PROBABILITY

Ever wondered if it'll rain tomorrow or if you'll win a game? That's probability! It's a math way to guess how likely things are to happen. Like when the weather app says there's a 90% chance of rain, that's probability telling us it's very likely. What else do you think we could use probability for?

A SAMPLE SPACE

Have you ever flipped a coin and wondered if it'd land on heads or tails? Or rolled a die and guessed what number you'd get? These are random experiments—things we do where the result isn't certain until it happens.

Definition Outcome

An **outcome** is one possible result of a random experiment.

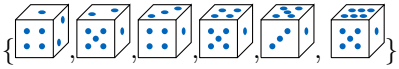
Definition Sample Space

The **sample space** is the list of all possible outcomes of a random experiment.

Ex: What's the sample space when you flip a coin?

Answer: It's {Heads, Tails} = {  }, or just {H, T} for short.

Ex: What's the sample space when you roll a six-sided die?

Answer: It's {1, 2, 3, 4, 5, 6} = {  }.

B EVENTS

Definition Event

An **event** is a set of outcomes from all possible outcomes.

In math, we use capital letters like E to name events. So, we might say E is the event "it's sunny tomorrow."

Ex: You roll a die. Let E be the event of rolling an even number. What's E?

Answer: The sample space is $\{1, 2, 3, 4, 5, 6\}$, and $E = \{2, 4, 6\}$ because those are the even numbers.

C COMPLEMENTARY EVENT

Definition Complementary Event

The **complementary event** of an event E is everything in the sample space that isn't in E. We call it E' (E-prime).

Ex: You roll a die, and E is rolling an even number. What's E'?

Answer: $E = \{2, 4, 6\}$, so E' is all the other numbers: $E' = \{1, 3, 5\}$. These are the odd numbers!

D PROBABILITY

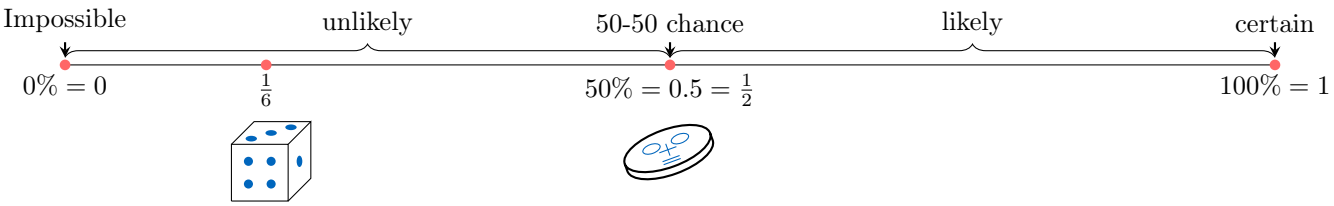
Definition Probability

The **probability** of an event E, written $P(E)$, tells us how likely E is to happen. It's always a number between 0 and 1:

- 0 means impossible (0% chance).
- 1 means certain (100% chance).

For example, the probability the sun rises tomorrow morning is 1!

We can show probability as a fraction, decimal, or percentage. Like a 50-50 chance is $\frac{1}{2}$, 0.5, or 50%. Picture it on a number line: 0 is impossible, 1 is certain, and 0.5 is right in the middle!



Ex: What's the probability the sun rises tomorrow morning?

Answer: It's 1 (or 100%) because it's certain!

E CALCULATE PROBABILITIES

Sometimes, every outcome in an experiment has the same chance—like flipping a fair coin or rolling a fair die. We call these equally likely outcomes.

Definition Equally Likely

If all outcomes are equally likely, the probability of an event, E, is given by

$$P(E) = \frac{\text{number of outcomes in the event}}{\text{total number of all possible outcomes}}$$

Ex: What's the probability of rolling an even number with a fair six-sided die?

Answer:

- Sample space $= \{1, 2, 3, 4, 5, 6\}$ (6 outcomes).
- $E = \{2, 4, 6\}$ (3 outcomes).
- $$\begin{aligned} P(E) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

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So, there's a $\frac{1}{2}$ chance (or 50%) of rolling an even number!

F COMPLEMENT RULE

Proposition Complement rule

For any event E and its complementary event E',

$$P(E) + P(E') = 1 \quad \text{or} \quad P(E') = 1 - P(E).$$

Ex: Farid has a 0.8 (80%) chance of finishing his homework on time tonight (event E). What's the chance he doesn't finish on time?

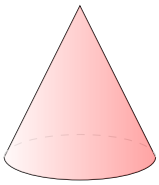
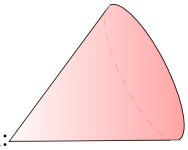
Answer: The complementary event E' represents the scenario where Farid does not complete his homework on time tonight. As $P(E) = 0.8$, by the complement rule, we get:

$$\begin{aligned} P(E') &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

So, there's a 20 % chance he doesn't finish on time!

G EXPERIMENTAL PROBABILITY

Isaac wants to know how a cone lands when he tosses it—base down or point down? Here's what it can do:

- Base down: 
- Point down: 

Since the cone's shape might make one more likely, Isaac can't guess. So, he tosses it 50 times and counts:

- Base down: 30 times.

- Point down: 20 times.

He estimates:

- $P(\text{"base down"}) = \frac{30}{50} = 0.6$ (60%).
- $P(\text{"point down"}) = \frac{20}{50} = 0.4$ (40%).

The more he tosses, the closer he gets to the real chances!

Definition **Experimental Probability**

The probability $P(E)$ of an event E can be estimated using the formula:

$$P(E) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}}$$