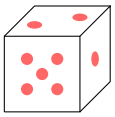
PROBABILITY

A SAMPLE SPACE

A.1 FINDING THE SAMPLE SPACES

MCQ 1: A fair six-sided die is rolled once.



Find the sample space.

☐ $\{1, 2, 3, 4, 5\}$

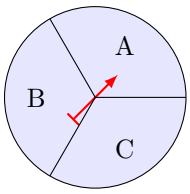
☐ $\{1, 2, 3, 4, 5, 6, 7\}$

☒ $\{1, 2, 3, 4, 5, 6\}$

Answer:

- The sample space is all possible outcomes.
- When rolling a fair six-sided die, the possible outcomes are the numbers on the die's faces.
- So, the sample space is $\{1, 2, 3, 4, 5, 6\}$.

MCQ 2: Find the sample space that the spinner can land on:



☒ $\{A, B, C\}$

☐ $\{A, B\}$

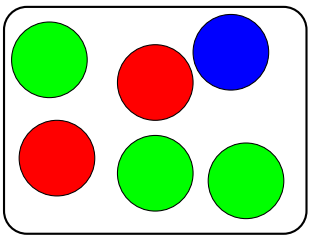
☐ $\{A, C\}$

Answer:

- The sample space is all possible outcomes.
- The spinner has three distinct regions labeled A, B, and C.
- So, the sample space is: $\{A, B, C\}$.

MCQ 3: A ball is chosen randomly from a bag containing 2 red balls, 1 blue ball, and 3 green balls.

Bag



Find the sample space.

☒ {Red, Blue, Green}

☐ {2 Red, 1 Blue, 3 Green}

☐ {Red, Red, Blue, Green, Green, Green}

Answer:

- When choosing a ball randomly from the bag containing 2 red balls, 1 blue ball, and 3 green balls, the balls are identical in color, so we do not distinguish between them based on quantity.
- So, the sample space (all possible outcomes) is: {Red, Blue, Green}

MCQ 4: A letter is chosen randomly from the word BANANA. Find all possible outcomes for the chosen letter.

☒ {B, N, A}

☐ {B, A, N, A, N, A}

☐ {A, B, N, A, B, N}

Answer:

- When choosing a letter randomly from the word "BANANA", the possible outcomes are the distinct letters in the word.
- So, the sample space (all possible outcomes) is: {B, A, N} or {B, N, A}. The order in which the letters are listed does not matter.

B EVENTS

B.1 FINDING THE EVENTS

MCQ 5: A letter is chosen randomly from the word ORANGE. Find the event where the chosen letter is a vowel.

☐ {O, R, A, N, G, E}

☒ {O, A, E}

☐ {R, G, N}

☐ {A, G, E}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When choosing a letter randomly from the word "ORANGE", the event where the chosen letter is a vowel consists of the vowels in the word.
- So, the event is: {O, A, E}.

MCQ 6: A fair six-sided dice is rolled once.
Find the event where the outcome is an even number.

☐ {1, 3, 5}

☒ {2, 4, 6 }

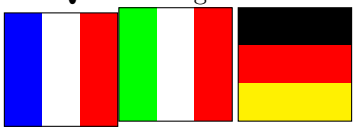
☐ {1, 2, 3, 4, 5, 6}

☐ {2, 3, 4, 5}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- When rolling a fair six-sided dice, the event where the outcome is an even number consists of the even numbers on the dice.
- So, the event is: {2, 4, 6}.

MCQ 7: A flag is chosen randomly from:



France Italy Germany

Find the event where the outcome is a flag with blue in them.

☒ {France }

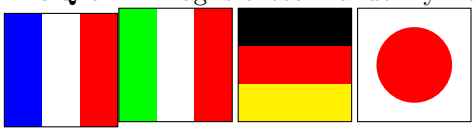
☐ {Italy, France}

☐ {Italy, France, Germany}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, only the French flag has blue in it.
- So, the correct answer is: {France}.

MCQ 8: A flag is chosen randomly from:



France Italy Germany Japan

Find the event where the outcome is a flag with red in them.

☐ {France, Japan}

☐ {Italy, France}

☒ {Italy, France, Germany, Japan}

Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, France, Italy, Germany, and Japan have red in them.
- So, the correct answer is: {Italy, France, Germany, Japan}.

MCQ 9: A flag is chosen randomly from:

France Italy Germany Nigeria

Find the event where the outcome is a flag with green in them.

☐ {France, Nigeria}

☒ {Italy, Nigeria}

☐ {Italy, France, Germany}

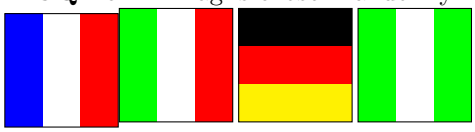
Answer:

- An event represents some outcomes from the sample space (all possible outcomes).
- Among the given flags, Italy and Nigeria have green in them.
- So, the correct answer is: {Italy, Nigeria}.

C COMPLEMENTARY EVENT

C.1 FINDING THE COMPLEMENTARY EVENTS

MCQ 10: A flag is chosen randomly from the following:



France Italy Germany Nigeria

Let E be the event where the selected flag contains green. Find the complement of event E, denoted as E'.

☒ $E' = \{\text{France, Germany}\}$

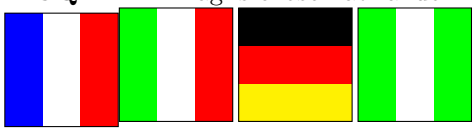
☐ $E' = \{\text{Italy, Nigeria}\}$

☐ $E' = \{\text{Italy, France, Germany}\}$

Answer:

- The event E includes flags with the color green: {Italy, Nigeria}.
- The complement event E' consists of flags that do not have green.
- So, the complement event $E' = \{\text{France, Germany}\}$.

MCQ 11: A flag is chosen at random from the following set:



France Italy Germany Nigeria

Let E be the event where the chosen flag contains the color red. Find the complement of event E, denoted E'.

☐ $E' = \{\text{France, Germany}\}$

☒ $E' = \{\text{Nigeria}\}$

☐ $E' = \{\text{Italy, France, Germany}\}$

Answer:

- The event E consists of flags that contain the color red: {France, Italy, Germany}.
- The complement of event E, denoted E', consists of flags that do not contain the color red.

- Since Nigeria is the only flag without the color red, $E' = \{\text{Nigeria}\}$.
- So, the correct answer is $E' = \{\text{Nigeria}\}$.

MCQ 12: A child's name is chosen randomly from the following list:

- Emily (girl's name)
- James (boy's name)
- Ava (girl's name)
- Sophia (girl's name)

Let E be the event where the selected name is a boy's name.
Find the complement of event E, denoted as E'.

☒ $E' = \{\text{Emily, Ava, Sophia}\}$

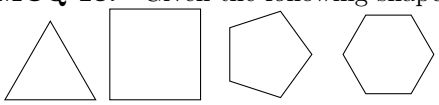
☐ $E' = \{\text{James}\}$

☐ $E' = \{\text{James, Ava}\}$

Answer:

- The event E includes boy's names: $\{\text{James}\}$.
- The complement event E' consists of names that are not boy's names (i.e., girl's names).
- So, the complement event $E' = \{\text{Emily, Ava, Sophia}\}$.

MCQ 13: Given the following shapes:



Triangle Square Pentagon Hexagon

Let E be the event where a polygon with an even number of sides is chosen.
Find the complement of event E, denoted as E'.

☐ $E' = \{\text{Square, Hexagon}\}$

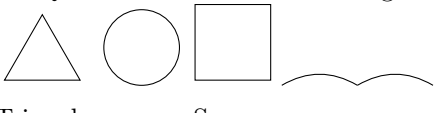
☒ $E' = \{\text{Triangle, Pentagon}\}$

☐ $E' = \{\text{Triangle, Square, Pentagon, Hexagon}\}$

Answer:

- Triangle: three sides. Square: four sides. Pentagon: five sides. Hexagon: six sides.
- The event E includes polygons with an even number of sides: $\{\text{Square, Hexagon}\}$.
- The complementary event E' consists of polygons that do not have an even number of sides (have an odd number of sides).
- Therefore, the complementary event $E' = \{\text{Triangle, Pentagon}\}$.

MCQ 14: Consider the following shapes:



Triangle Circle Square Curve

Let E be the event where the shape is a polygon.
Find the complement of event E, denoted as E'.

☐ $E' = \{\text{Triangle, Square}\}$

☐ $E' = \{\text{Triangle, Circle, Square, Curve}\}$

☒ $E' = \{\text{Circle, Curve}\}$

Answer:

- The event E includes shapes that are polygons: $\{\text{Triangle, Square}\}$.
- The complementary event E' consists of shapes that are not polygons.
- Therefore, the complementary event $E' = \{\text{Circle, Curve}\}$.

D PROBABILITY

D.1 DETERMINING THE PROBABILITY

MCQ 15: Keziah eats rice often. Let E be the event that Keziah eats rice this week. Find $P(E)$, the probability that Keziah eats rice this week.

☐ $P(E) = 1\%$

☐ $P(E) = 50\%$

☒ $P(E) = 99\%$

Answer:

- Since he eats rice often, it's very likely.
- So, the probability that Keziah eats rice this week is $P(E) = 99\%$.

MCQ 16: Emily drinks water every day. Let E be the event that Emily drinks water tomorrow. Find $P(E)$, the probability that Emily drinks water tomorrow.

☐ $P(E) = 50\%$

☐ $P(E) = 90\%$

☒ $P(E) = 100\%$

Answer:

- She drinks water every day, so it's certain.
- So, the probability that Emily drinks water tomorrow is $P(E) = 100\%$.

MCQ 17: It almost never snows in July in the Sahara Desert. Let E be the event that it snows this July in the Sahara Desert. Find $P(E)$, the probability that it snows this July.

☒ $P(E) = 0.01\%$

☐ $P(E) = 5\%$

☐ $P(E) = 99.9\%$

Answer:

- It's extremely rare, so very unlikely.
- So, the probability that it snows this July is $P(E) = 0.01\%$.

MCQ 18: Samuel loves playing basketball. Let E be the event that Samuel plays basketball this weekend. Find $P(E)$, the probability that Samuel plays this weekend.

- ☐ $P(E) = 5\%$
- ☐ $P(E) = 20\%$
- ☒ $P(E) = 90\%$

Answer:

- He loves it, so it's very likely.
- So, the probability that Samuel plays this weekend is $P(E) = 90\%$.

MCQ 19: Benjamin rolls a die. Let E be the event that Benjamin rolls a number bigger than 7. Find $P(E)$, the probability that Benjamin rolls a number bigger than 7.

- ☒ $P(E) = 0\%$
- ☐ $P(E) = 50\%$
- ☐ $P(E) = 100\%$

Answer:

- A die only has numbers 1 to 6, so it's impossible.
- So, the probability that Benjamin rolls a number bigger than 7 is $P(E) = 0\%$.

E CALCULATE PROBABILITIES

E.1 DETERMINING THE PROBABILITY

Ex 20: A ball is chosen randomly from a bag containing 2 red balls, 3 blue balls.
Find the probability that we choose a red ball.

$$P(\text{"choosing a red ball"}) = \boxed{\frac{2}{5}}$$

Answer:

- To find the probability of choosing a red ball, divide the number of red balls by the total number of balls.
- $$P(\text{"choosing a red ball"}) = \frac{\text{number of red balls}}{\text{total number of balls}} = \frac{2}{5}$$

Ex 21: A card is drawn at random from a standard deck of 52 playing cards. Determine the probability of drawing an Ace and express your answer as a simplified fraction.

$$P(\text{"drawing an Ace"}) = \boxed{\frac{1}{13}}$$

Answer:

- To find the probability of drawing an Ace, divide the number of Aces by the total number of cards.
- There are 4 Aces in a standard deck of 52 playing cards.
- $$P(\text{"drawing an Ace"}) = \frac{\text{number of Aces}}{\text{total number of cards}} = \frac{4}{52} = \frac{1 \times \cancel{4}}{13 \times \cancel{4}} = \frac{1}{13}$$

Ex 22: A six-sided die is rolled once. Determine the probability of obtaining an even number.

$$P(\text{"rolling an even number"}) = \boxed{\frac{1}{2}}$$

Answer:

- To find the probability of rolling an even number, divide the number of even numbers by the total number of sides.
- There are 3 even numbers on a six-sided die (2, 4, and 6).
- $$P(\text{rolling an even number}) = \frac{\text{number of even numbers}}{\text{total number of sides}} = \frac{3}{6} = \frac{1 \times \cancel{3}}{2 \times \cancel{3}} = \frac{1}{2}$$

MCQ 23: A fruit is selected randomly from a basket containing 3 apples, 2 oranges, and 5 bananas.
Find the probability that the selected fruit is an orange (simplify the fraction).

$$P(\text{"selecting an orange"}) = \boxed{\frac{1}{5}}$$

Answer:

- To find the probability of selecting an orange, divide the number of oranges by the total number of fruits.
- There are 2 oranges out of 10 fruits in total.
- $$P(\text{"selecting an orange"}) = \frac{\text{number of oranges}}{\text{total number of fruits}} = \frac{2}{10} = \frac{1 \times \cancel{2}}{5 \times \cancel{2}} = \frac{1}{5}$$

F COMPLEMENT RULE

F.1 APPLYING THE COMPLEMENT RULE

Ex 24: I toss a fair coin. The probability of getting heads is $\frac{1}{2}$. Find the probability of getting tails.

$$P(\text{"Getting tails"}) = \boxed{\frac{1}{2}}$$

Answer:

- The probability of getting heads is $\frac{1}{2}$.
- The event "Getting tails" is the complement of "Getting heads."
- Using the complement rule:

$$\begin{aligned} P(\text{"Getting tails"}) &= 1 - P(\text{"Getting heads"}) \\ &= 1 - \frac{1}{2} \\ &= \frac{2}{2} - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

- So, the probability of getting tails is $\frac{1}{2} = 50\%$.

Ex 25: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

$$P(\text{"Not laughing"}) = \boxed{30}\%$$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing."
- Using the complement rule:

$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not laugh at the joke is 30%.

Ex 26: I randomly select a student in the class. The probability that a girl is selected is $\frac{9}{10}$.
Find the probability that a boy is selected.

$$P(\text{"Selecting a boy"}) = \frac{\boxed{1}}{\boxed{10}}$$

Answer:

- The probability that a girl is selected is $\frac{9}{10}$.
- The event "Selecting a boy" is the complement of "Selecting a girl."
- Using the complement rule:

$$\begin{aligned} P(\text{"Selecting a boy"}) &= 1 - P(\text{"Selecting a girl"}) \\ &= 1 - \frac{9}{10} \\ &= \frac{10}{10} - \frac{9}{10} \\ &= \frac{1}{10} \end{aligned}$$

- So, the probability that a boy is selected is $\frac{1}{10} = 10\%$.

Ex 27: The weather forecast predicts that there is a 70% chance of rain tomorrow.
Find the probability that it will not rain tomorrow.

$$P(\text{"No rain"}) = \boxed{30}\%$$

Answer:

- The probability that it will rain tomorrow is 70%.
- The event "No rain" is the complement of the event "Rain".
- Using the complement rule:

$$\begin{aligned} P(\text{"No rain"}) &= 1 - P(\text{"Rain"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that it will not rain tomorrow is 30%.

Ex 28: A survey shows that 70% of the students in a school love Math.
Find the probability that a randomly chosen student does not love Math.

$$P(\text{"Not loving Math"}) = \boxed{30}\%$$

Answer:

- The probability that a student loves Math is 70%.
- The event "Not loving Math" is the complement event of "Loving Math."
- Using the complement rule:

$$\begin{aligned} P(\text{"Not loving Math"}) &= 1 - P(\text{"Loving Math"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- Therefore, the probability that a student does not love Math is 30%.

MCQ 29: A teacher told a joke in class: "Why was the math book sad? Because it had too many problems!" The probability that a student laughs at the joke is 70%.
Find the probability that a student does not laugh at the joke.

- ☒ $P(\text{"Not laughing"}) = 30\%$
- ☐ $P(\text{"Not laughing"}) = 70\%$
- ☐ $P(\text{"Not laughing"}) = 50\%$

Answer:

- The probability that a student laughs at the joke is 70%.
- The event "Not laughing" is the complement event of "Laughing".

- Using the complement rule:

$$\begin{aligned} P(\text{"Not laughing"}) &= 1 - P(\text{"Laughing"}) \\ &= 1 - 70\% \\ &= 100\% - 70\% \\ &= 30\% \end{aligned}$$

- So, the probability that a student does not laugh at the joke is 30%.

G EXPERIMENTAL PROBABILITY

G.1 SOLVING REAL-WORLD PROBLEMS

Ex 30: During a week of basketball practice, Mia made 45 out of 60 free-throw attempts. Estimate the experimental probability that Mia will make her next free-throw attempt (you can use a calculator).

$$P(\text{"Making the next attempt"}) \approx \boxed{75}\ \%$$

Answer:

- The experimental probability of Mia making a free-throw is the ratio of successful free-throws to the total number of attempts.
- $$\begin{aligned} P(\text{"Making the next attempt"}) &\approx \frac{\text{Number of successes}}{\text{Total attempts}} \\ &\approx \frac{45}{60} \\ &\approx 0.75 \\ &\approx 75\% \end{aligned}$$
- So, the estimated probability that Mia will make her next free-throw attempt is 75%.

Ex 31: During a week, the school cafeteria recorded that out of 150 students, 120 chose a vegetarian meal. Estimate the probability that the next student will choose a vegetarian meal based on this experimental probability(you can use a calculator).

$$P(\text{"Choosing a Vegetarian meal"}) \approx \boxed{80}\ \%$$

Answer:

- The probability $P(\text{"Vegetarian meal"})$ is estimated by the ratio of times a vegetarian meal was chosen to the total number of trials.
- $$\begin{aligned} P(\text{"Vegetarian meal"}) &\approx \frac{\text{Number of vegetarian meals}}{\text{Total number of students}} \\ &\approx \frac{120}{150} \\ &\approx 0.8 \\ &\approx 80\% \end{aligned}$$
- So, the estimated probability that the next student will choose a vegetarian meal is 80%.

Ex 32: Over the course of a year, it rained on 120 days out of 300 recorded days. Estimate the experimental probability that it will rain (you can use a calculator).

$$P(\text{"Raining"}) \approx \boxed{40}\ \%$$

Answer:

- The experimental probability of raining is the ratio of rainy days to the total number of recorded days.
- $$\begin{aligned} P(\text{"Raining"}) &\approx \frac{\text{Number of rainy days}}{\text{Total recorded days}} \\ &\approx \frac{120}{300} \\ &\approx 0.4 \\ &\approx 40\% \end{aligned}$$
- So, the estimated probability that it will rain is 40%.

Ex 33: A local bakery found that out of 200 customers, 150 ordered a croissant. Estimate the experimental probability that the next customer will order a croissant (you can use a calculator).

$$P(\text{"Ordering a croissant"}) \approx \boxed{75}\ \%$$

Answer:

- The experimental probability of a customer ordering a croissant is the ratio of customers who ordered a croissant to the total number of customers.
- $$\begin{aligned} P(\text{"Ordering a croissant"}) &\approx \frac{\text{Number of customers croissant}}{\text{Total number of customers}} \\ &\approx \frac{150}{200} \\ &\approx 0.75 \\ &\approx 75\% \end{aligned}$$
- So, the estimated probability that the next customer will order a croissant is 75%.