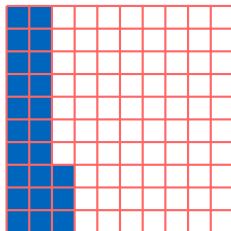# PERCENTAGES

A WHAT IS A PERCENTAGE?

Definition Percentage

A **percentage** is a ratio out of 100.
The symbol $\%$ means "percent," which comes from the phrase "per centum," meaning "out of one hundred."

Ex: This grid has 100 squares. Since 23 out of 100 squares are colored, we say that 23% of the grid is colored.

$$23\% = \frac{23}{100}$$

B CONVERTING BETWEEN FORMS

Method Percentage to Fraction

To convert a percentage to a fraction, write it as a fraction over 100, then simplify if possible.

$$40\% = \frac{40}{100} = \frac{2}{5} \qquad (\div 20)$$

Method Fraction to Percentage

To convert a fraction like $\frac{3}{4}$ to a percentage, you have two common methods:

- **Method 1: Equivalent Fractions.** Find an equivalent fraction with a denominator of 100.

$$\frac{3}{4} = \frac{75}{100} = 75\% \qquad (\times 25)$$

- **Method 2: Multiply by** 100%**.** This works because multiplying by 100% is the same as multiplying by 1.

$$\frac{3}{4} = 0.75 \quad (\text{since } 3 \div 4 = 0.75)$$
$$= 0.75 \times 100\%$$
$$= 75\%$$

Method Percentage to Decimal

To convert a percentage to a decimal, divide by 100. A quick way to do this is to move the decimal point two places to the left.

$$45\% = 45 \div 100 = 0.45$$

Method Decimal to Percentage

To convert a decimal to a percentage, multiply by 100. A quick way to do this is to move the decimal point two places to the right and add the percent sign (%).

$$0.68 = 0.68 \times 100\% = 68\%$$

C RATIO TO PERCENTAGE

Method Ratio to Percentage

To convert a part-to-whole ratio into a percentage, use the following formula:

$$\text{Percentage} = \frac{\text{part}}{\text{whole}} \times 100\%$$

Ex: You took a math quiz and answered 21 questions correctly out of a total of 24 questions. Calculate your percentage score.

Answer:

- The part is the number of correct answers: 21.

- The whole is the total number of questions: 24.

- Percentage Score $= \dfrac{21}{24} \times 100\%$

$$= 0.875 \times 100\%$$
$$= 87.5\%$$

D COMPARING RATIOS USING PERCENTAGES

Method Comparing with Percentages

When comparing different part-to-whole ratios, converting them to percentages provides a common baseline (out of 100), which allows for a fair and direct comparison.

- **Step 1: Calculate the percentage for each group.**

- **Step 2: Compare the percentages to draw a conclusion.**

E FINDING THE PART OR THE WHOLE

Method Finding the Part

To find a part of a total, multiply the percentage by the whole.

$$\text{Part} = \text{Percentage} \times \text{Whole}$$

Remember to convert the percentage to a decimal or fraction before calculating.

Ex: In a school with 200 students, 60% are girls. Calculate the number of girls.

Answer: **Method 1: Using the formula**

$$\text{Number of girls} = 60\% \times 200$$
$$= 0.60 \times 200$$
$$= 120$$

There are 120 girls in the school. **Method 2: Cross-Multiplication**
Set up a proportion where x is the number of girls.

$$\frac{60}{100} = \frac{x}{200}$$
$$100 \times x = 60 \times 200$$
$$x = \frac{12000}{100} = 120$$

Method Finding the Whole

To find the whole when you know a part and its percentage, divide the part by the percentage.

$$\text{Whole} = \frac{\text{Part}}{\text{Percentage}}$$

Remember to convert the percentage to a decimal or fraction before calculating.

Ex: In a class, 40% of the students are girls. If there are 14 girls, what is the total number of students?

Answer: **Method 1: Using the formula**

$$\text{Total students} = \frac{14}{40\%}$$
$$= \frac{14}{0.40}$$
$$= 35$$

There are 35 students in the class. **Method 2: Cross-Multiplication**
Set up a proportion where x is the total number of students.

$$\frac{40}{100} = \frac{14}{x}$$
$$40 \times x = 14 \times 100$$
$$x = \frac{1400}{40} = 35$$

F PERCENTAGE INCREASE AND DECREASE

> **Method Two-Step Method for Percentage Change**
>
> 1. **Calculate the change amount:**
>
> $$\text{Change} = \text{Percentage} \times \text{Original Value}$$
>
> 2. **Calculate the new value:**
>
>   - For an **increase**:
>
> $$\text{New Value} = \text{Original Value} + \text{Change}$$
>
>   - For a **decrease**:
>
> $$\text{New Value} = \text{Original Value} - \text{Change}$$

Ex: The original price of a shirt is $50. Calculate the final price after a 20% discount.

Answer:

1. **Calculate the decrease amount:**

$$\text{Decrease} = 20\% \text{ of } \$50$$
$$= 20\% \times \$50$$
$$= 0.20 \times \$50$$
$$= \$10$$

2. **Calculate the new price:**

$$\text{New Price} = \$50 - \$10 = \$40$$

G PERCENTAGE CHANGE

> **Definition Percentage Change**
>
> **Percentage change** is a signed value that indicates both the direction and magnitude of a change.
>
> - If a quantity **increases**, the percentage change is **positive**. An increase of 15% means a percentage change of +15%.
> - If a quantity **decreases**, the percentage change is **negative**. A decrease of 15% means a percentage change of −15%.

> **Method Calculating New Value with a Multiplier**
>
> A fast way to find the new value after a percentage change is to use a multiplier.
>
> $$\text{New Value} = \text{Original Value} \times (1 + \text{Percentage Change})$$

The term $(1 + \text{Percentage Change})$ is the **multiplier**. Remember to express the percentage change as a decimal in this formula.

Ex: Find the new amount for increasing \$200 by 10%.

Answer: The percentage change is $+10\% = +0.10$.

$$\begin{aligned} \text{New amount} &= \$200 \times (1 + 0.10) \\ &= \$200 \times 1.10 \\ &= \$220 \end{aligned}$$

Ex: Find the new amount for decreasing \$200 by 25%.

Answer: The percentage change is $-25\% = -0.25$.

$$\begin{aligned} \text{New amount} &= \$200 \times (1 - 0.25) \\ &= \$200 \times 0.75 \\ &= \$150 \end{aligned}$$

H CALCULATING THE PERCENTAGE CHANGE

Method **Formula for Percentage Change**

To find the percentage change when you know the original and new values, use this formula:

$$\text{Percentage Change} = \frac{\text{Change in Value}}{\text{Original Value}} \times 100\% = \frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} \times 100\%$$

Ex: Find the percentage change when a weight increases from 25 kg to 28 kg.

Answer: The weight increases, so we expect a positive result.

$$\begin{aligned} \text{Percentage Change} &= \frac{28 - 25}{25} \times 100\% \\ &= \frac{3}{25} \times 100\% \\ &= +12\% \end{aligned}$$

This is a 12% increase.

Ex: Find the percentage change when a price drops from \$500 to \$420.

Answer: The price decreases, so we expect a negative result.

$$\begin{aligned} \text{Percentage Change} &= \frac{420 - 500}{500} \times 100\% \\ &= \frac{-80}{500} \times 100\% \\ &= -16\% \end{aligned}$$

This is a 16% decrease.