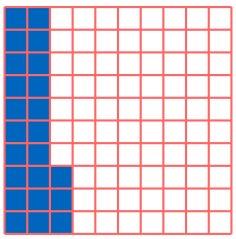
PERCENTAGES

A WHAT IS A PERCENTAGE?

Definition Percentage

A **percentage** is a ratio out of 100.
The symbol % means "percent," which comes from the phrase "per centum," meaning "out of one hundred."

Ex: This grid has 100 squares. Since 23 out of 100 squares are colored, we say that 23% of the grid is colored.



$$23\% = \frac{23}{100}$$

B CONVERTING BETWEEN FORMS

Discover: Percentages, fractions, and decimals are three different ways to talk about the same value. For example, 50%, the fraction $\frac{1}{2}$, and the decimal 0.5 all mean "half." Being able to convert between these forms is a very useful skill.

Remark: A very important idea is that $100\% = \frac{100}{100} = 1$.
Since 100% is just another way of writing the number 1, you can multiply any number by 100% without changing its value. This is a very useful trick for converting decimals and fractions into percentages.

Method Percentage to Fraction

To convert a percentage to a fraction, write it as a fraction over 100, then simplify if possible.

$$40\% = \frac{40}{100} \overset{\div 20}{=} \frac{2}{5}$$

Method Fraction to Percentage

To convert a fraction like $\frac{3}{4}$ to a percentage, you have two common methods:

- **Method 1: Equivalent Fractions.** Find an equivalent fraction with a denominator of 100.

$$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100} = 75\%$$

- **Method 2: Multiply by 100%.** This works because multiplying by 100% is the same as multiplying by 1.

$$\begin{aligned} \frac{3}{4} &= 0.75 \quad (\text{since } 3 \div 4 = 0.75) \\ &= 0.75 \times 100\% \\ &= 75\% \end{aligned}$$

Method Percentage to Decimal

To convert a percentage to a decimal, divide by 100. A quick way to do this is to move the decimal point two places to the left.

$$45\% = 45 \div 100 = 0.45$$

Method Decimal to Percentage

To convert a decimal to a percentage, multiply by 100. A quick way to do this is to move the decimal point two places to the right and add the percent sign (%).

$$0.68 = 0.68 \times 100\% = 68\%$$

C RATIO TO PERCENTAGE

Discover: Percentages are one of the best ways to compare a part to a whole.
For example, if a class of 20 students has 12 girls, what percentage of the class is female? To solve this, we can set up a proportion to find an equivalent fraction with a denominator of 100:

$$\begin{aligned} \frac{\text{part}}{\text{whole}} = \frac{12}{20} &= \frac{x}{100} \\ x &= \frac{12}{20} \times 100 \quad \text{(multiplying both sides by 100)} \\ x &= 60 \end{aligned}$$

The percentage of girls in the class is 60%. This means for every 100 students, 60 would be girls.

Method Ratio to Percentage

To convert a part-to-whole ratio into a percentage, use the following formula:

$$\text{Percentage} = \frac{\text{part}}{\text{whole}} \times 100\%$$

Ex: You took a math quiz and answered 21 questions correctly out of a total of 24 questions. Calculate your percentage score.

Answer:

- The part is the number of correct answers: 21.
- The whole is the total number of questions: 24.
- Percentage Score $= \frac{21}{24} \times 100\%$
  $= 0.875 \times 100\%$
  $= 87.5\%$

D COMPARING RATIOS USING PERCENTAGES

Discover: In Parliament A, there are 26 women out of 50 members. In Parliament B, there are 30 women out of 80 members. Hugo says, "Since there are more women in Parliament B, women have better representation there."
Is this statement a fair comparison?

Answer: To make a fair comparison, we must compare the percentages, not the absolute numbers.

- **Parliament A:**
  Percentage of women $= \frac{26}{50} \times 100\% = 52\%$
- **Parliament B:**
  Percentage of women $= \frac{30}{80} \times 100\% = 37.5\%$

Conclusion: Since $52\% > 37.5\%$, women are proportionally better represented in Parliament A.

Method Comparing with Percentages

When comparing different part-to-whole ratios, converting them to percentages provides a common baseline (out of 100), which allows for a fair and direct comparison.

- **Step 1: Calculate the percentage for each group.**
- **Step 2: Compare the percentages to draw a conclusion.**

E FINDING THE PART OR THE WHOLE

Method **Finding the Part**

To find a part of a total, multiply the percentage by the whole.

$$\text{Part} = \text{Percentage} \times \text{Whole}$$

Remember to convert the percentage to a decimal or fraction before calculating.

Ex: In a school with 200 students, 60% are girls. Calculate the number of girls.

Answer: **Method 1: Using the formula**

$$\begin{aligned}\text{Number of girls} &= 60\% \times 200 \\ &= 0.60 \times 200 \\ &= 120\end{aligned}$$

There are 120 girls in the school.**Method 2: Cross-Multiplication**
Set up a proportion where x is the number of girls.

$$\begin{aligned}\frac{60}{100} &= \frac{x}{200} \\ 100 \times x &= 60 \times 200 \\ x &= \frac{12000}{100} = 120\end{aligned}$$

Method **Finding the Whole**

To find the whole when you know a part and its percentage, divide the part by the percentage.

$$\text{Whole} = \frac{\text{Part}}{\text{Percentage}}$$

Remember to convert the percentage to a decimal or fraction before calculating.

Ex: In a class, 40% of the students are girls. If there are 14 girls, what is the total number of students?

Answer: **Method 1: Using the formula**

$$\begin{aligned}\text{Total students} &= \frac{14}{40\%} \\ &= \frac{14}{0.40} \\ &= 35\end{aligned}$$

There are 35 students in the class.**Method 2: Cross-Multiplication**
Set up a proportion where x is the total number of students.

$$\begin{aligned}\frac{40}{100} &= \frac{14}{x} \\ 40 \times x &= 14 \times 100 \\ x &= \frac{1400}{40} = 35\end{aligned}$$

F PERCENTAGE INCREASE AND DECREASE

Discover: Quantities often change by a certain percentage. For example:

- A store offers a 20% discount (a decrease).
- A salary increases by 7%.
- A city's population grows by 10%.

Method **Two-Step Method for Percentage Change**

1. **Calculate the change amount:**

$$\text{Change} = \text{Percentage} \times \text{Original Value}$$

2. **Calculate the new value:**

- For an **increase**:
$$\text{New Value} = \text{Original Value} + \text{Change}$$
- For a **decrease**:
$$\text{New Value} = \text{Original Value} - \text{Change}$$

Ex: The original price of a shirt is \$50. Calculate the final price after a 20% discount.

Answer:

1. **Calculate the decrease amount:**
$$\begin{aligned}\text{Decrease} &= 20\% \text{ of } \$50 \\ &= 20\% \times \$50 \\ &= 0.20 \times \$50 \\ &= \$10\end{aligned}$$

2. **Calculate the new price:**
$$\text{New Price} = \$50 - \$10 = \$40$$

G PERCENTAGE CHANGE

Discover: **Percentage Change** is a way to express a change in a quantity as a percentage of the original amount. This allows us to understand the scale of the change, whether it's an increase or a decrease.

Definition **Percentage Change**

Percentage change is a signed value that indicates both the direction and magnitude of a change.

- If a quantity **increases**, the percentage change is **positive**. An increase of 15% means a percentage change of +15%.
- If a quantity **decreases**, the percentage change is **negative**. A decrease of 15% means a percentage change of −15%.

Method **Calculating New Value with a Multiplier**

A fast way to find the new value after a percentage change is to use a multiplier.

$$\text{New Value} = \text{Original Value} \times (1 + \text{Percentage Change})$$

The term $(1 + \text{Percentage Change})$ is the **multiplier**. Remember to express the percentage change as a decimal in this formula.

Ex: Find the new amount for increasing \$200 by 10%.

Answer: The percentage change is $+10\% = +0.10$.

$$\begin{aligned}\text{New amount} &= \$200 \times (1 + 0.10) \\ &= \$200 \times 1.10 \\ &= \$220\end{aligned}$$

Ex: Find the new amount for decreasing \$200 by 25%.

Answer: The percentage change is $-25\% = -0.25$.

$$\begin{aligned}\text{New amount} &= \$200 \times (1 - 0.25) \\ &= \$200 \times 0.75 \\ &= \$150\end{aligned}$$

H CALCULATING THE PERCENTAGE CHANGE

Method **Formula for Percentage Change**

To find the percentage change when you know the original and new values, use this formula:

$$\text{Percentage Change} = \frac{\text{Change in Value}}{\text{Original Value}} \times 100\% = \frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} \times 100\%$$

Proof

We start from the multiplier formula:

$$\begin{aligned}
\text{New Value} &= \text{Original Value} \times (1 + \text{Percentage Change}) \\
\frac{\text{New Value}}{\text{Original Value}} &= 1 + \text{Percentage Change} && \text{(Divide by Original Value)} \\
\frac{\text{New Value}}{\text{Original Value}} - 1 &= \text{Percentage Change} && \text{(Subtract 1)} \\
\frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} &= \text{Percentage Change} && \text{(Combine into a single fraction)}
\end{aligned}$$

In this final step, the percentage change is in decimal form. We multiply by 100% to express it as a percentage.

Ex: Find the percentage change when a weight increases from 25 kg to 28 kg.

Answer: The weight increases, so we expect a positive result.

$$\begin{aligned}
\text{Percentage Change} &= \frac{28 - 25}{25} \times 100\% \\
&= \frac{3}{25} \times 100\% \\
&= +12\%
\end{aligned}$$

This is a 12% increase.

Ex: Find the percentage change when a price drops from \$500 to \$420.

Answer: The price decreases, so we expect a negative result.

$$\begin{aligned}
\text{Percentage Change} &= \frac{420 - 500}{500} \times 100\% \\
&= \frac{-80}{500} \times 100\% \\
&= -16\%
\end{aligned}$$

This is a 16% decrease.