test result is positive.
- **Define the probabilities**:
  - $P(E) = \frac{1}{1000} = 0.001$, thus $P(E') = 1 - 0.001 = 0.999$.
  - $P(F \mid E) = 0.99$, hence $P(F' \mid E) = 1 - 0.99 = 0.01$.
  - $P(F' \mid E') = 0.95$, hence $P(F \mid E') = 1 - 0.95 = 0.05$.

- **Draw the probability tree**:



- **Calculate** $P(F)$:

$$\begin{aligned} P(F) &= P(E)P(F \mid E) + P(E')P(F \mid E') \\ &= (0.001 \times 0.99) + (0.999 \times 0.05) \\ &= 0.00099 + 0.04995 \\ &= 0.05094. \end{aligned}$$

- **Calculate** $P(E \mid F)$: Using Bayes' theorem, we have:

$$\begin{aligned} P(E \mid F) &= \frac{P(E)P(F \mid E)}{P(F)} \\ &= \frac{0.00099}{0.05094} \\ &\approx 0.019 \end{aligned}$$

- Therefore, the probability that a person actually has the disease, given a positive test result, is approximately 1.94%. This underscores a key issue with screening tests for rare conditions: even highly accurate tests can yield a significant proportion of false positives.

Ex 156: Consider a rare alien signal that is present in approximately 1 out of every 10,000 radio scans conducted by a space observatory. A signal detector has the following characteristics:

- Sensitivity: If an alien signal is present, the detector correctly identifies it as positive 98% of the time.
- Specificity: If no alien signal is present, the detector correctly identifies it as negative 96% of the time.

Find the probability in percent that an alien signal is actually present if the detector returns a positive result (round to 1 decimal place):

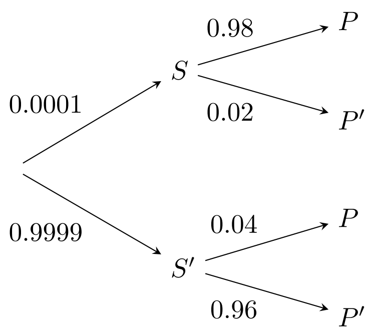
$$P(\text{Signal} \mid \text{Positive}) = \boxed{0.2}\%$$

Answer:

- **Define the events:**
  - S: An alien signal is present.
  - P: The detector returns a positive result.
- **Define the probabilities:**
  - $P(S) = \frac{1}{10000} = 0.0001$, thus $P(S') = 1 - 0.0001 = 0.9999$.
  - $P(P|S) = 0.98$, hence $P(P'|S) = 1 - 0.98 = 0.02$.

– $P(P'|S') = 0.96$, hence $P(P|S') = 1 - 0.96 = 0.04$.

- **Draw the probability tree:**



- **Calculate $P(P)$:**

$$\begin{aligned}P(P) &= P(S) \times P(P|S) + P(S') \times P(P|S') \\ &= (0.0001 \times 0.98) + (0.9999 \times 0.04) \\ &= 0.000098 + 0.039996 \\ &= 0.040094\end{aligned}$$

- **Calculate $P(S \mid P)$:** Using Bayes' theorem:

$$\begin{aligned}P(S \mid P) &= \frac{P(S) \times P(P|S)}{P(P)} \\ &= \frac{0.0001 \times 0.98}{0.040094} \\ &\approx 0.002445 \\ &\approx 0.2445\% \quad \text{(convert to percent)} \\ &\approx 0.2\% \quad \text{(round to 1 decimal place)}\end{aligned}$$

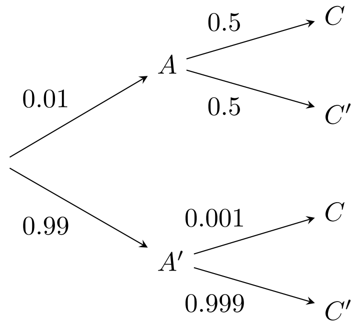
- The probability that an alien signal is actually present, given a positive detector result, is approximately 0.2%. This demonstrates how rare events, even with a highly accurate detector, can lead to many false positives.

Ex 157: In a city, 1 out of every 100 drivers drives with alcohol in their system. The probability of having an accident given that a driver has alcohol is $1/2$, and the probability of having an accident given that a driver has no alcohol is $1/1000$. Find the probability in percent that a driver has alcohol in their system if they have had an accident (round to 1 decimal place):

$$P(\text{Alcohol} \mid \text{Accident}) = \boxed{83.5}\%$$

Answer:

- **Define the events:**
  – A: The driver has alcohol in their system.
  – C: The driver has an accident.
- **Define the probabilities:**
  – $P(A) = \frac{1}{100} = 0.01$, thus $P(A') = 1 - 0.01 = 0.99$.
  – $P(C|A) = \frac{1}{2} = 0.5$, hence $P(C'|A) = 1 - 0.5 = 0.5$.
  – $P(C|A') = \frac{1}{1000} = 0.001$, hence $P(C'|A') = 1 - 0.001 = 0.999$.
- **Draw the probability tree:**



- **Calculate $P(C)$:**

$$\begin{aligned}P(C) &= P(A) \times P(C|A) + P(A') \times P(C|A') \\ &= (0.01 \times 0.5) + (0.99 \times 0.001) \\ &= 0.005 + 0.00099 \\ &= 0.00599\end{aligned}$$

- **Calculate $P(A \mid C)$:** Using Bayes' theorem:

$$\begin{aligned}P(A \mid C) &= \frac{P(A) \times P(C|A)}{P(C)} \\ &= \frac{0.01 \times 0.5}{0.00599} \\ &\approx 0.8347 \\ &\approx 83.47\% \quad \text{(convert to percent)} \\ &\approx 83.5\% \quad \text{(round to 1 decimal place)}\end{aligned}$$

- The probability that a driver has alcohol in their system, given they had an accident, is approximately 83.5%. This shows that while alcohol significantly increases accident risk, most accidents still occur among sober drivers due to their larger population.

Ex 158: In a futuristic society, 1 out of every 500 devices contains a rare quantum crystal as its power source. A crystal detector has been invented with the following properties:

- Sensitivity: If a device has a quantum crystal, the detector correctly registers it as active 90% of the time.
- Specificity: If a device does not have a quantum crystal, the detector correctly registers it as inactive 97% of the time.

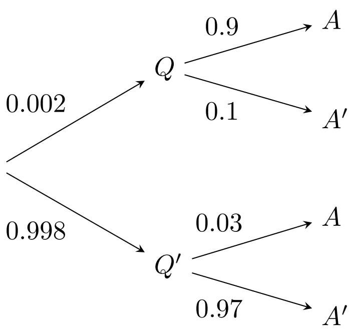
Find the probability in percent that a device actually has a quantum crystal if the detector registers it as active (round to 1 decimal place):

$$P(\text{Crystal} \mid \text{Active}) = \boxed{5.7}\%$$

Answer:

- **Define the events:**
  – Q: The device has a quantum crystal.
  – A: The detector registers the device as active.
- **Define the probabilities:**
  – $P(Q) = \frac{1}{500} = 0.002$, thus $P(Q') = 1 - 0.002 = 0.998$.
  – $P(A|Q) = 0.9$, hence $P(A'|Q) = 1 - 0.9 = 0.1$.
  – $P(A'|Q') = 0.97$, hence $P(A|Q') = 1 - 0.97 = 0.03$.

- **Draw the probability tree:**



- **Calculate** $P(A)$**:**

$$\begin{aligned}P(A) &= P(Q) \times P(A|Q) + P(Q') \times P(A|Q') \\ &= (0.002 \times 0.9) + (0.998 \times 0.03) \\ &= 0.0018 + 0.02994 \\ &= 0.03174\end{aligned}$$

- **Calculate** $P(Q \mid A)$**:**Using Bayes' theorem:

$$\begin{aligned}P(Q \mid A) &= \frac{P(Q) \times P(A|Q)}{P(A)} \\ &= \frac{0.002 \times 0.9}{0.03174} \\ &\approx 0.056710 \\ &\approx 5.6710\% \quad \text{(convert to percent)} \\ &\approx 5.7\% \quad \text{(round to 1 decimal place)}\end{aligned}$$

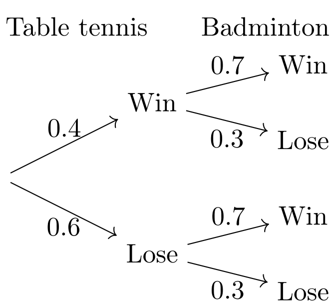
- The probability that a device actually has a quantum crystal, given the detector registers it as active, is approximately 5.9%. This reflects how the rarity of quantum crystals leads to a low probability of a true positive despite a reliable detector.

D PROBABILITY OF INDEPENDENT EVENTS

D.1 DEFINITION

D.1.1 READING PROBABILITY TREE

Ex 159: Niamh plays a game of table tennis on Saturday and a game of badminton on Sunday. The probability tree is represented:



Calculate the probability that Niamh wins both games.

$$P(\text{"Win both"}) = \boxed{0.28}$$

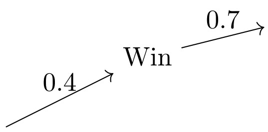
Answer: Let's figure this out step by step:

- Since the table tennis game on Saturday and the badminton game on Sunday are independent events—winning one doesn't affect the other—we multiply the probabilities along the path where Niamh wins both.

- The probability of winning the table tennis game is 0.4 (from the first branch).

- The probability of winning the badminton game, given a win in table tennis, is 0.7 (from the second branch).

- Here's the relevant path on the tree diagram:

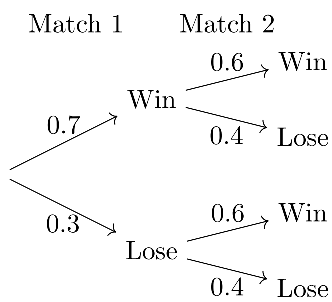


- Now, calculate the probability of winning both games:

$$\begin{aligned}P(\text{"Win both"}) &= P(\text{"Win table tennis"}) \times \\ &\quad P(\text{"Win badminton"}) \\ &= 0.4 \times 0.7 \\ &= 0.28\end{aligned}$$

- So, the probability that Niamh wins both games is 0.28.

Ex 160: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.



Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \boxed{0.12}$$

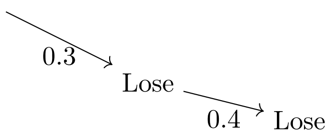
Answer: Let's figure this out step by step:

- Since losing the first match and losing the second match are independent events—one loss doesn't affect the other—we multiply their individual probabilities of losing to find the chance of both losing.

- The probability that Sam loses the first match is $1 - 0.7 = 0.3$.

- The probability that Sam loses the second match is $1 - 0.6 = 0.4$.

- Here's the relevant path on the tree diagram (following the bottom branches):

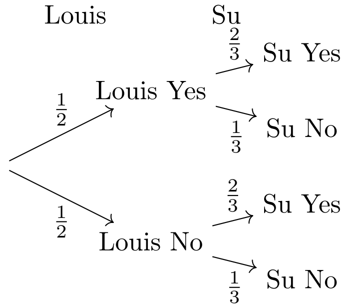


- Now, calculate the probability that both lose:

$$\begin{aligned} P(\text{"Both lose"}) &= P(\text{"Lose first match"}) \times P(\text{"Lose second match"}) \\ &= 0.3 \times 0.4 \\ &= 0.12 \end{aligned}$$

- So, the probability that Sam loses both the first and second matches is 0.12.

Ex 161: A party is happening this weekend! Louis might come with a probability of $\frac{1}{2}$, and Su might come with a probability of $\frac{2}{3}$.

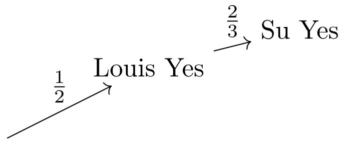


Calculate the probability that both Louis and Su come to the party (simplify the fraction).

$$P(\text{"Both come"}) = \frac{\boxed{1}}{\boxed{3}}$$

Answer: Let's figure this out step by step:

- Since Louis coming to the party and Su coming to the party are independent events—Louis's decision doesn't affect Su's—we multiply their individual probabilities to find the chance of both happening.
- The probability that Louis comes is $\frac{1}{2}$.
- The probability that Su comes is $\frac{2}{3}$.
- Here's the relevant path on the tree diagram:

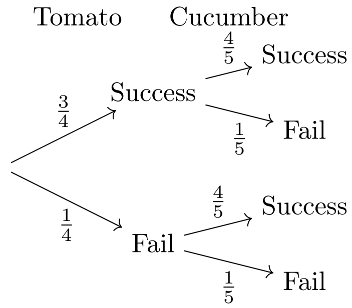


- Now, calculate the probability that both come:

$$\begin{aligned} P(\text{"Both come"}) &= P(\text{"Louis comes"}) \times P(\text{"Su comes"}) \\ &= \frac{1}{2} \times \frac{2}{3} \\ &= \frac{1 \times 2}{2 \times 3} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

- The fraction $\frac{2}{6}$ simplifies to $\frac{1}{3}$ by dividing both numerator and denominator by 2.
- So, the probability that both Louis and Su come to the party is $\frac{1}{3}$.

Ex 162: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.

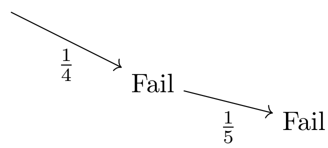


Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\boxed{1}}{\boxed{20}}$$

Answer: Let's figure this out step by step:

- Since the failure of the tomato plants and the failure of the cucumber plants are independent events—one's failure doesn't affect the other—we multiply their individual probabilities of failing to find the chance of both failing.
- The probability that the tomato plants fail is $1 - \frac{3}{4} = \frac{1}{4}$.
- The probability that the cucumber plants fail is $1 - \frac{4}{5} = \frac{1}{5}$.
- Here's the relevant path on the tree diagram (following the



bottom branches):

- Now, calculate the probability that both fail:

$$\begin{aligned} P(\text{"Both fail"}) &= P(\text{"Tomato fail"}) \times P(\text{"Cucumber fail"}) \\ &= \frac{1}{4} \times \frac{1}{5} \\ &= \frac{1 \times 1}{4 \times 5} \\ &= \frac{1}{20} \end{aligned}$$

- So, the probability that both the tomato plants and the cucumber plants fail to grow is $\frac{1}{20}$.

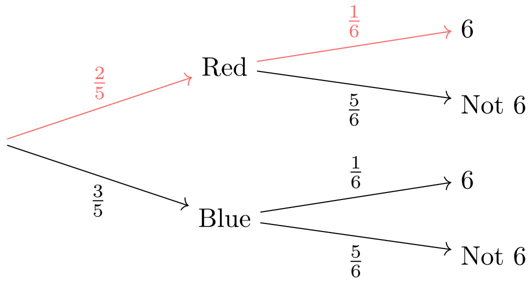
D.1.2 FINDING THE PROBABILITY WITH INDEPENDENT EVENTS

Ex 163: Imagine you're at a carnival playing a game. You pick a ball from a bag containing 2 red balls and 3 blue balls, then roll a fair six-sided die. Find the probability of choosing a red ball **and** rolling a 6 (simplify the fraction).

$$P(\text{"Red" and "6"}) = \frac{\boxed{1}}{\boxed{15}}$$

Answer: Let's figure this out step by step:

- Since picking a ball and rolling a die are independent events—one doesn't change the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of choosing a red ball is $\frac{2}{5}$ because there are 2 red balls out of 5 total balls.
- The probability of rolling a 6 on a six-sided die is $\frac{1}{6}$ since there's only one 6 out of six possible outcomes.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Red" and "6"}) &= P(\text{"Red"}) \times P(\text{"6"}) \\ &= \frac{2}{5} \times \frac{1}{6} \\ &= \frac{2}{30} \\ &= \frac{1 \times \cancel{2}}{15 \times \cancel{2}} \\ &= \frac{1}{15} \end{aligned}$$

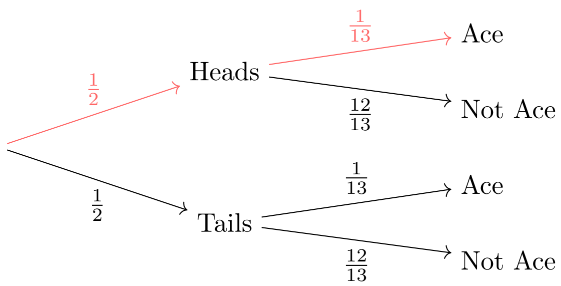
- So, the probability is $\frac{1}{15}$.

Ex 164: Imagine you're at a carnival playing another game. You flip a fair coin and draw a card from a standard deck of 52 playing cards. Find the probability of getting heads **and** drawing an Ace (simplify the fraction).

$$P(\text{"Heads" and "Ace"}) = \frac{\boxed{1}}{\boxed{26}}$$

Answer: Let's figure this out step by step:

- Since flipping a coin and drawing a card are independent events—one doesn't change the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of getting heads is $\frac{1}{2}$ because a fair coin has two equally likely outcomes: heads or tails.
- The probability of drawing an Ace from a standard deck is $\frac{4}{52} = \frac{1}{13}$, since there are 4 Aces (one for each suit: hearts, diamonds, clubs, spades) in 52 cards.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Heads" and "Ace"}) &= P(\text{"Heads"}) \times P(\text{"Ace"}) \\ &= \frac{1}{2} \times \frac{1}{13} \\ &= \frac{1}{26} \end{aligned}$$

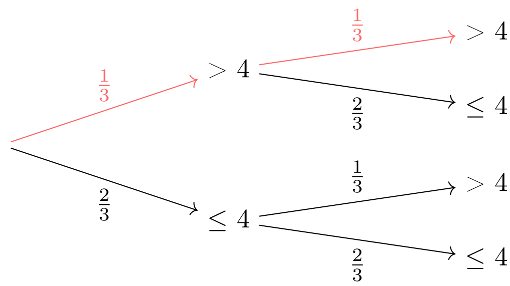
- The fraction $\frac{1}{26}$ is already in its simplest form, as 1 and 26 have no common factors other than 1.
- So, the probability is $\frac{1}{26}$.

Ex 165: Imagine you're at a carnival playing a dice game. You roll a fair six-sided die two times in a row. Find the probability of getting a number greater than 4 (a 5 or 6) on both rolls (simplify the fraction).

$$P(\text{"Number} > 4\text{" and "Number} > 4\text{"}) = \frac{\boxed{1}}{\boxed{9}}$$

Answer: Let's figure this out step by step:

- Since rolling the die the first time and rolling it the second time are independent events—each roll doesn't affect the other—we multiply their individual probabilities to find the chance of both happening.
- The probability of getting a number greater than 4 (a 5 or 6) on one roll is $\frac{2}{6} = \frac{1}{3}$, because there are 2 favorable outcomes (5 or 6) out of 6 possible outcomes (1, 2, 3, 4, 5, 6).
- Since it's the same die rolled twice, the probability is the same for the second roll: $\frac{1}{3}$.
- Here's the probability tree showing all possibilities:



- Now, calculate the probability of both events:

$$\begin{aligned} P(\text{"Nbr 1} > 4\text{" and "Nbr 2} > 4\text{"}) &= P(\text{"Nbr 1} > 4\text{"}) \times P(\text{"Nb} \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

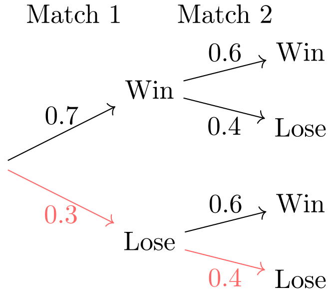
- The fraction $\frac{1}{9}$ is already in its simplest form, as 1 and 9 have no common factors other than 1.
- So, the probability is $\frac{1}{9}$.

Ex 166: Sam is playing an online multiplayer game. The probability that Sam wins their first match is 0.7, and the probability that Sam wins their second match is 0.6.
Calculate the probability that Sam loses both the first and second matches.

$$P(\text{"Both lose"}) = \boxed{0.12}$$

Answer: Let's figure this out step by step:

- Since losing the first match and losing the second match are independent events—one loss doesn't affect the other—we multiply their individual probabilities of losing to find the chance of both losing.
- The probability that Sam loses the first match is $1 - 0.7 = 0.3$.
- The probability that Sam loses the second match is $1 - 0.6 = 0.4$.
- The probability tree is:



- Now, calculate the probability that both lose:

$$\begin{aligned} P(\text{"Both lose"}) &= P(\text{"Lose first match"}) \times P(\text{"Lose second match"}) \\ &= 0.3 \times 0.4 \\ &= 0.12 \end{aligned}$$

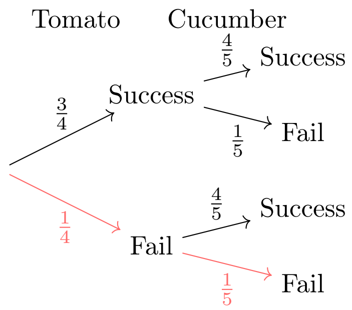
- So, the probability that Sam loses both the first and second matches is 0.12.

Ex 167: Mia takes care of her garden. The probability that her tomato plants grow is $\frac{3}{4}$, and the probability that her cucumber plants grow is $\frac{4}{5}$.
Calculate the probability that both the tomato plants and the cucumber plants fail to grow (simplify the fraction).

$$P(\text{"Both fail"}) = \frac{\boxed{1}}{\boxed{20}}$$

Answer: Let's figure this out step by step:

- Since the failure of the tomato plants and the failure of the cucumber plants are independent events—one's failure doesn't affect the other—we multiply their individual probabilities of failing to find the chance of both failing.
- The probability that the tomato plants fail is $1 - \frac{3}{4} = \frac{1}{4}$.
- The probability that the cucumber plants fail is $1 - \frac{4}{5} = \frac{1}{5}$.
- The probability tree is



- Now, calculate the probability that both fail:

$$\begin{aligned} P(\text{"Both fail"}) &= P(\text{"Tomato fail"}) \times P(\text{"Cucumber fail"}) \\ &= \frac{1}{4} \times \frac{1}{5} \\ &= \frac{1 \times 1}{4 \times 5} \\ &= \frac{1}{20} \end{aligned}$$

- So, the probability that both the tomato plants and the cucumber plants fail to grow is $\frac{1}{20}$.