# PERCENTAGES

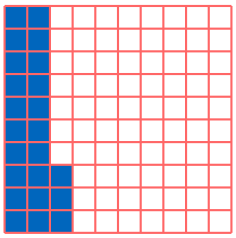
## A DEFINITION

**Definition Percentage**

A **percentage** is a ratio out of 100.
% reads "percent," which is short for the Latin term "per centum," meaning "by the hundred."

$$x\% = \frac{x}{100}$$

**Ex:**



As 23 parts out of 100 are colored, this is $23\% = \frac{23}{100}$.

## B PERCENTAGE AS A NUMBER

**Discover:** By definition, the percentage $5\%$ is equal to the fraction $\frac{5}{100}$. Also, the fraction $\frac{5}{100}$ is equal to the decimal number 0.05, since $5 \div 100 = 0.05$. It is important to be able to convert between these different representations of a percentage.

**Method Converting a percentage to a fraction**

To convert the percentage $5\%$ into a fraction, use the definition:

$$5\% = \frac{5}{100}$$

**Method Converting a fraction to a percentage**

To convert the fraction $\frac{3}{4}$ into a percentage:

- **Method 1 (Equivalent Fractions)**

$$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100} \overset{\div 25}{=} 75\%$$

- **Method 2 (Formula)**

$$\frac{3}{4} = \left(\frac{3}{4} \times 100\right)\% = 75\% \quad (\text{compute } 3 \div 4 \times 100 = 75)$$

Proof

To prove the formula:

$$\begin{aligned} x\% &= \frac{3}{4} \\ \frac{x}{100} &= \frac{3}{4} \\ x &= \frac{3}{4} \times 100 \quad (\text{multiplying both sides by } 100) \\ &= 75 \end{aligned}$$

**Method Converting a percentage to a decimal**

To convert the percentage $5\%$ into a decimal:

$$\begin{aligned} 5\% &= \frac{5}{100} \\ &= 5 \div 100 \\ &= 0.05 \end{aligned}$$

**Method Converting a decimal to a percentage**

To convert the decimal 0.05 into a percentage:

$$\begin{aligned} 0.05 &= 0.05 \times 100\% \\ &= 5\% \end{aligned}$$

*Proof*

To prove the formula:

$$\begin{aligned} 0.05 &= 0.05 \times \frac{100}{100} \\ &= 0.05 \times 100\% \end{aligned}$$

## C RATIO TO PERCENTAGE

**Discover:** We use percentages to compare a part with a whole.
For example, in a class with $20$ students, there are $12$ girls. To find the percentage of girls in the class, we find an equivalent fraction with a denominator of 100:

$$\begin{aligned} \frac{x}{100} &= \frac{12}{20} \\ x &= \frac{12}{20} \times 100 \quad \text{(multiplying both sides by } 100\text{)} \\ x &= 60 \end{aligned}$$

There are $60\%$ girls in the class. In this class, for every $100$ students, there would be $60$ girls in proportion.

**Method Ratio to Percentage**

To calculate the percentage of a ratio of a part to the whole, use the following formula:

$$x\% = \left(\frac{\text{part}}{\text{whole}} \times 100\right)\%$$

**Ex:** You took a math quiz and answered $21$ questions correctly out of $24$ questions total.
Calculate the percentage of correct answers.

*Solution:*

- The part is the number of correct answers: $21$.
- The whole is the total number of questions: $24$.
- $$\begin{aligned} \text{Percentage of correct answers} &= \left(\frac{\text{part}}{\text{whole}} \times 100\right)\% \\ &= \left(\frac{21}{24} \times 100\right)\% \\ &= 87.5\% \quad \text{(compute: } 21 \div 24 \times 100{=}87.5\text{)} \end{aligned}$$

## D COMPARING PERCENTAGES

**Discover:** In Parliament A of Country A, there are $26$ girls out of $50$ deputies. In Parliament B of Country B, there are $30$ girls out of $80$ deputies. Hugo says, "Since there are more girls in Parliament B, girls are better represented in this parliament." Do you agree with that statement?

*Solution:*

- Percentage of girls in Parliament A $= \left(\frac{26}{50} \times 100\right)\%$
  $= 52\%$
  In Parliament A, for every $100$ deputies, there would be $52$ girls in proportion.
- Percentage of girls in Parliament B $= \left(\frac{30}{80} \times 100\right)\%$
  $= 37.5\%$
  In Parliament B, for every $100$ deputies, there would be $37.5$ girls in proportion.

**Conclusion:** Since $52\% > 37.5\%$, girls are better represented in Parliament A.

Method **Comparing Percentages**

- **Step 1: Calculate the percentage for each group.**
- **Step 2: Compare the percentages and conclude.**

## E FORMULA TO FIND A PART USING PERCENTAGES

Method **Finding a Part Using Percentages**

To calculate a part in a part-to-whole relationship, multiply the percentage by the whole:

$$\text{part} = x\% \times \text{whole}$$

Proof

$$\frac{\text{part}}{\text{whole}} = x\%$$
$$\text{part} = x\% \times \text{whole} \quad \text{(multiplying both sides by whole)}$$

**Ex:** In a school with $200$ students, $60\%$ are girls.
Calculate the number of girls in the school.

*Solution:*

- Method 1 (using the formula)

$$\begin{aligned} \text{number of girls} &= 60\% \text{ of } 200 \text{ students} \\ &= 60\% \times 200 \\ &= \frac{60}{100} \times 200 \\ &= 120 \qquad \text{(compute: } 60 \div 100 \times 200 = 120\text{)} \end{aligned}$$

- Method 2 (cross-multiplication)

$$\begin{aligned} \frac{60}{100} &= \frac{\text{number of girls}}{200} \\ 100 \times \text{number of girls} &= 60 \times 200 \qquad \text{(cross-multiplication)} \\ \text{number of girls} &= \frac{60 \times 200}{100} \qquad \text{(dividing both sides by 100)} \\ \text{number of girls} &= 120 \qquad \text{(compute: } (60 \times 200) \div 100 = 120\text{)} \end{aligned}$$

## F FORMULA TO FIND THE WHOLE USING PERCENTAGES

Method **Finding the Whole Using Percentages**

To calculate the whole in a part-to-whole relationship, divide the part by the percentage (as a decimal):

$$\text{whole} = \frac{\text{part}}{x\%}$$

Proof

$$\frac{\text{part}}{\text{whole}} = x\%$$
$$\text{part} = x\% \times \text{whole}$$
$$\text{whole} = \frac{\text{part}}{x\%} \qquad \text{(dividing both sides by } x\%\text{)}$$

**Ex:** In a class, 40% of the students are girls, and there are 14 girls in total.
Find the total number of students in the class.

*Solution:*

- Method 1 (using the formula)

$$\begin{aligned}\text{total students} &= \frac{14}{40\%} \\ &= \frac{14}{\left(\frac{40}{100}\right)} \\ &= 35 \qquad \text{(compute: } 14 \div (14 \div 100) = 35\text{)}\end{aligned}$$

- Method 2 (cross-multiplication)

$$\begin{aligned}\frac{40}{100} &= \frac{14}{\text{total students}} \\ 40 \times \text{total students} &= 100 \times 14 \qquad \text{(cross-multiplication)} \\ \text{total students} &= \frac{100 \times 14}{40} \qquad \text{(dividing both sides by } 40\text{)} \\ \text{total students} &= 35 \qquad \text{(compute: } (100 \times 14) \div 40 = 35\text{)}\end{aligned}$$

## G PERCENTAGE INCREASE OR DECREASE

**Discover:** In everyday life, there are many situations where quantities are either increased or decreased by a certain percentage. For example:

- A store offers a 20% discount on all items during a sale.
- A worker receives a salary increase of 7%.
- A person on a diet reduces their weight by 10%.

Definition **Percentage Increase or Decrease**

- A **percentage increase** occurs when a quantity is raised by a certain percentage.
- A **percentage decrease** occurs when a quantity is reduced by a certain percentage.

Method **Calculating the New Quantity for a Percentage Increase in Two Steps**

1. **Calculate the increase:**

$$\begin{aligned}\text{Increase} &= \text{Percentage of the original quantity} \\ &= \text{Percentage} \times \text{Original quantity}\end{aligned}$$

2. **Calculate the new quantity:**

$$\text{New quantity} = \text{Original quantity} + \text{Increase}$$

**Ex:** If the original price of a shirt is \$50 and it is increased by 20%, find the new price.

*Solution:*

- **Calculate the increase:**

$$\begin{aligned}\text{Increase} &= 20\% \times \$50 \\ &= \frac{20}{100} \times \$50 \\ &= \$10\end{aligned}$$

- **Calculate the new price:**

$$\text{New price} = \$50 + \$10 = \$60$$

Method **Calculating the New Quantity for a Percentage Decrease in Two Steps**

1. **Calculate the decrease:**

$$\begin{aligned}\text{Decrease} &= \text{Percentage of the original quantity}\\ &= \text{Percentage} \times \text{Original quantity}\end{aligned}$$

2. **Calculate the new quantity:**

$$\text{New quantity} = \text{Original quantity} - \text{Decrease}$$

**Ex:** If the original price of a shirt is \$ 50 and it is decreased by 20%, find the new price.

*Solution:*

- **Calculate the decrease:**

$$\begin{aligned}\text{Decrease} &= 20\% \times \$50\\ &= \frac{20}{100} \times \$50\\ &= \$10\end{aligned}$$

- **Calculate the new price:**

$$\text{New price} = \$50 - \$10 = \$40$$

## H PERCENTAGE CHANGE

**Discover:** **Percentage change** measures how much a quantity has increased or decreased relative to its original value. It can be expressed as either a **percentage increase** or a **percentage decrease**, depending on the nature of the change.

Definition **Percentage Change**

- If the quantity **increases**, the percentage change is positive:

$$\text{Percentage Change} = \text{Percentage Increase}$$

- If the quantity **decreases**, the percentage change is negative:

$$\text{Percentage Change} = -\text{Percentage Decrease}$$

**Ex:** A store offers a discount of 25%. Determine the percentage change.

*Solution:* Since a discount represents a reduction, the percentage change is negative. Therefore, the percentage change is $-25\%$.

The relationship between the new quantity and the original quantity can be expressed as:

Proposition **Percentage Increase or Decrease in One Step**

$$\begin{aligned}\text{New Quantity} &= \text{Original Quantity} + \text{Percentage Change} \times \text{Original Quantity}\\ &= (1 + \text{Percentage Change}) \times \text{Original Quantity}\end{aligned}$$

Definition **Multiplier**

$(1 + \text{Percentage Change})$ is the **multiplier**.

**Ex:** Increase \$200 by 10%.

*Solution:* It is an increase, so the percentage change is 10%.

$$\begin{aligned}\text{New amount} &= \left(1 + \frac{10}{100}\right) \times \$200\\ &= 1.10 \times \$200\\ &= \$220\end{aligned}$$

## I FORMULA TO FIND THE PERCENTAGE CHANGE

Method **Calculating the Percentage Change**

$$\text{Percentage Change} = \frac{\text{New Quantity} - \text{Original Quantity}}{\text{Original Quantity}} \times 100\%$$

Proof

$$\begin{aligned}
\text{Original Quantity} + \text{Percentage Change} \times \text{Original Quantity} &= \text{New Quantity} \\
\text{Percentage Change} \times \text{Original Quantity} &= \text{New Quantity} - \text{Original Quantity} \\
\text{Percentage Change} &= \frac{\text{New Quantity} - \text{Original Quantity}}{\text{Original Quantity}} \\
\text{Percentage Change} &= \frac{\text{New Quantity} - \text{Original Quantity}}{\text{Original Quantity}} \times 100\%
\end{aligned}$$

**Ex:** Find the percentage change in Louis's weight from 25 kg to 28 kg.

*Solution:*

$$\begin{aligned}
\text{Percentage Change} &= \frac{28 - 25}{25} \times 100\% \\
&= \frac{3}{25} \times 100\% \\
&= 12\%
\end{aligned}$$

This is a 12% increase.