

PERCENTAGES

A DEFINITION

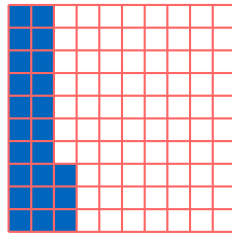
Definition Percentage

A **percentage** is a ratio out of 100.

% reads "percent," which is short for the Latin term "per centum," meaning "by the hundred."

$$x\% = \frac{x}{100}$$

Ex:



As 23 parts out of 100 are colored, this is $23\% = \frac{23}{100}$.

B PERCENTAGE AS A NUMBER

Discover: By definition, the percentage 5% is equal to the fraction $\frac{5}{100}$. Also, the fraction $\frac{5}{100}$ is equal to the decimal number 0.05, since $5 \div 100 = 0.05$. It is important to be able to convert between these different representations of a percentage.

Method Converting a percentage to a fraction

To convert the percentage 5% into a fraction, use the definition:

$$5\% = \frac{5}{100}$$

Method Converting a fraction to a percentage

To convert the fraction $\frac{3}{4}$ into a percentage:

- Method 1 (Equivalent Fractions)

$$\frac{3}{4} = \frac{75}{100} = 75\%$$

(Note: An arrow labeled $\times 25$ points from $\frac{3}{4}$ to $\frac{75}{100}$, and an arrow labeled $\div 25$ points from $\frac{75}{100}$ to 75% .)

- Method 2 (Formula)

$$\begin{aligned} \frac{3}{4} &= \left(\frac{3}{4} \times 100 \right) \% \\ &= 75\% \quad (\text{compute } 3 \div 4 \times 100 = 75) \end{aligned}$$

Proof

To prove the formula:

$$\begin{aligned} x\% &= \frac{3}{4} \\ \frac{x}{100} &= \frac{3}{4} \\ x &= \frac{3}{4} \times 100 \quad (\text{multiplying both sides by } 100) \\ &= 75 \end{aligned}$$

Method Converting a percentage to a decimal

To convert the percentage 5% into a decimal:

$$\begin{aligned}5\% &= \frac{5}{100} \\ &= 5 \div 100 \\ &= 0.05\end{aligned}$$

Method Converting a decimal to a percentage

To convert the decimal 0.05 into a percentage:

$$\begin{aligned}0.05 &= 0.05 \times 100\% \\ &= 5\%\end{aligned}$$

Proof

To prove the formula:

$$\begin{aligned}0.05 &= 0.05 \times \frac{100}{100} \\ &= 0.05 \times 100\%\end{aligned}$$

C RATIO TO PERCENTAGE

Discover: We use percentages to compare a part with a whole.

For example, in a class with 20 students, there are 12 girls. To find the percentage of girls in the class, we find an equivalent fraction with a denominator of 100:

$$\begin{aligned}\frac{x}{100} &= \frac{12}{20} \\ x &= \frac{12}{20} \times 100 \quad (\text{multiplying both sides by } 100) \\ x &= 60\end{aligned}$$

There are 60% girls in the class. In this class, for every 100 students, there would be 60 girls in proportion.

Method Ratio to Percentage

To calculate the percentage of a ratio of a part to the whole, use the following formula:

$$x\% = \left(\frac{\text{part}}{\text{whole}} \times 100 \right) \%$$

Ex: You took a math quiz and answered 21 questions correctly out of 24 questions total. Calculate the percentage of correct answers.

Solution:

- The **part** is the number of correct answers: 21.
- The **whole** is the total number of questions: 24.
- Percentage of correct answers = $\left(\frac{\text{part}}{\text{whole}} \times 100 \right) \%$

$$\begin{aligned}&= \left(\frac{21}{24} \times 100 \right) \% \\ &= 87.5\% \quad (\text{compute: } 21 \div 24 \times 100 = 87.5)\end{aligned}$$

D COMPARING PERCENTAGES

Discover: In Parliament A of Country A, there are 26 girls out of 50 deputies. In Parliament B of Country B, there are 30 girls out of 80 deputies. Hugo says, "Since there are more girls in Parliament B, girls are better represented in this parliament." Do you agree with that statement?

Solution:

- Percentage of girls in Parliament A = $\left(\frac{26}{50} \times 100\right)\%$
= 52%

In Parliament A, for every 100 deputies, there would be 52 girls in proportion.

- Percentage of girls in Parliament B = $\left(\frac{30}{80} \times 100\right)\%$
= 37.5%

In Parliament B, for every 100 deputies, there would be 37.5 girls in proportion.

Conclusion: Since 52% > 37.5%, girls are better represented in Parliament A.

Method Comparing Percentages

- Step 1: Calculate the percentage for each group.
- Step 2: Compare the percentages and conclude.

E FORMULA TO FIND A PART USING PERCENTAGES

Method Finding a Part Using Percentages

To calculate a part in a part-to-whole relationship, multiply the percentage by the whole:

$$\text{part} = x\% \times \text{whole}$$

Proof

$$\frac{\text{part}}{\text{whole}} = x\%$$

$$\text{part} = x\% \times \text{whole} \quad (\text{multiplying both sides by whole})$$

Ex: In a school with 200 students, 60% are girls.
Calculate the number of girls in the school.

Solution:

- Method 1 (using the formula)

$$\begin{aligned} \text{number of girls} &= 60\% \text{ of } 200 \text{ students} \\ &= 60\% \times 200 \\ &= \frac{60}{100} \times 200 \\ &= 120 \end{aligned} \quad (\text{compute: } 60 \div 100 \times 200 = 120)$$

- Method 2 (cross-multiplication)

$$\frac{60}{100} = \frac{\text{number of girls}}{200}$$

$$100 \times \text{number of girls} = 60 \times 200 \quad (\text{cross-multiplication})$$

$$\text{number of girls} = \frac{60 \times 200}{100} \quad (\text{dividing both sides by } 100)$$

$$\text{number of girls} = 120 \quad (\text{compute: } (60 \times 200) \div 100 = 120)$$

F FORMULA TO FIND THE WHOLE USING PERCENTAGES

Method Finding the Whole Using Percentages

To calculate the whole in a part-to-whole relationship, divide the part by the percentage (as a decimal):

$$\text{whole} = \frac{\text{part}}{x\%}$$

Proof

$$\begin{aligned}\frac{\text{part}}{\text{whole}} &= x\% \\ \text{part} &= x\% \times \text{whole} \\ \text{whole} &= \frac{\text{part}}{x\%} \quad (\text{dividing both sides by } x\%)\end{aligned}$$

Ex: In a class, 40% of the students are girls, and there are 14 girls in total. Find the total number of students in the class.

Solution:

- Method 1 (using the formula)

$$\begin{aligned}\text{total students} &= \frac{14}{40\%} \\ &= \frac{14}{\left(\frac{40}{100}\right)} \\ &= 35 \quad (\text{compute: } 14 \div (14 \div 100) = 35)\end{aligned}$$

- Method 2 (cross-multiplication)

$$\begin{aligned}\frac{40}{100} &= \frac{14}{\text{total students}} \quad (\text{cross-multiplication}) \\ 40 \times \text{total students} &= 100 \times 14 \\ \text{total students} &= \frac{100 \times 14}{40} \quad (\text{dividing both sides by } 40) \\ \text{total students} &= 35 \quad (\text{compute: } (100 \times 14) \div 40 = 35)\end{aligned}$$

G PERCENTAGE INCREASE OR DECREASE

Discover: In everyday life, there are many situations where quantities are either increased or decreased by a certain percentage. For example:

- A store offers a 20% discount on all items during a sale.
- A worker receives a salary increase of 7%.
- A person on a diet reduces their weight by 10%.

Definition Percentage Increase or Decrease

- A **percentage increase** occurs when a quantity is raised by a certain percentage.
- A **percentage decrease** occurs when a quantity is reduced by a certain percentage.

Method Calculating the New Quantity for a Percentage Increase in Two Steps

1. **Calculate the increase:**

$$\begin{aligned}\text{Increase} &= \text{Percentage of the original quantity} \\ &= \text{Percentage} \times \text{Original quantity}\end{aligned}$$

2. **Calculate the new quantity:**

$$\text{New quantity} = \text{Original quantity} + \text{Increase}$$

Ex: If the original price of a shirt is \$50 and it is increased by 20%, find the new price.

Solution:

- **Calculate the increase:**

$$\begin{aligned}\text{Increase} &= 20\% \times \$50 \\ &= \frac{20}{100} \times \$50 \\ &= \$10\end{aligned}$$

- **Calculate the new price:**

$$\text{New price} = \$50 + \$10 = \$60$$

Method Calculating the New Quantity for a Percentage Decrease in Two Steps

1. Calculate the decrease:

$$\begin{aligned}\text{Decrease} &= \text{Percentage of the original quantity} \\ &= \text{Percentage} \times \text{Original quantity}\end{aligned}$$

2. Calculate the new quantity:

$$\text{New quantity} = \text{Original quantity} - \text{Decrease}$$

Ex: If the original price of a shirt is \$ 50 and it is decreased by 20%, find the new price.

Solution:

• Calculate the decrease:

$$\begin{aligned}\text{Decrease} &= 20\% \times \$50 \\ &= \frac{20}{100} \times \$50 \\ &= \$10\end{aligned}$$

• Calculate the new price:

$$\text{New price} = \$50 - \$10 = \$40$$