PROBABILITY

Ever wondered if it'll rain tomorrow or if you'll win a game? That's probability! It's a math way to guess how likely things are to happen. Like when the weather app says there's a 90% chance of rain, that's probability telling us it's very likely. What else do you think we could use probability for?

A ALGEBRA OF EVENTS

A.1 SAMPLE SPACE

Have you ever flipped a coin and wondered if it'd land on heads or tails? Or rolled a die and guessed what number you'd get? These are random experiments—things we do where the result isn't certain until it happens.

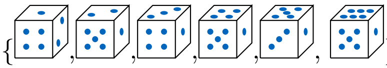
> Definition **Outcome**
>
> An **outcome** is one possible result of a random experiment.

> Definition **Sample Space**
>
> The **sample space** is the list of all possible outcomes of a random experiment.

Ex: What's the sample space when you flip a coin?

Answer: It's {Heads, Tails} = {}, or just {H, T} for short.

Ex: What's the sample space when you roll a six-sided die?

Answer: It's {1, 2, 3, 4, 5, 6} = {}.

A.2 EVENTS

> Definition **Event**
>
> An **event** is a set of outcomes from all possible outcomes.

In math, we use capital letters like E to name events. So, we might say E is the event "it's sunny tomorrow."

Ex: You roll a die. Let E be the event of rolling an even number. What's E?

Answer: The sample space is $\{1, 2, 3, 4, 5, 6\}$, and $E = \{2, 4, 6\}$ because those are the even numbers.

A.3 COMPLEMENTARY EVENT

> Definition **Complementary Event**
>
> The **complementary event** of an event E is everything in the sample space that isn't in E. We call it E' (E-prime).

Ex: You roll a die, and E is rolling an even number. What's E'?

Answer: $E = \{2, 4, 6\}$, so E' is all the other numbers: $E' = \{1, 3, 5\}$. These are the odd numbers!

A.4 MULTI-STEP RANDOM EXPERIMENTS

A multi-step random experiment is one that involves a sequence of actions, where each action (or step) has its own set of possible outcomes. In our example of tossing two coins, the experiment is multi-step because it involves two separate coin tosses:

- The first coin toss (step 1) can result in Heads (H) or Tails (T).
- The second coin toss (step 2) can also result in Heads (H) or Tails (T).

The overall outcome of the experiment is given by combining the outcomes of each step. For instance, the outcome HT means that the first coin landed on Heads and the second coin landed on Tails.Using different representations—such as grids, tables, tree diagrams, or simply listing the outcomes—helps us organize and visualize all the possible combinations that result from the multiple steps.

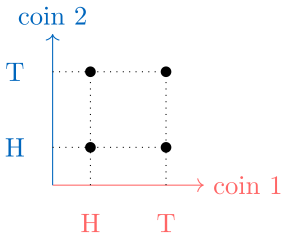
> Method **Representations of Multi-Step Random Experiment**
>
> When an experiment involves more than one step, we can represent the sample space (the set of all possible outcomes) in several ways:

- using a **grid** (to visually map combinations along two axes),
- using a **table** (to organize outcomes in rows and columns),
- using a **tree diagram** (to follow each sequential step), or
- by **listing** all possible outcomes.

Ex: For the random experiment of tossing two coins, display the sample space by:

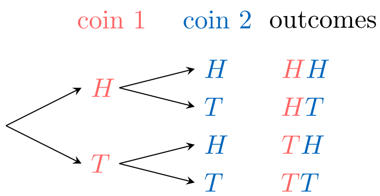
1. using a grid
2. using a table
3. using a tree diagram
4. listing all possible outcomes

Answer:

1. 

2.

coin 2 / coin 1	H	T
H	HH	HT
T	TH	TT

3. 

4. $\{HH, HT, TH, TT\}$

B PROBABILITY

B.1 DEFINITION

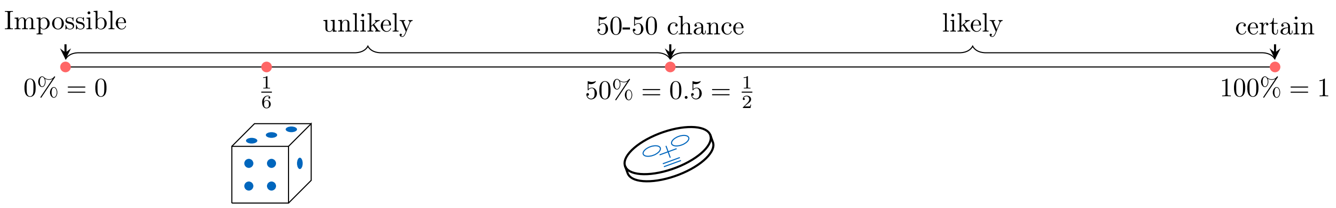
Definition Probability

The **probability** of an event E, written $P(E)$, tells us how likely E is to happen. It's always a number between 0 and 1:

- 0 means impossible (0% chance).
- 1 means certain (100% chance).

For example, the probability the sun rises tomorrow morning is 1!

We can show probability as a fraction, decimal, or percentage. Like a 50-50 chance is $\frac{1}{2}$, 0.5, or 50%. Picture it on a number line: 0 is impossible, 1 is certain, and 0.5 is right in the middle!



Ex: What's the probability the sun rises tomorrow morning?

Answer: It's 1 (or 100%) because it's certain!

B.2 EQUALLY LIKELY

Sometimes, every outcome in an experiment has the same chance—like flipping a fair coin or rolling a fair die. We call these equally likely outcomes.

Definition Equally Likely

If all outcomes are equally likely, the probability of an event, E, is given by

$$P(E) = \frac{\text{number of outcomes in the event}}{\text{total number of all possible outcomes}}$$

Ex: What's the probability of rolling an even number with a fair six-sided die?

Answer:

- Sample space = $\{1, 2, 3, 4, 5, 6\}$ (6 outcomes).
- $E = \{2, 4, 6\}$ (3 outcomes).
- $$\begin{aligned} P(E) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$
  .

So, there's a $\frac{1}{2}$ chance (or 50%) of rolling an even number!

B.3 COMPLEMENT RULE

Proposition Complement rule

For any event E and its complementary event E',

$$P(E) + P(E') = 1 \quad \text{or} \quad P(E') = 1 - P(E).$$

Ex: Farid has a 0.8 (80%) chance of finishing his homework on time tonight (event E). What's the chance he doesn't finish on time (E')?

Answer: As $P(E) = 0.8$, by the Complement rule, we get

$$\begin{aligned} P(E') &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

So, there's a 20% chance he doesn't finish on time!

B.4 PROBABILITY OF INDEPENDENT EVENTS

Independent events are like you choosing an ice cream flavor and your friend picking a movie—your choice doesn't affect theirs, and theirs doesn't affect yours! For example, flipping a coin and rolling a die at the same time: whether you get heads or tails has no effect on what number you roll. Let's explore this idea together!

Definition Independent Events

Two events, A and B, are **independent** if the chance of both happening is just the product of their individual chances. Mathematically:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Ex: Imagine you do two totally separate actions:

1. Flipping a fair coin (heads or tails).
2. Rolling a fair six-sided die (1, 2, 3, 4, 5, or 6).

What's the probability of getting tails **and** rolling a number greater than 4 (like a 5 or 6)?

Answer: Let's break it down:

- These events are independent, so we multiply their probabilities.
- For the coin: You've got two options—heads or tails—and they're equally likely. So, $P(\text{"tails"}) = \frac{1}{2}$.
- For the die: There are six sides, and "greater than 4" means 5 or 6. That's 2 out of 6 possibilities, so $P(\text{"number} > 4\text{"}) = \frac{2}{6} = \frac{1}{3}$.
- Now, combine them:

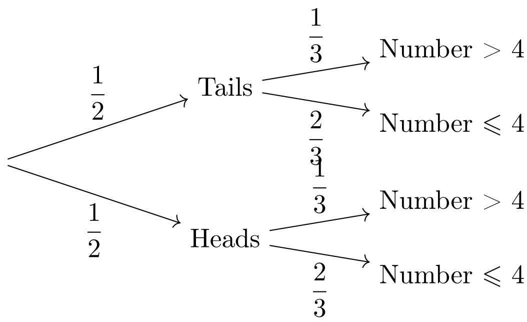
$$\begin{aligned} P(\text{"tails" and "number} > 4\text{"}) &= P(\text{"tails"}) \times P(\text{"number} > 4\text{"}) \\ &= \frac{1}{2} \times \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

- Result: There's a $\frac{1}{6}$ chance of landing tails and rolling a 5 or 6.

Method **Finding the probability of two independent events using a probability tree diagram**

Let's use a probability tree for our coin flip and die roll:

1. **Draw the Tree:** Start with two branches for the coin: "Heads" and "Tails." Then, from each, draw two more branches for the die: "Number > 4" (5 or 6) and "Number ⩽ 4" (1, 2, 3, or 4).
2. **Add Probabilities:** Label each branch. The coin gives $\frac{1}{2}$ for "Tails" and $\frac{1}{2}$ for "Heads." For the die, "Number > 4" is $\frac{1}{3}$ (2 out of 6), and "Number ⩽ 4" is $\frac{2}{3}$ (4 out of 6).



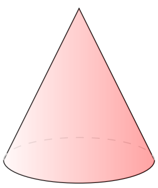
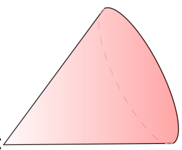
3. **Follow the Path:** To find $P(\text{tails and number} > 4)$, trace the "Tails" branch, then the "Number > 4" branch, and multiply:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

4. **Wrap It Up:** The tree confirms our answer—$\frac{1}{6}$!

B.5 EXPERIMENTAL PROBABILITY

Isaac wants to know how a cone lands when he tosses it—base down or point down? Here's what it can do:

- Base down: 
- Point down: 

Since the cone's shape might make one more likely, Isaac can't guess. So, he tosses it 50 times and counts:

- Base down: 30 times.

- Point down: 20 times.

He estimates:

- $P(\text{"base down"}) = \frac{30}{50} = 0.6$ (60%).
- $P(\text{"point down"}) = \frac{20}{50} = 0.4$ (40%).

The more he tosses, the closer he gets to the real chances!

Definition **Experimental Probability**

The probability $P(E)$ of an event E can be estimated using the formula:

$$P(E) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}}$$