PROBABILITY

Ever wondered if it'll rain tomorrow or if you'll win a game? That's probability! It's a math way to guess how likely things are to happen.

A SAMPLE SPACES

Definition Outcome

An **outcome** is one possible result of a random experiment.

Ex: What are the outcomes when you flip a coin? 

Answer: The outcomes are Heads (H)=  and Tails (T)=  .

Ex: What are the outcomes when you roll a six-sided die?

Answer: The outcomes are 1=  ,2=  ,3=  ,4=  ,5=  ,and 6=  .

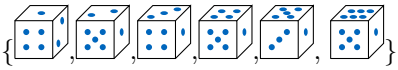
Definition Sample Space

The **sample space** is the set of all possible outcomes of a random experiment.

Ex: What's the sample space when you flip a coin?

Answer: The sample space is {Heads, Tails} = {  ,  }, or just {H, T} for short.

Ex: What's the sample space when you roll a six-sided die?

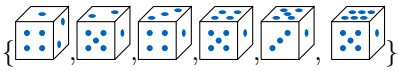
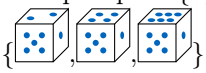
Answer: The sample space is {1, 2, 3, 4, 5, 6} = {  }.

B EVENTS

Definition Event

An **event** is a set of outcomes from the outcomes of the sample space. We write it E.

Ex: In the experiment of rolling a die, find E the event of rolling an even number.

Answer: Among the outcomes of the sample space {1, 2, 3, 4, 5, 6} = {  } ,the event of rolling an even number is $E = \{2, 4, 6\} =$ {  }.

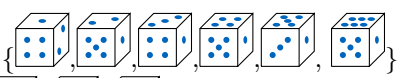
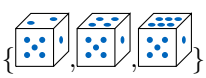
C COMPLEMENTARY EVENTS

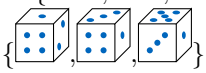
Ever wonder what happens if you look for everything **except** a certain event? That's where the complementary event comes in! It's just everything in the sample space that isn't in your event. We usually write it as E' ("E-prime").

Definition Complementary Event

The **complementary event** of an event E is all the outcomes in the sample space that are **not** in E. We write it E'.

Ex: In the experiment of rolling a die, let E the event of rolling an even number. Find E'.

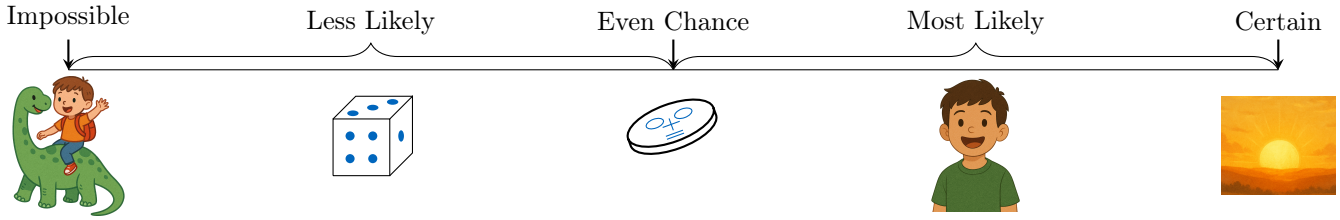
Answer: The sample space is $\{1, 2, 3, 4, 5, 6\} =$ {  } and $E = \{2, 4, 6\} =$ {  }.

So E' is all the other numbers: $\{1, 3, 5\} =$ {  }. These are the odd numbers.

D USING WORDS TO DESCRIBE PROBABILITY

We often use words to talk about probability. If something will never happen, it's impossible. If it will definitely happen, it's certain. In between, we say things like 'likely,' '50-50 chance,' or 'unlikely.' We can line them up from least to most likely.

Definition **Probability Line**



- **Impossible**: It can't happen.
  Example: Riding a dinosaur.
- **Less likely**: It probably won't happen.
  Example: Rolling a die and getting a 3.
- **Even chance**: It has the same chance to happen or not.
  Example: Tossing a coin and getting head.
- **Most likely**: It will probably happen.
  Example: Smiling at school today.
- **Certain**: It will happen.
  Example: The sun will rise tomorrow.

E PROBABILITY

When you flip a coin, there are two possible outcomes: heads or tails. The chance of getting heads is the same as getting tails—it's 1 out of 2! In math, we write:

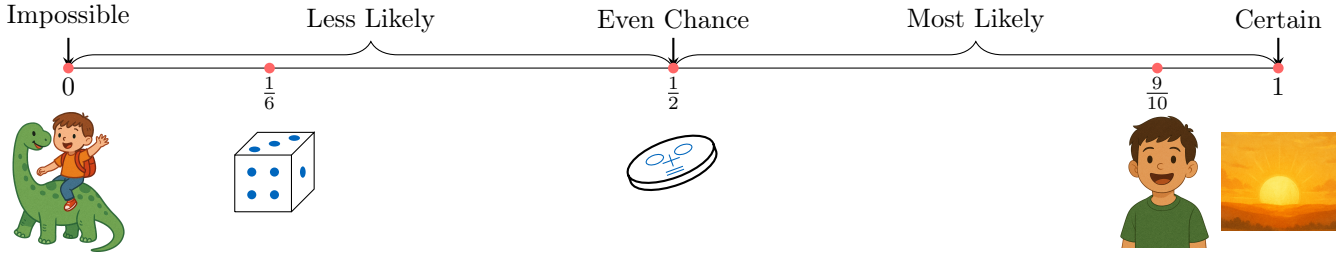
$$P\ (\text{"Getting Heads"}) = \frac{1}{2}$$

The probability — of getting heads — is equal — 1 chance out of 2

This means heads will happen about half the time!

Definition **Probability**

The **probability** of an event E, written $P(E)$, is a number that tells us how likely the event is to happen. It's always between 0 (impossible) and 1 (certain).



Ex: The probability of an event "even chance" can be represented as:

- **Fraction**: $\frac{1}{2}$
- **Decimal**: To convert the fraction to a decimal, divide the numerator by the denominator: $1 \div 2 = 0.5$.
- **Percentage**: To convert the decimal to a percentage, multiply by 100: $0.5 \times 100 = 50\%$.

F EQUALLY LIKELY

Discover: Have you ever flipped a fair coin or rolled a fair die? In these experiments, each outcome is just as likely as the others. We call these equally likely outcomes.

Definition **Equally Likely**

When all outcomes are **equally likely**, the probability of an event E is:

$$P(E) = \frac{\textbf{number of outcomes in the event}}{\textbf{number of outcomes in the sample space}}$$

Ex: What's the probability of rolling an even number with a fair six-sided die?

Answer:

- Sample space $= \{1, 2, 3, 4, 5, 6\}$ (6 outcomes).
- $E = \{2, 4, 6\}$ (3 outcomes).
- $$\begin{aligned} P(E) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

So, there's a $\frac{1}{2}$ chance (or 50%) of rolling an even number!

G COMPLEMENT RULE

Discover: Ever wondered how to quickly find the chance that something **doesn't** happen? There's a shortcut for that! It's called the complement rule.

Proposition **Complement Rule**

For any event E and its complementary event E', the probabilities always add up to 1:

$$P(E) + P(E') = 1 \quad \text{or} \quad P(E') = 1 - P(E).$$

Ex: Farid has a 0.8 (80%) chance of finishing his homework on time tonight (event E). What's the chance he **doesn't** finish on time?

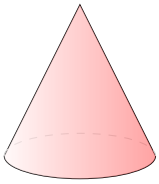
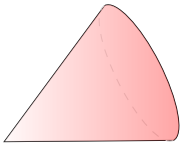
Answer: The complementary event E' is "Farid does **not** finish his homework on time." By the complement rule:

$$\begin{aligned} P(E') &= 1 - P(E) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

So, there's a 0.2 (or 20%) chance he doesn't finish on time!

H EXPERIMENTAL PROBABILITY

Discover: Isaac wishes to determine how a cone lands when tossed—base down or point down? The possible outcomes are as follows:

- Base down: 
- Point down: 

Due to the cone's shape potentially favoring one outcome, Isaac cannot predict the probabilities. He conducts 50 trials and records the results:

- Base down: 30 times.
- Point down: 20 times.

The chance of landing base down is approximately 30 times over 50 times. So, he estimates:

- $P(\text{"base down"}) = \frac{30}{50} = 0.6$ (60%).
- $P(\text{"point down"}) = \frac{20}{50} = 0.4$ (40%).

The more trials he conducts, the closer his estimates approach the true probabilities.

Theorem Law of Large Numbers

The probability of an event E can be estimated using the formula:

$$P(E) \approx \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Here, "trials" refer to the number of times the experiment is repeated.